Student Welcome Letter

Dear Student:

Have you heard others say, “You’re taking statistics? That’s the hardest course I ever took!” They say that, because they probably spent the entire course confused and struggling. They were probably lectured to and never had the chance to experience the subject. You will not have that problem. Let’s find out why.

There is a Chinese Proverb that describes our feelings about the field of statistics:

I HEAR, AND I FORGET

I SEE, AND I REMEMBER

I DO, AND I UNDERSTAND

Statistics is a “do” field. In order to learn it, you must “do” it. We have structured this book so that you will have hands-on experiences. They will enable you to truly understand the concepts instead of merely going through the requirements for the course.

What makes this book different from other texts? First, we have eliminated the drudgery of tedious calculations. You might be using computers or graphing calculators so that you do not need to struggle with algebraic manipulations. Second, this course is taught as a collaborative activity. With others in your class, you will work toward the common goal of learning this material.

Here are some hints for success in your class:

- Work hard and work every night.
- Form a study group and learn together.
- Don’t get discouraged - you can do it!
- As you solve problems, ask yourself, “Does this answer make sense?”
- Many statistics words have the same meaning as in everyday English.

- Go to your teacher for help as soon as you need it.
- Don’t get behind.
- Read the newspaper and ask yourself, “Does this article make sense?”
- Draw pictures - they truly help!

Good luck and don’t give up!

Sincerely,
Susan Dean and Barbara Illowsky

De Anza College
21250 Stevens Creek Blvd.
Cupertino, California 95014

Introduction
This module provides a brief introduction to the field of statistics, including examples of how these topics shows up in a variety of real-life examples.

Student Learning Objectives

By the end of this chapter, the student should be able to:

- Recognize and differentiate between key terms.
- Apply various types of sampling methods to data collection.

Introduction

You are probably asking yourself the question, "When and where will I use statistics?". If you read any newspaper or watch television, or use the Internet, you will see statistical information. There are statistics about crime, sports, education, politics, and real estate. Typically, when you read a newspaper article or watch a news program on television, you are given sample information. With this information, you may make a decision about the correctness of a statement, claim, or "fact." Statistical methods can help you make the "best educated guess."

Since you will undoubtedly be given statistical information at some point in your life, you need to know some techniques to analyze the information thoughtfully. Think about buying a house or managing a budget. Think about your chosen profession. The fields of economics, business, psychology, education, biology, law, computer science, police science, and early childhood development require at least one course in statistics.

Included in this chapter are the basic ideas and words of probability and statistics. You will soon understand that statistics and probability work together. You will also learn how data are gathered and what "good" data are.

Statistics
This module introduces the concept of statistics, specifically the ability to use statistics to describe data (descriptive statistics) as well as draw conclusions (inferential statistics). An optional classroom exercise is included.

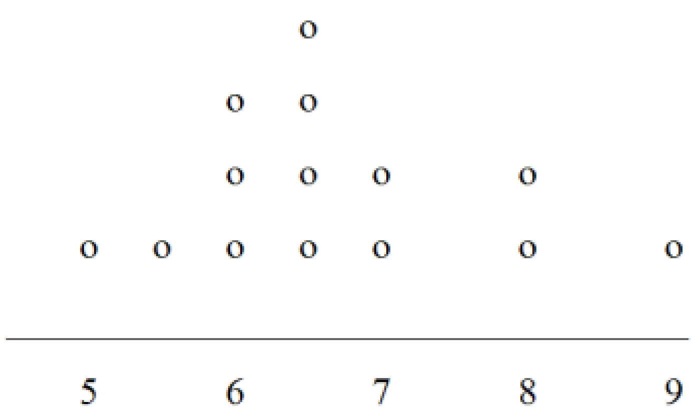
The science of **statistics** deals with the collection, analysis, interpretation, and presentation of **data**. We see and use data in our everyday lives.

Optional Collaborative Classroom Exercise

In your classroom, try this exercise. Have class members write down the average time (in hours, to the nearest half-hour) they sleep per night. Your instructor will record the data. Then create a simple graph (called a **dot plot**) of the data. A dot plot consists of a number line and dots (or points) positioned above the number line. For example, consider the following data:

5 5.5 6 6 6 6.5 6.5 6.5 6.5 7 7 8 8 9

The dot plot for this data would be as follows:
Frequency of Average Time (in Hours) Spent Sleeping per Night



Does your dot plot look the same as or different from the example? Why? If you did the same example in an English class with the same number of students, do you think the results would be the same? Why or why not?

Where do your data appear to cluster? How could you interpret the clustering?

The questions above ask you to analyze and interpret your data. With this example, you have begun your study of statistics.

In this course, you will learn how to organize and summarize data. Organizing and summarizing data is called **descriptive statistics**. Two ways to summarize data are by graphing and by numbers (for example, finding an average). After you have studied probability and probability distributions, you will use formal methods for drawing conclusions from "good" data. The formal methods are called **inferential statistics**. Statistical inference uses probability to determine how confident we can be that the conclusions are correct.

Effective interpretation of data (inference) is based on good procedures for producing data and thoughtful examination of the data. You will encounter what will seem to be too many mathematical formulas for interpreting data. The goal of statistics is not to perform numerous calculations using the formulas, but to gain an understanding of your data. The calculations can be done using a calculator or a computer. The understanding must come from you. If you can thoroughly grasp the basics of statistics, you can be more confident in the decisions you make in life.

Levels of Measurement and Statistical Operations

The way a set of data is measured is called its level of measurement. Correct statistical procedures depend on a researcher being familiar with levels of measurement. Not every statistical operation can be used with every set of data. Data can be classified into four levels of measurement. They are (from lowest to highest level):

- Nominal scale level
- Ordinal scale level
- Interval scale level
- Ratio scale level

Data that is measured using a **nominal scale** is qualitative. Categories, colors, names, labels and favorite foods along with yes or no responses are examples of nominal level data. Nominal scale data are not ordered. For example, trying to classify people according to their favorite food does not make any sense. Putting pizza first and sushi second is not meaningful.

Smartphone companies are another example of nominal scale data. Some examples are Sony, Motorola, Nokia, Samsung and Apple. This is just a list and there is no agreed upon order. Some people may favor Apple but that is a matter of opinion. Nominal scale data cannot be used in calculations.

Data that is measured using an **ordinal scale** is similar to nominal scale data but there is a big difference. The ordinal scale data can be ordered. An example of ordinal scale data is a list of the top five national parks in the United States. The top five national parks in the United States can be ranked from one to five but we cannot measure differences between the data.

Another example using the ordinal scale is a cruise survey where the responses to questions about the cruise are “excellent,” “good,” “satisfactory” and “unsatisfactory.” These responses are ordered from the most desired response by the cruise lines to the least desired. But the differences between two pieces of data cannot be measured. Like the nominal scale data, ordinal scale data cannot be used in calculations.

Data that is measured using the **interval scale** is similar to ordinal level data because it has a definite ordering but there is a difference between data. The differences between interval scale data can be measured though the data does not have a starting point.

Temperature scales like Celsius (C) and Fahrenheit (F) are measured by using the interval scale. In both temperature measurements, 40 degrees is equal to 100 degrees minus 60 degrees. Differences make sense. But 0 degrees does not because, in both scales, 0 is not the absolute lowest temperature. Temperatures like -10° F and -15° C exist and are colder than 0.

Interval level data can be used in calculations but one type of comparison cannot be done. Eighty degrees C is not 4 times as hot as 20° C (nor is 80° F 4 times as hot as 20° F). There is no meaning to the ratio of 80 to 20 (or 4 to 1).

Data that is measured using the **ratio scale** takes care of the ratio problem and gives you the most information. Ratio scale data is like interval scale data but, in addition, it has a 0 point and ratios can be calculated. For example, four multiple choice statistics final exam scores are 80, 68, 20 and 92 (out of a possible 100 points). The exams were machine-graded.

The data can be put in order from lowest to highest: 20, 68, 80, 92.

The differences between the data have meaning. The score 92 is more than the score 68 by 24 points.

Ratios can be calculated. The smallest score for ratio data is 0. So 80 is 4 times 20. The score of 80 is 4 times better than the score of 20.

Exercises

What type of measure scale is being used? Nominal, Ordinal, Interval or Ratio.

1. High school men soccer players classified by their athletic ability: Superior, Average, Above average.
2. Baking temperatures for various main dishes: 350, 400, 325, 250, 300
3. The colors of crayons in a 24-crayon box.
4. Social security numbers.
5. Incomes measured in dollars
6. A satisfaction survey of a social website by number: 1 = very satisfied, 2 = somewhat satisfied, 3 = not satisfied.
7. Political outlook: extreme left, left-of-center, right-of-center, extreme right.
8. Time of day on an analog watch.
9. The distance in miles to the closest grocery store.
10. The dates 1066, 1492, 1644, 1947, 1944.

11. The heights of 21 – 65 year-old women.
12. Common letter grades A, B, C, D, F.

Answers 1. ordinal, 2. interval, 3. nominal, 4. nominal, 5. ratio, 6. ordinal, 7. nominal, 8. interval, 9. ratio, 10. interval, 11. ratio, 12. ordinal

Glossary

Data
: A set of observations (a set of possible outcomes). Most data can be put into two groups: **qualitative** (hair color, ethnic groups and other **attributes** of the population) and **quantitative** (distance traveled to college, number of children in a family, etc.). Quantitative data can be separated into two subgroups: **discrete** and **continuous**. Data is discrete if it is the result of counting (the number of students of a given ethnic group in a class, the number of books on a shelf, etc.). Data is continuous if it is the result of measuring (distance traveled, weight of luggage, etc.)

Statistic
: A numerical characteristic of the sample. A statistic estimates the corresponding population parameter. For example, the average number of full-time students in a 7:30 a.m. class for this term (statistic) is an estimate for the average number of full-time students in any class this term (parameter).

Probability
This module introduces the concept of probability as a mathematical measure of randomness, including a number of real-world applications.

Probability is a mathematical tool used to study randomness. It deals with the chance (the likelihood) of an event occurring. For example, if you toss a **fair** coin 4 times, the outcomes may not be 2 heads and 2 tails. However, if you toss the same coin 4,000 times, the outcomes will be close to half heads and half tails. The expected theoretical probability of heads in any one toss is $\frac{1}{2}$ or 0.5. Even though the outcomes of a few repetitions are uncertain, there is a regular pattern of outcomes when there are many repetitions. After reading about the English statistician Karl Pearson who tossed a coin 24,000 times with a result of 12,012 heads, one of the authors tossed a coin 2,000 times. The results were 996 heads. The fraction $\frac{996}{2000}$ is equal to 0.498 which is very close to 0.5, the expected probability.

The theory of probability began with the study of games of chance such as poker. Predictions take the form of probabilities. To predict the likelihood of an earthquake, of rain, or whether you will get an A in this course, we use probabilities. Doctors use probability to determine the chance of a vaccination causing the disease the vaccination is supposed to prevent. A stockbroker uses probability to determine the rate of return on a client's investments. You might use probability to decide to buy a lottery ticket or not. In your study of statistics, you will use the power of mathematics through probability calculations to analyze and interpret your data.

Glossary

Probability
: A number between 0 and 1, inclusive, that gives the likelihood that a specific event will occur. The foundation of statistics is given by the following 3 axioms (by A. N. Kolmogorov, 1930's): Let S denote the sample space and A and B are two events in S . Then:

- $0 \leq P(A) \leq 1;$.
- If A and B are any two mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.

- $P(S) = 1$.

Key Terms
This module introduces a number of key terms related to statistical sampling and data.

In statistics, we generally want to study a **population**. You can think of a population as an entire collection of persons, things, or objects under study. To study the larger population, we select a **sample**. The idea of **sampling** is to select a portion (or subset) of the larger population and study that portion (the sample) to gain information about the population. Data are the result of sampling from a population.

Because it takes a lot of time and money to examine an entire population, sampling is a very practical technique. If you wished to compute the overall grade point average at your school, it would make sense to select a sample of students who attend the school. The data collected from the sample would be the students' grade point averages. In presidential elections, opinion poll samples of 1,000 to 2,000 people are taken. The opinion poll is supposed to represent the views of the people in the entire country. Manufacturers of canned carbonated drinks take samples to determine if a 16 ounce can contains 16 ounces of carbonated drink.

From the sample data, we can calculate a statistic. A **statistic** is a number that is a property of the sample. For example, if we consider one math class to be a sample of the population of all math classes, then the average number of points earned by students in that one math class at the end of the term is an example of a statistic. The statistic is an estimate of a population parameter. A **parameter** is a number that is a property of the population. Since we considered all math classes to be the population, then the average number of points earned per student over all the math classes is an example of a parameter.

One of the main concerns in the field of statistics is how accurately a statistic estimates a parameter. The accuracy really depends on how well the sample represents the population. The sample must contain the characteristics of the population in order to be a **representative sample**. We are interested in both the sample statistic and the population parameter in inferential statistics. In a later chapter, we will use the sample statistic to test the validity of the established population parameter.

A **variable**, notated by capital letters like X and Y, is a characteristic of interest for each person or thing in a population. Variables may be **numerical** or **categorical**. **Numerical variables** take on values with equal units such as weight in pounds and time in hours. **Categorical variables** place the person or thing into a category. If we let X equal the number of points earned by one math student at the end of a term, then X is a numerical variable. If we let Y be a person's party affiliation, then examples of Y include Republican, Democrat, and Independent. Y is a categorical variable. We could do some math with values of X (calculate the average number of points earned, for example), but it makes no sense to do math with values of Y (calculating an average party affiliation makes no sense).

Data are the actual values of the variable. They may be numbers or they may be words. Datum is a single value.

Two words that come up often in statistics are **mean** and **proportion**. If you were to take three exams in your math classes and obtained scores of 86, 75, and 92, you calculate your mean score by adding the three exam scores and dividing by three (your mean score would be 84.3 to one decimal place). If, in your math class, there are 40 students and 22 are men and 18 are women, then the proportion of men students is $\frac{22}{40}$ and the proportion of women students is $\frac{18}{40}$. Mean and proportion are discussed in more detail in later chapters.

Note:
Mean and Average
The words "mean" and "average" are often used interchangeably. The substitution of one word for the other is common practice. The technical term is "arithmetic mean" and "average" is technically a center location. However, in practice among non-statisticians, "average" is commonly accepted for "arithmetic mean."

Example:

Exercise:

Problem:

Define the key terms from the following study: We want to know the average (mean) amount of money first year college students spend at ABC College on school supplies that do not include books. We randomly survey 100 first year students at the college. Three of those students spent $150, $200, and $225, respectively.

Solution:

The **population** is all first year students attending ABC College this term.

The **sample** could be all students enrolled in one section of a beginning statistics course at ABC College (although this sample may not represent the entire population).

The **parameter** is the average (mean) amount of money spent (excluding books) by first year college students at ABC College this term.

The **statistic** is the average (mean) amount of money spent (excluding books) by first year college students in the sample.

The **variable** could be the amount of money spent (excluding books) by one first year student. Let X = the amount of money spent (excluding books) by one first year student attending ABC College.

The **data** are the dollar amounts spent by the first year students. Examples of the data are $150, $200, and $225.

Optional Collaborative Classroom Exercise

Do the following exercise collaboratively with up to four people per group. Find a population, a sample, the parameter, the statistic, a variable, and data for the following study: You want to determine the average (mean) number of glasses of milk college students drink per day. Suppose yesterday, in your English class, you asked five students how many glasses of milk they drank the day before. The answers were 1, 0, 1, 3, and 4 glasses of milk.

Glossary

Average
: A number that describes the central tendency of the data. There are a number of specialized averages, including the arithmetic mean, weighted mean, median, mode, and geometric mean.

Data
: A set of observations (a set of possible outcomes). Most data can be put into two groups: **qualitative** (hair color, ethnic groups and other **attributes** of the population) and **quantitative** (distance traveled to college, number of children in a family, etc.). Quantitative data can be separated into two subgroups: **discrete** and **continuous**. Data is discrete if it is the result of counting (the number of students of a given ethnic group in a class, the number of books on a shelf, etc.). Data is continuous if it is the result of measuring (distance traveled, weight of luggage, etc.)

Proportion

- As a number: A proportion is the number of successes divided by the total number in the sample.
- As a probability distribution: Given a binomial random variable (RV), $X \sim B(n, p)$, consider the ratio of the number X of successes in n Bernouli trials to the number n of trials. $P\prime = \frac{X}{n}$. This new RV is called a proportion, and if the number of trials, n, is large enough, P' $\sim N\left(p, \frac{\text{pq}}{n}\right)$.

Data
This module introduces the concepts of qualitative data, quantitative continuous data, and quantitative discrete data as used in statistics. Sample problems are included.

Data may come from a population or from a sample. Small letters like x or y generally are used to represent data values. Most data can be put into the following categories:

- Qualitative
- Quantitative

Qualitative data are the result of categorizing or describing attributes of a population. Hair color, blood type, ethnic group, the car a person drives, and the street a person lives on are examples of qualitative data. Qualitative data are generally described by words or letters. For instance, hair color might be black, dark brown, light brown, blonde, gray, or red. Blood type might be AB+, O-, or B+. Researchers often prefer to use quantitative data over qualitative data because it lends itself more easily to mathematical analysis. For example, it does not make sense to find an average hair color or blood type.

Quantitative data are always numbers. Quantitative data are the result of **counting** or **measuring** attributes of a population. Amount of money, pulse rate, weight, number of people living in your town, and the number of students who take statistics are examples of quantitative data. Quantitative data may be either **discrete** or **continuous**.

All data that are the result of counting are called **quantitative discrete data**. These data take on only certain numerical values. If you count the number of phone calls you receive for each day of the week, you might get 0, 1, 2, 3, etc.

All data that are the result of measuring are **quantitative continuous data** assuming that we can measure accurately. Measuring angles in radians might result in the numbers $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, π, $\frac{3\pi}{4}$, etc. If you and your friends carry backpacks with books in them to school, the numbers of books in the

backpacks are discrete data and the weights of the backpacks are continuous data.

Note: In this course, the data used is mainly quantitative. It is easy to calculate statistics (like the mean or proportion) from numbers. In the chapter **Descriptive Statistics**, you will be introduced to stem plots, histograms and box plots all of which display quantitative data. Qualitative data is discussed at the end of this section through graphs.

Example:
Data Sample of Quantitative Discrete Data
The data are the number of books students carry in their backpacks. You sample five students. Two students carry 3 books, one student carries 4 books, one student carries 2 books, and one student carries 1 book. The numbers of books (3, 4, 2, and 1) are the quantitative discrete data.

Example:
Data Sample of Quantitative Continuous Data
The data are the weights of the backpacks with the books in it. You sample the same five students. The weights (in pounds) of their backpacks are 6.2, 7, 6.8, 9.1, 4.3. Notice that backpacks carrying three books can have different weights. Weights are quantitative continuous data because weights are measured.

Example:
Data Sample of Qualitative Data
The data are the colors of backpacks. Again, you sample the same five students. One student has a red backpack, two students have black backpacks, one student has a green backpack, and one student has a gray backpack. The colors red, black, black, green, and gray are qualitative data.

Note: You may collect data as numbers and report it categorically. For example, the quiz scores for each student are recorded throughout the term. At the end of the term, the quiz scores are reported as A, B, C, D, or F.

Example:
Exercise:

Problem:

Work collaboratively to determine the correct data type (quantitative or qualitative). Indicate whether quantitative data are continuous or discrete. Hint: Data that are discrete often start with the words "the number of."

1. The number of pairs of shoes you own.
2. The type of car you drive.
3. Where you go on vacation.
4. The distance it is from your home to the nearest grocery store.
5. The number of classes you take per school year.
6. The tuition for your classes
7. The type of calculator you use.
8. Movie ratings.
9. Political party preferences.
10. Weight of sumo wrestlers.
11. Amount of money won playing poker.
12. Number of correct answers on a quiz.
13. Peoples' attitudes toward the government.
14. IQ scores. (This may cause some discussion.)

Solution:

Items 1, 5, 11, and 12 are quantitative discrete; items 4, 6, 10, and 14 are quantitative continuous; and items 2, 3, 7, 8, 9, and 13 are qualitative.

Qualitative Data Discussion
Below are tables of part-time vs full-time students at De Anza College in Cupertino, CA and Foothill College in Los Altos, CA for the Spring 2010 quarter. The tables display counts (frequencies) and percentages or proportions (relative frequencies). The percent columns make comparing the same categories in the colleges easier. Displaying percentages along with the numbers is often helpful, but it is particularly important when comparing sets of data that do not have the same totals, such as the total enrollments for both colleges in this example. Notice how much larger the percentage for part-time students at Foothill College is compared to De Anza College.

	Number	Percent
Full-time	9,200	40.9%
Part-time	13,296	59.1%
Total	22,496	100%

De Anza College

	Number	Percent
Full-time	4,059	28.6%
Part-time	10,124	71.4%

Total	14,183	100%

Foothill College

Tables are a good way of organizing and displaying data. But graphs can be even more helpful in understanding the data. There are no strict rules concerning what graphs to use. Below are pie charts and bar graphs, two graphs that are used to display qualitative data.

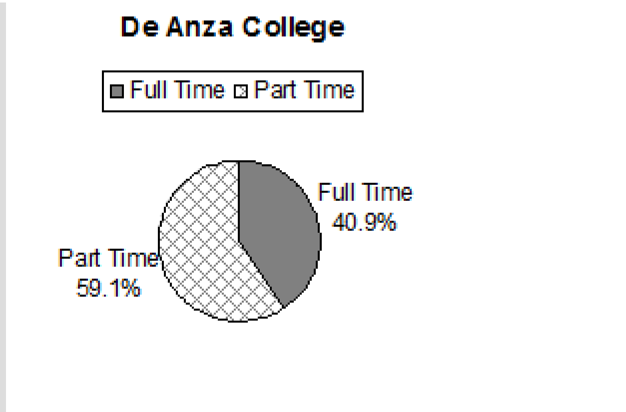
In a **pie chart**, categories of data are represented by wedges in the circle and are proportional in size to the percent of individuals in each category.

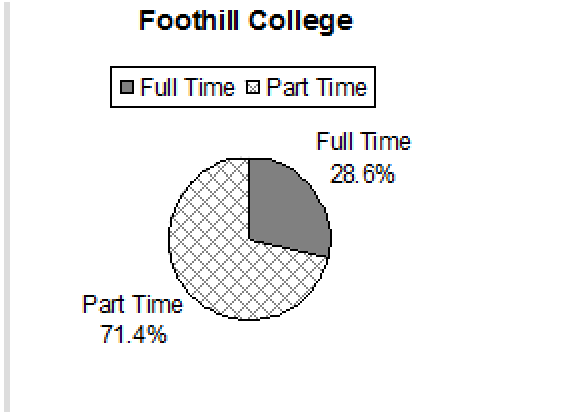
In a **bar graph**, the length of the bar for each category is proportional to the number or percent of individuals in each category. Bars may be vertical or horizontal.

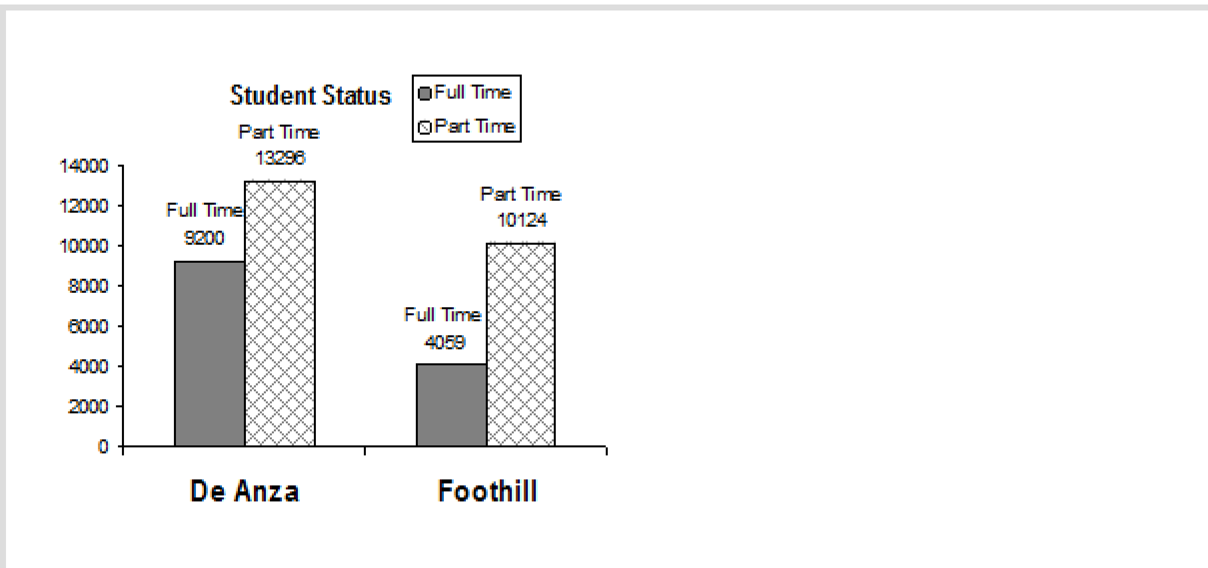
A **Pareto chart** consists of bars that are sorted into order by category size (largest to smallest).

Look at the graphs and determine which graph (pie or bar) you think displays the comparisons better. This is a matter of preference.

It is a good idea to look at a variety of graphs to see which is the most helpful in displaying the data. We might make different choices of what we think is the "best" graph depending on the data and the context. Our choice also depends on what we are using the data for.



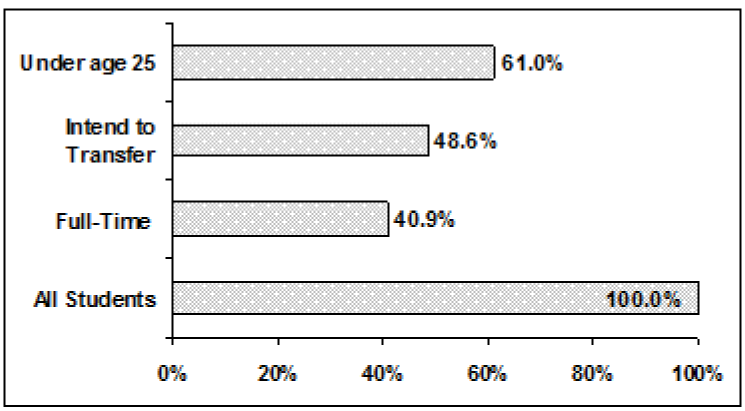




Percentages That Add to More (or Less) Than 100%
Sometimes percentages add up to be more than 100% (or less than 100%). In the graph, the percentages add to more than 100% because students can be in more than one category. A bar graph is appropriate to compare the relative size of the categories. A pie chart cannot be used. It also could not be used if the percentages added to less than 100%.

Characteristic/Category	Percent
Full-time Students	40.9%
Students who intend to transfer to a 4-year educational institution	48.6%
Students under age 25	61.0%
TOTAL	150.5%

De Anza College Spring 2010

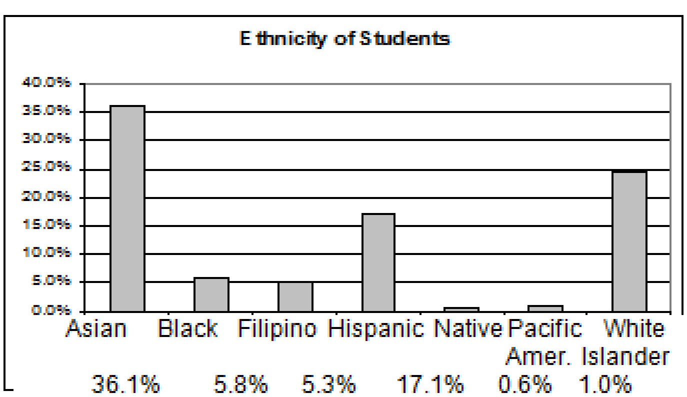


Omitting Categories/Missing Data

The table displays Ethnicity of Students but is missing the "Other/Unknown" category. This category contains people who did not feel they fit into any of the ethnicity categories or declined to respond. Notice that the frequencies do not add up to the total number of students. Create a bar graph and not a pie chart.

	Frequency	Percent
Asian	8,794	36.1%
Black	1,412	5.8%
Filipino	1,298	5.3%
Hispanic	4,180	17.1%
Native American	146	0.6%
Pacific Islander	236	1.0%
White	5,978	24.5%
TOTAL	22,044 out of 24,382	90.4% out of 100%

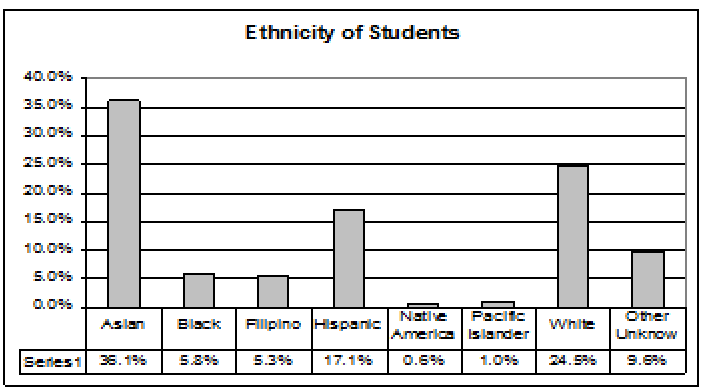
Missing Data: Ethnicity of Students De Anza College Fall Term 2007 (Census Day)



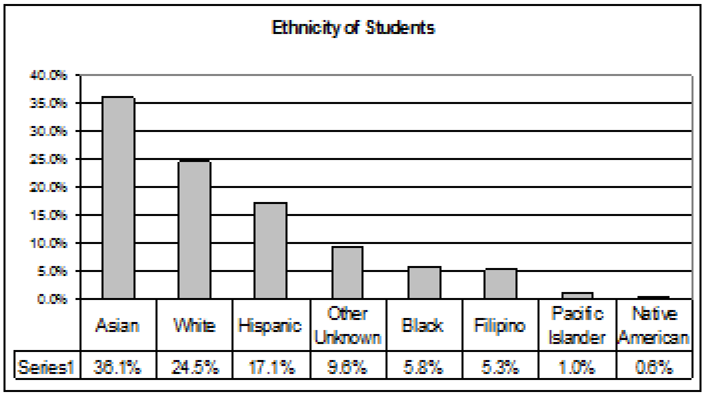
Bar graph Without Other/Unknown Category

The following graph is the same as the previous graph but the "Other/Unknown" percent (9.6%) has been added back in. The "Other/Unknown" category is large compared to some of the other categories (Native American, 0.6%, Pacific Islander 1.0% particularly). This is important to know when we think about what the data are telling us.

This particular bar graph can be hard to understand visually. The graph below it is a Pareto chart. The Pareto chart has the bars sorted from largest to smallest and is easier to read and interpret.



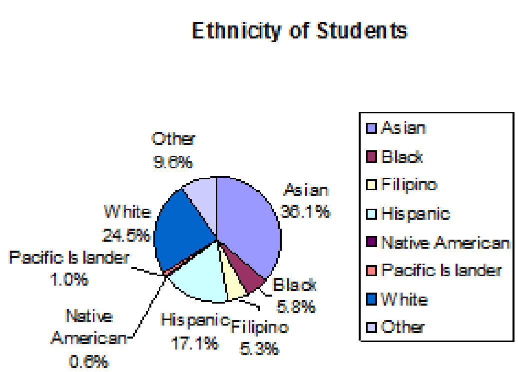
Bar Graph With Other/Unknown Category

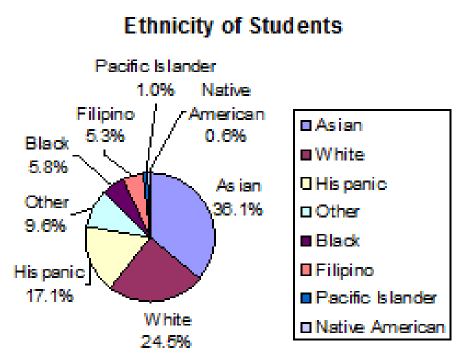


Pareto Chart With Bars Sorted By Size

Pie Charts: No Missing Data

The following pie charts have the "Other/Unknown" category added back in (since the percentages must add to 100%). The chart on the right is organized having the wedges by size and makes for a more visually informative graph than the unsorted, alphabetical graph on the left.





Glossary

Continuous Random Variable

A random variable (RV) whose outcomes are measured.

Example:
The height of trees in the forest is a continuous RV.

Data

A set of observations (a set of possible outcomes). Most data can be put into two groups: **qualitative** (hair color, ethnic groups and other **attributes** of the population) and **quantitative** (distance traveled to college, number of children in a family, etc.). Quantitative data can be separated into two subgroups: **discrete** and **continuous**. Data is discrete if it is the result of counting (the number of students of a given ethnic group in a class, the number of books on a shelf, etc.). Data is continuous if it is the result of measuring (distance traveled, weight of luggage, etc.)

Discrete Random Variable

A random variable (RV) whose outcomes are counted.

Qualitative Data

See **Data**.

Quantitative Data

See **Data**.

Sampling
This module introduces the concept of statistical sampling. Students are taught the difference between a simple random sample, stratified sample, cluster sample, systematic sample, and convenience sample. Example problems are provided, including an optional classroom activity.

Gathering information about an entire population often costs too much or is virtually impossible. Instead, we use a sample of the population. **A sample should have the same characteristics as the population it is representing.** Most statisticians use various methods of random sampling in an attempt to achieve this goal. This section will describe a few of the most common methods.

There are several different methods of **random sampling**. In each form of random sampling, each member of a population initially has an equal chance of being selected for the sample. Each method has pros and cons. The easiest method to describe is called a **simple random sample**. Any group of n individuals is equally likely to be chosen by any other group of n individuals if the simple random sampling technique is used. In other words, each sample of the same size has an equal chance of being selected. For example, suppose Lisa wants to form a four-person study group (herself and three other people) from her pre-calculus class, which has 31 members not including Lisa. To choose a simple random sample of size 3 from the other members of her class, Lisa could put all 31 names in a hat, shake the hat, close her eyes, and pick out 3 names. A more technological way is for Lisa to first list the last names of the members of her class together with a two-digit number as shown below.

ID	Name
00	Anselmo

ID	Name
01	Bautista
02	Bayani
03	Cheng
04	Cuarismo
05	Cuningham
06	Fontecha
07	Hong
08	Hoobler
09	Jiao
10	Khan
11	King
12	Legeny
13	Lundquist
14	Macierz
15	Motogawa
16	Okimoto
17	Patel

ID	Name
18	Price
19	Quizon
20	Reyes
21	Roquero
22	Roth
23	Rowell
24	Salangsang
25	Slade
26	Stracher
27	Tallai
28	Tran
29	Wai
30	Wood

Class Roster

Lisa can either use a table of random numbers (found in many statistics books as well as mathematical handbooks) or a calculator or computer to generate random numbers. For this example, suppose Lisa chooses to generate random numbers from a calculator. The numbers generated are:

.94360 .99832 .14669 .51470 .40581 .73381 .04399

Lisa reads two-digit groups until she has chosen three class members (that is, she reads .94360 as the groups 94, 43, 36, 60). Each random number may only contribute one class member. If she needed to, Lisa could have generated more random numbers.

The random numbers .94360 and .99832 do not contain appropriate two digit numbers. However the third random number, .14669, contains 14 (the fourth random number also contains 14), the fifth random number contains 05, and the seventh random number contains 04. The two-digit number 14 corresponds to Macierz, 05 corresponds to Cunningham, and 04 corresponds to Cuarismo. Besides herself, Lisa's group will consist of Marcierz, and Cunningham, and Cuarismo.

Besides simple random sampling, there are other forms of sampling that involve a chance process for getting the sample. **Other well-known random sampling methods are the stratified sample, the cluster sample, and the systematic sample.**

To choose a **stratified sample**, divide the population into groups called strata and then take a **proportionate** number from each stratum. For example, you could stratify (group) your college population by department and then choose a proportionate simple random sample from each stratum (each department) to get a stratified random sample. To choose a simple random sample from each department, number each member of the first department, number each member of the second department and do the same for the remaining departments. Then use simple random sampling to choose proportionate numbers from the first department and do the same for each of the remaining departments. Those numbers picked from the first department, picked from the second department and so on represent the members who make up the stratified sample.

To choose a **cluster sample**, divide the population into clusters (groups) and then randomly select some of the clusters. All the members from these clusters are in the cluster sample. For example, if you randomly sample four departments from your college population, the four departments make up the cluster sample. For example, divide your college faculty by department. The departments are the clusters. Number each department and then choose

four different numbers using simple random sampling. All members of the four departments with those numbers are the cluster sample.

To choose a **systematic sample**, randomly select a starting point and take every nth piece of data from a listing of the population. For example, suppose you have to do a phone survey. Your phone book contains 20,000 residence listings. You must choose 400 names for the sample. Number the population 1 - 20,000 and then use a simple random sample to pick a number that represents the first name of the sample. Then choose every 50th name thereafter until you have a total of 400 names (you might have to go back to the of your phone list). Systematic sampling is frequently chosen because it is a simple method.

A type of sampling that is nonrandom is convenience sampling. **Convenience sampling** involves using results that are readily available. For example, a computer software store conducts a marketing study by interviewing potential customers who happen to be in the store browsing through the available software. The results of convenience sampling may be very good in some cases and highly biased (favors certain outcomes) in others.

Sampling data should be done very carefully. Collecting data carelessly can have devastating results. Surveys mailed to households and then returned may be very biased (for example, they may favor a certain group). It is better for the person conducting the survey to select the sample respondents.

True random sampling is done **with replacement**. That is, once a member is picked that member goes back into the population and thus may be chosen more than once. However for practical reasons, in most populations, simple random sampling is done **without replacement**. Surveys are typically done without replacement. That is, a member of the population may be chosen only once. Most samples are taken from large populations and the sample tends to be small in comparison to the population. Since this is the case, sampling without replacement is approximately the same as sampling with replacement because the chance of picking the same individual more than once using with replacement is very low.

For example, in a college population of 10,000 people, suppose you want to randomly pick a sample of 1000 for a survey. **For any particular sample of 1000**, if you are sampling **with replacement**,

- the chance of picking the first person is 1000 out of 10,000 (0.1000);
- the chance of picking a different second person for this sample is 999 out of 10,000 (0.0999);
- the chance of picking the same person again is 1 out of 10,000 (very low).

If you are sampling **without replacement**,

- the chance of picking the first person for any particular sample is 1000 out of 10,000 (0.1000);
- the chance of picking a different second person is 999 out of 9,999 (0.0999);
- you do not replace the first person before picking the next person.

Compare the fractions 999/10,000 and 999/9,999. For accuracy, carry the decimal answers to 4 place decimals. To 4 decimal places, these numbers are equivalent (0.0999).

Sampling without replacement instead of sampling with replacement only becomes a mathematics issue when the population is small which is not that common. For example, if the population is 25 people, the sample is 10 and you are sampling **with replacement for any particular sample**,

- the chance of picking the first person is 10 out of 25 and a different second person is 9 out of 25 (you replace the first person).

If you sample **without replacement**,

- the chance of picking the first person is 10 out of 25 and then the second person (which is different) is 9 out of 24 (you do not replace the first person).

Compare the fractions 9/25 and 9/24. To 4 decimal places, 9/25 = 0.3600 and 9/24 = 0.3750. To 4 decimal places, these numbers are not equivalent.

When you analyze data, it is important to be aware of **sampling errors** and nonsampling errors. The actual process of sampling causes sampling errors. For example, the sample may not be large enough. Factors not related to the sampling process cause **nonsampling errors**. A defective counting device can cause a nonsampling error.

In reality, a sample will never be exactly representative of the population so there will always be some sampling error. As a rule, the larger the sample, the smaller the sampling error.

In statistics, **a sampling bias** is created when a sample is collected from a population and some members of the population are not as likely to be chosen as others (remember, each member of the population should have an equally likely chance of being chosen). When a sampling bias happens, there can be incorrect conclusions drawn about the population that is being studied.

Example:
Exercise:

Problem:

Determine the type of sampling used (simple random, stratified, systematic, cluster, or convenience).

1. A soccer coach selects 6 players from a group of boys aged 8 to 10, 7 players from a group of boys aged 11 to 12, and 3 players from a group of boys aged 13 to 14 to form a recreational soccer team.
2. A pollster interviews all human resource personnel in five different high tech companies.
3. A high school educational researcher interviews 50 high school female teachers and 50 high school male teachers.
4. A medical researcher interviews every third cancer patient from a list of cancer patients at a local hospital.

5. A high school counselor uses a computer to generate 50 random numbers and then picks students whose names correspond to the numbers.
6. A student interviews classmates in his algebra class to determine how many pairs of jeans a student owns, on the average.

Solution:

1. stratified
2. cluster
3. stratified
4. systematic
5. simple random
6. convenience

If we were to examine two samples representing the same population, even if we used random sampling methods for the samples, they would not be exactly the same. Just as there is variation in data, there is variation in samples. As you become accustomed to sampling, the variability will seem natural.

Example:
Suppose ABC College has 10,000 part-time students (the population). We are interested in the average amount of money a part-time student spends on books in the fall term. Asking all 10,000 students is an almost impossible task.
Suppose we take two different samples.
First, we use convenience sampling and survey 10 students from a first term organic chemistry class. Many of these students are taking first term calculus in addition to the organic chemistry class . The amount of money they spend is as follows:
$128 $87 $173 $116 $130 $204 $147 $189 $93 $153

The second sample is taken by using a list from the P.E. department of senior citizens who take P.E. classes and taking every 5th senior citizen on the list, for a total of 10 senior citizens. They spend:
$50 $40 $36 $15 $50 $100 $40 $53 $22 $22

Exercise:

Problem:

Do you think that either of these samples is representative of (or is characteristic of) the entire 10,000 part-time student population?

Solution:

No. The first sample probably consists of science-oriented students. Besides the chemistry course, some of them are taking first-term calculus. Books for these classes tend to be expensive. Most of these students are, more than likely, paying more than the average part-time student for their books. The second sample is a group of senior citizens who are, more than likely, taking courses for health and interest. The amount of money they spend on books is probably much less than the average part-time student. Both samples are biased. Also, in both cases, not all students have a chance to be in either sample.

Exercise:

Problem:

Since these samples are not representative of the entire population, is it wise to use the results to describe the entire population?

Solution:

No. For these samples, each member of the population did not have an equally likely chance of being chosen.

Now, suppose we take a third sample. We choose ten different part-time students from the disciplines of chemistry, math, English, psychology, sociology, history, nursing, physical education, art, and early childhood development. (We assume that these are the only disciplines in which part-time students at ABC College are enrolled and that an equal number of

part-time students are enrolled in each of the disciplines.) Each student is chosen using simple random sampling. Using a calculator, random numbers are generated and a student from a particular discipline is selected if he/she has a corresponding number. The students spend:
$180 $50 $150 $85 $260 $75 $180 $200 $200 $150

Exercise:

Problem: Is the sample biased?

Solution:

The sample is unbiased, but a larger sample would be recommended to increase the likelihood that the sample will be close to representative of the population. However, for a biased sampling technique, even a large sample runs the risk of not being representative of the population.

Students often ask if it is "good enough" to take a sample, instead of surveying the entire population. If the survey is done well, the answer is yes.

Optional Collaborative Classroom Exercise

Exercise:

Problem:

As a class, determine whether or not the following samples are representative. If they are not, discuss the reasons.

1. To find the average GPA of all students in a university, use all honor students at the university as the sample.
2. To find out the most popular cereal among young people under the age of 10, stand outside a large supermarket for three hours and speak to every 20th child under age 10 who enters the supermarket.

3. To find the average annual income of all adults in the United States, sample U.S. congressmen. Create a cluster sample by considering each state as a stratum (group). By using simple random sampling, select states to be part of the cluster. Then survey every U.S. congressman in the cluster.
4. To determine the proportion of people taking public transportation to work, survey 20 people in New York City. Conduct the survey by sitting in Central Park on a bench and interviewing every person who sits next to you.
5. To determine the average cost of a two day stay in a hospital in Massachusetts, survey 100 hospitals across the state using simple random sampling.

Variation
This module discusses statistical variability within data and samples. Students will be given the opportunity to see this variability in action through participation in an optional classroom exercise. This module also has a section that discusses Critical Evaluation.

Variation in Data

Variation is present in any set of data. For example, 16-ounce cans of beverage may contain more or less than 16 ounces of liquid. In one study, eight 16 ounce cans were measured and produced the following amount (in ounces) of beverage:

15.8 16.1 15.2 14.8 15.8 15.9 16.0 15.5

Measurements of the amount of beverage in a 16-ounce can may vary because different people make the measurements or because the exact amount, 16 ounces of liquid, was not put into the cans. Manufacturers regularly run tests to determine if the amount of beverage in a 16-ounce can falls within the desired range.

Be aware that as you take data, your data may vary somewhat from the data someone else is taking for the same purpose. This is completely natural. However, if two or more of you are taking the same data and get very different results, it is time for you and the others to reevaluate your data-taking methods and your accuracy.

Variation in Samples

It was mentioned previously that two or more **samples** from the same **population**, taken randomly, and having close to the same characteristics of the population are different from each other. Suppose Doreen and Jung both decide to study the average amount of time students at their college sleep each night. Doreen and Jung each take samples of 500 students. Doreen uses systematic sampling and Jung uses cluster sampling. Doreen's sample will be different from Jung's sample. Even if Doreen and Jung used the

same sampling method, in all likelihood their samples would be different. Neither would be wrong, however.

Think about what contributes to making Doreen's and Jung's samples different.

If Doreen and Jung took larger samples (i.e. the number of data values is increased), their sample results (the average amount of time a student sleeps) might be closer to the actual population average. But still, their samples would be, in all likelihood, different from each other. This **variability in samples** cannot be stressed enough.

Size of a Sample

The size of a sample (often called the number of observations) is important. The examples you have seen in this book so far have been small. Samples of only a few hundred observations, or even smaller, are sufficient for many purposes. In polling, samples that are from 1200 to 1500 observations are considered large enough and good enough if the survey is random and is well done. You will learn why when you study confidence intervals.

Be aware that many large samples are biased. For example, call-in surveys are invariable biased because people choose to respond or not.

Optional Collaborative Classroom Exercise

Exercise:

Problem:

Divide into groups of two, three, or four. Your instructor will give each group one 6-sided die. **Try this experiment twice.** Roll one fair die (6-sided) 20 times. Record the number of ones, twos, threes, fours, fives, and sixes you get below ("frequency" is the number of times a particular face of the die occurs):

Face on Die	**Frequency**
1	
2	
3	
4	
5	
6	

First Experiment (20 rolls)

Face on Die	**Frequency**
1	
2	
3	
4	
5	
6	

Second Experiment (20 rolls)

Did the two experiments have the same results? Probably not. If you did the experiment a third time, do you expect the results to be identical to the first or second experiment? (Answer yes or no.) Why or why not?

Which experiment had the correct results? They both did. The job of the statistician is to see through the variability and draw appropriate conclusions.

Critical Evaluation

We need to critically evaluate the statistical studies we read about and analyze before accepting the results of the study. Common problems to be aware of include

- Problems with Samples: A sample should be representative of the population. A sample that is not representative of the population is biased. Biased samples that are not representative of the population give results that are inaccurate and not valid.
- Self-Selected Samples: Responses only by people who choose to respond, such as call-in surveys are often unreliable.
- Sample Size Issues: Samples that are too small may be unreliable. Larger samples are better if possible. In some situations, small samples are unavoidable and can still be used to draw conclusions, even though larger samples are better. Examples: Crash testing cars, medical testing for rare conditions.
- Undue influence: Collecting data or asking questions in a way that influences the response.
- Non-response or refusal of subject to participate: The collected responses may no longer be representative of the population. Often, people with strong positive or negative opinions may answer surveys, which can affect the results.
- Causality: A relationship between two variables does not mean that one causes the other to occur. They may both be related (correlated) because of their relationship through a different variable.
- Self-Funded or Self-Interest Studies: A study performed by a person or organization in order to support their claim. Is the study impartial?

Read the study carefully to evaluate the work. Do not automatically assume that the study is good but do not automatically assume the study is bad either. Evaluate it on its merits and the work done.

- Misleading Use of Data: Improperly displayed graphs, incomplete data, lack of context.
- Confounding: When the effects of multiple factors on a response cannot be separated. Confounding makes it difficult or impossible to draw valid conclusions about the effect of each factor.

Glossary

Population
: The collection, or set, of all individuals, objects, or measurements whose properties are being studied.

Sample
: A portion of the population understudy. A sample is representative if it characterizes the population being studied.

Answers and Rounding Off

This module briefly explains the correct way to round off answers when working with statistical data.

A simple way to round off answers is to carry your final answer one more decimal place than was present in the original data. Round only the final answer. Do not round any intermediate results, if possible. If it becomes necessary to round intermediate results, carry them to at least twice as many decimal places as the final answer. For example, the average of the three quiz scores 4, 6, 9 is 6.3, rounded to the nearest tenth, because the data are whole numbers. Most answers will be rounded in this manner.

It is not necessary to reduce most fractions in this course. Especially in Probability Topics, the chapter on probability, it is more helpful to leave an answer as an unreduced fraction.

Summary
This module provides an outline/review of key concepts related to statistical sampling and data.

Statistics

- Deals with the collection, analysis, interpretation, and presentation of data

Probability

- Mathematical tool used to study randomness

Key Terms

- Population
- Parameter
- Sample
- Statistic
- Variable
- Data

Types of Data

- Quantitative Data (a number)
    - Discrete (You count it.)
    - Continuous (You measure it.)
- Qualitative Data (a category, words)

Sampling

- **With Replacement**: A member of the population may be chosen more than once
- **Without Replacement**: A member of the population may be chosen only once

Random Sampling

- Each member of the population has an equal chance of being selected

Sampling Methods

- Random
  - Simple random sample
  - Stratified sample
  - Cluster sample
  - Systematic sample
- Not Random
  - Convenience sample

Note: Samples must be representative of the population from which they come. They must have the same characteristics. However, they may vary but still represent the same population.

Practice: Sampling and Data
This module provides an opportunity for students to practice concepts related to statistical sampling and data. Given a sample data set, the student will practice constructing frequency tables, differentiating between key terms, and comparing sampling techniques.

Student Learning Outcomes

- The student will differentiate between key terms.
- The student will compare sampling techniques.

Given

Studies are often done by pharmaceutical companies to determine the effectiveness of a treatment program. Suppose that a new AIDS antibody drug is currently under study. It is given to patients once the AIDS symptoms have revealed themselves. Of interest is the average length of time in months patients live once starting the treatment. Two researchers each follow a different set of 40 AIDS patients from the start of treatment until their deaths. The following data (in months) are collected.

Researcher 13 4 11 15 16 17 22 44 37 16 14 24 25 15 26 27 33 29 35 44 13 21 22 10 12 8 40 32 26 27 31 34 29 17 8 24 18 47 33 34

Researcher 23 14 11 5 16 17 28 41 31 18 14 14 26 25 21 22 31 2 35 44 23 21 21 16 12 18 41 22 16 25 33 34 29 13 18 24 23 42 33 29

Key Terms

Define the key terms based upon the above example for Researcher 1.
Exercise:

Problem: Population

Exercise:

Problem: Sample

Exercise:

Problem: Parameter

Exercise:

Problem: Statistic

Exercise:

Problem: Variable

Exercise:

Problem: Data

Discussion Questions

Discuss the following questions and then answer in complete sentences.

Exercise:

Problem: List two reasons why the data may differ.

Exercise:

Problem:

Can you tell if one researcher is correct and the other one is incorrect? Why?

Exercise:

Problem: Would you expect the data to be identical? Why or why not?

Exercise:

Problem: How could the researchers gather random data?

Exercise:

Problem:

Suppose that the first researcher conducted his survey by randomly choosing one state in the nation and then randomly picking 40 patients from that state. What sampling method would that researcher have used?

Exercise:

Problem:

Suppose that the second researcher conducted his survey by choosing 40 patients he knew. What sampling method would that researcher have used? What concerns would you have about this data set, based upon the data collection method?

Exercises

This module presents students with a number of problems related to statistical sampling and data. In particular, students are asked to demonstrate understanding of concepts such as frequency, relative frequency, and cumulative relative frequency, random samples, quantitative vs. qualitative data, continuous vs. discrete data, and other key terms related to sampling and data.

Exercise:

Problem: For each item below:

- **i**Identify the type of data (quantitative - discrete, quantitative - continuous, or qualitative) that would be used to describe a response.
- **ii**Give an example of the data.

- **a**Number of tickets sold to a concert
- **b**Amount of body fat
- **c**Favorite baseball team
- **d**Time in line to buy groceries
- **e**Number of students enrolled at Evergreen Valley College
- **f**Most–watched television show
- **g**Brand of toothpaste
- **h**Distance to the closest movie theatre
- **i**Age of executives in Fortune 500 companies
- **j**Number of competing computer spreadsheet software packages

---

Solution:

- **a**quantitative - discrete
- **b**quantitative - continuous
- **c**qualitative
- **d**quantitative - continuous
- **e**quantitative - discrete
- **f**qualitative
- **g**qualitative

- **h**quantitative - continuous
- **i**quantitative - continuous
- **j**quantitative - discrete

Exercise:

Problem:

A fitness center is interested in the average amount of time a client exercises in the center each week. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter
- **d**Statistic
- **e**Variable
- **f**Data

Exercise:

Problem:

Ski resorts are interested in the average age that children take their first ski and snowboard lessons. They need this information to optimally plan their ski classes. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter
- **d**Statistic
- **e**Variable
- **f**Data

Solution:

- **a**Children who take ski or snowboard lessons
- **b**A group of these children
- **c**The population average
- **d**The sample average
- **e**X = the age of one child who takes the first ski or snowboard lesson
- **f**A value for X, such as 3, 7, etc.

Exercise:

Problem:

A cardiologist is interested in the average recovery period for her patients who have had heart attacks. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter
- **d**Statistic
- **e**Variable
- **f**Data

Exercise:

Problem:

Insurance companies are interested in the average health costs each year for their clients, so that they can determine the costs of health insurance. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter
- **d**Statistic
- **e**Variable
- **f**Data

Solution:

- **a**The clients of the insurance companies
- **b**A group of the clients
- **c**The average health costs of the clients
- **d**The average health costs of the sample
- **e**X = the health costs of one client
- **f**A value for X, such as 34, 9, 82, etc.

Exercise:

Problem:

A politician is interested in the proportion of voters in his district that think he is doing a good job. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter
- **d**Statistic
- **e**Variable
- **f**Data

Exercise:

Problem:

A marriage counselor is interested in the proportion the clients she counsels that stay married. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter
- **d**Statistic
- **e**Variable

- **f**Data

Solution:

- **a**All the clients of the counselor
- **b**A group of the clients
- **c**The proportion of all her clients who stay married
- **d**The proportion of the sample who stay married
- **e**X = the number of couples who stay married
- **f**yes, no

Exercise:

Problem:

Political pollsters may be interested in the proportion of people that will vote for a particular cause. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter
- **d**Statistic
- **e**Variable
- **f**Data

Exercise:

Problem:

A marketing company is interested in the proportion of people that will buy a particular product. Define the following in terms of the study. Give examples where appropriate.

- **a**Population
- **b**Sample
- **c**Parameter

- **d**Statistic
- **e**Variable
- **f**Data

Solution:

- **a**All people (maybe in a certain geographic area, such as the United States)
- **b**A group of the people
- **c**The proportion of all people who will buy the product
- **d**The proportion of the sample who will buy the product
- **e**X = the number of people who will buy it
- **f**buy, not buy

Exercise:

Problem:

Airline companies are interested in the consistency of the number of babies on each flight, so that they have adequate safety equipment. Suppose an airline conducts a survey. Over Thanksgiving weekend, it surveys 6 flights from Boston to Salt Lake City to determine the number of babies on the flights. It determines the amount of safety equipment needed by the result of that study.

- **a**Using complete sentences, list three things wrong with the way the survey was conducted.
- **b**Using complete sentences, list three ways that you would improve the survey if it were to be repeated.

Exercise:

Problem:

Suppose you want to determine the average number of students per statistics class in your state. Describe a possible sampling method in 3 – 5 complete sentences. Make the description detailed.

Exercise:

Problem:

Suppose you want to determine the average number of cans of soda drunk each month by persons in their twenties. Describe a possible sampling method in 3 - 5 complete sentences. Make the description detailed.

Exercise:

Problem:

A “random survey” was conducted of 3274 people of the “microprocessor generation” (people born since 1971, the year the microprocessor was invented). It was reported that 48% of those individuals surveyed stated that if they had $2000 to spend, they would use it for computer equipment. Also, 66% of those surveyed considered themselves relatively savvy computer users. (*Source: San Jose Mercury News*)

- **a**Do you consider the sample size large enough for a study of this type? Why or why not?
- **b**Based on your “gut feeling,” do you believe the percents accurately reflect the U.S. population for those individuals born since 1971? If not, do you think the percents of the population are actually higher or lower than the sample statistics? Why?

Additional information: The survey was reported by Intel Corporation of individuals who visited the Los Angeles Convention Center to see the Smithsonian Institure's road show called “America’s Smithsonian.”

- **c**With this additional information, do you feel that all demographic and ethnic groups were equally represented at the event? Why or why not?
- **d**With the additional information, comment on how accurately you think the sample statistics reflect the population parameters.

Exercise:

Problem:

- **a**List some practical difficulties involved in getting accurate results from a telephone survey.
- **b**List some practical difficulties involved in getting accurate results from a mailed survey.
- **c**With your classmates, brainstorm some ways to overcome these problems if you needed to conduct a phone or mail survey.

Try these multiple choice questions

The next four questions refer to the following: A Lake Tahoe Community College instructor is interested in the average number of days Lake Tahoe Community College math students are absent from class during a quarter.

Exercise:

Problem: What is the population she is interested in?

- **A**All Lake Tahoe Community College students
- **B**All Lake Tahoe Community College English students
- **C**All Lake Tahoe Community College students in her classes
- **D**All Lake Tahoe Community College math students

Solution:

D

Exercise:

Problem: Consider the following:

X **= number of days a Lake Tahoe Community College math student is absent**

In this case, X is an example of a:

- **A**Variable
- **B**Population
- **C**Statistic
- **D**Data

Solution:

A

Exercise:

Problem:

The instructor takes her sample by gathering data on 5 randomly selected students from each Lake Tahoe Community College math class. The type of sampling she used is

- **A**Cluster sampling
- **B**Stratified sampling
- **C**Simple random sampling
- **D**Convenience sampling

Solution:

B

Exercise:

Problem:

The instructor's sample produces an average number of days absent of 3.5 days. This value is an example of a

- **A**Parameter
- **B**Data
- **C**Statistic
- **D**Variable

Solution:

C

The next three questions refer to the following: A study was done to determine the age, number of times per week and the duration (amount of time) of resident use of a local park in San Jose. The first house in the neighborhood around the park was selected randomly and then every 8th house in the neighborhood around the park was interviewed.

Exercise:

Problem: "‘Number of times per week’" is what type of data?

- **A**qualitative
- **B**quantitative - discrete
- **C**quantitative - continuous

Solution:

B

Exercise:

Problem: The sampling method was:

- **A**simple random
- **B**systematic
- **C**stratified
- **D**cluster

Solution:

B

Exercise:

Problem: "‘Duration (amount of time)’" is what type of data?

- **A**qualitative
- **B**quantitative - discrete
- **C**quantitative - continuous

Solution:

C

Introduction

Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Display data graphically and interpret graphs: stemplots, histograms and boxplots.
- Recognize, describe, and calculate the measures of location of data: quartiles and percentiles.
- Recognize, describe, and calculate the measures of the center of data: mean, median, and mode.
- Recognize, describe, and calculate the measures of the spread of data: variance, standard deviation, and range.

Introduction

Once you have collected data, what will you do with it? Data can be described and presented in many different formats. For example, suppose you are interested in buying a house in a particular area. You may have no clue about the house prices, so you might ask your real estate agent to give you a sample data set of prices. Looking at all the prices in the sample often is overwhelming. A better way might be to look at the median price and the variation of prices. The median and variation are just two ways that you will learn to describe data. Your agent might also provide you with a graph of the data.

In this chapter, you will study numerical and graphical ways to describe and display your data. This area of statistics is called **"Descriptive Statistics"**. You will learn to calculate, and even more importantly, to interpret these measurements and graphs.

Displaying Data

This module provides a brief introduction into the ways graphs and charts can be used to provide visual representations of data.

A statistical graph is a tool that helps you learn about the shape or distribution of a sample. The graph can be a more effective way of presenting data than a mass of numbers because we can see where data clusters and where there are only a few data values. Newspapers and the Internet use graphs to show trends and to enable readers to compare facts and figures quickly.

Statisticians often graph data first to get a picture of the data. Then, more formal tools may be applied.

Some of the types of graphs that are used to summarize and organize data are the dot plot, the bar chart, the histogram, the stem-and-leaf plot, the frequency polygon (a type of broken line graph), pie charts, and the boxplot. In this chapter, we will briefly look at stem-and-leaf plots, line graphs and bar graphs. Our emphasis will be on histograms and boxplots.

Stem and Leaf Graphs (Stemplots)
This module introduces the use of stem-and-leaf graphs (stemplots), line graphs and bar graphs for describing a set of data visually.

One simple graph, the **stem-and-leaf graph** or **stem plot**, comes from the field of exploratory data analysis.It is a good choice when the data sets are small. To create the plot, divide each observation of data into a stem and a leaf. The leaf consists of a **final significant digit**. For example, 23 has stem 2 and leaf 3. Four hundred thirty-two (432) has stem 43 and leaf 2. Five thousand four hundred thirty-two (5,432) has stem 543 and leaf 2. The decimal 9.3 has stem 9 and leaf 3. Write the stems in a vertical line from smallest the largest. Draw a vertical line to the right of the stems. Then write the leaves in increasing order next to their corresponding stem.

Example:
For Susan Dean's spring pre-calculus class, scores for the first exam were as follows (smallest to largest):
33424949535555616367686869697273747880838888889092949494949 6
100

Stem	Leaf
3	3
4	299
5	355
6	1378899

Stem	Leaf
7	2348
8	03888
9	0244446
10	0

Stem-and-Leaf Diagram

The stem plot shows that most scores fell in the 60s, 70s, 80s, and 90s. Eight out of the 31 scores or approximately 26% of the scores were in the 90's or 100, a fairly high number of As.

The stem plot is a quick way to graph and gives an exact picture of the data. You want to look for an overall pattern and any outliers. An **outlier** is an observation of data that does not fit the rest of the data. It is sometimes called an **extreme value.** When you graph an outlier, it will appear not to fit the pattern of the graph. Some outliers are due to mistakes (for example, writing down 50 instead of 500) while others may indicate that something unusual is happening. It takes some background information to explain outliers. In the example above, there were no outliers.

Example:
Create a stem plot using the data:
1.11.52.32.52.73.23.33.33.53.84.0 4.24.54.54.74.85.55.66.56.712.3
The data are the distance (in kilometers) from a home to the nearest supermarket.
Exercise:

Problem:

1. Are there any values that might possibly be outliers?
2. Do the data seem to have any concentration of values?

Note: The leaves are to the right of the decimal.

Solution:

The value 12.3 may be an outlier. Values appear to concentrate at 3 and 4 kilometers.

Stem	Leaf
1	1 5
2	3 5 7
3	2 3 3 5 8
4	0 2 5 5 7 8
5	5 6
6	5 7
7	
8	

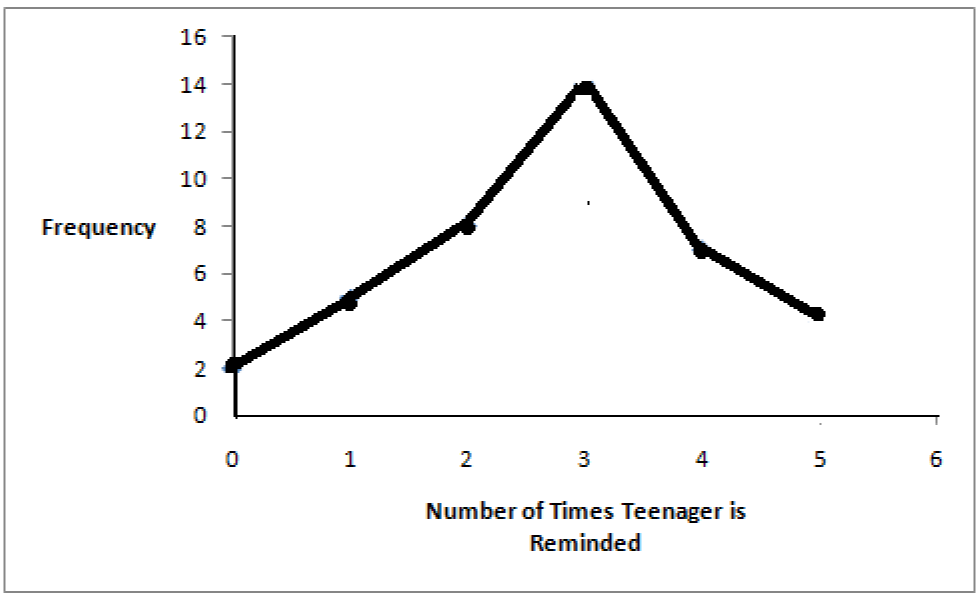
Stem	Leaf
9	
10	
11	
12	3

Another type of graph that is useful for specific data values is a **line graph**. In the particular line graph shown in the example, the **x-axis** consists of **data values** and the **y-axis** consists of **frequency points**. The frequency points are connected.

Example:
In a survey, 40 mothers were asked how many times per week a teenager must be reminded to do his/her chores. The results are shown in the table and the line graph.

Number of times teenager is reminded	Frequency
0	2
1	5

Number of times teenager is reminded	Frequency
2	8
3	14
4	7
5	4



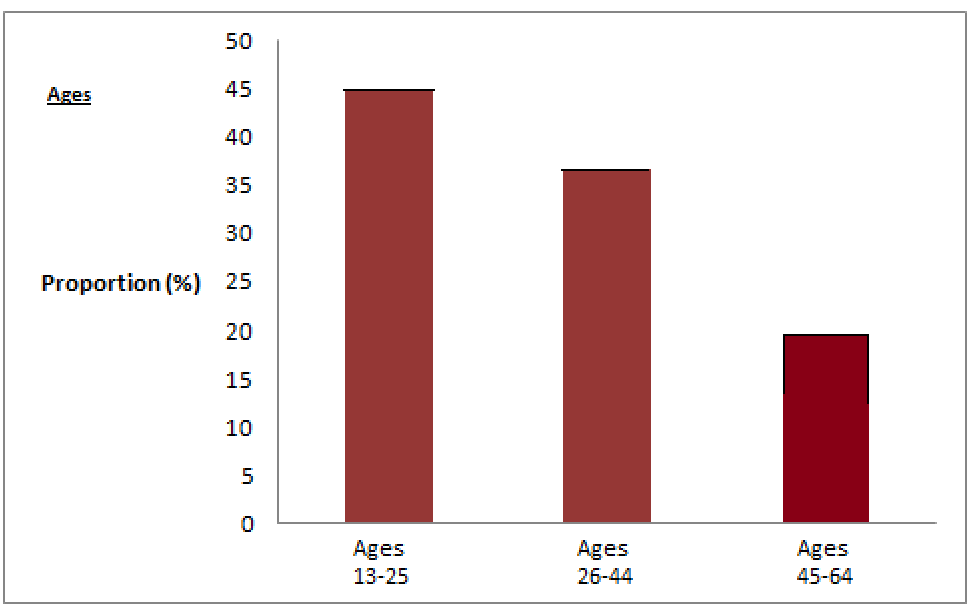
Bar graphs consist of bars that are separated from each other. The bars can be rectangles or they can be rectangular boxes and they can be vertical or horizontal.

The **bar graph** shown in **Example 4** has age groups represented on the **x-axis** and proportions on the **y-axis**.

Example:

By the end of 2011, in the United States, Facebook had over 146 million users. The table shows three age groups, the number of users in each age group and the proportion (%) of users in each age group. **Source:** ***http://www.kenburbary.com/2011/03/facebook-demographics-revisited-2011-statistics-2/***

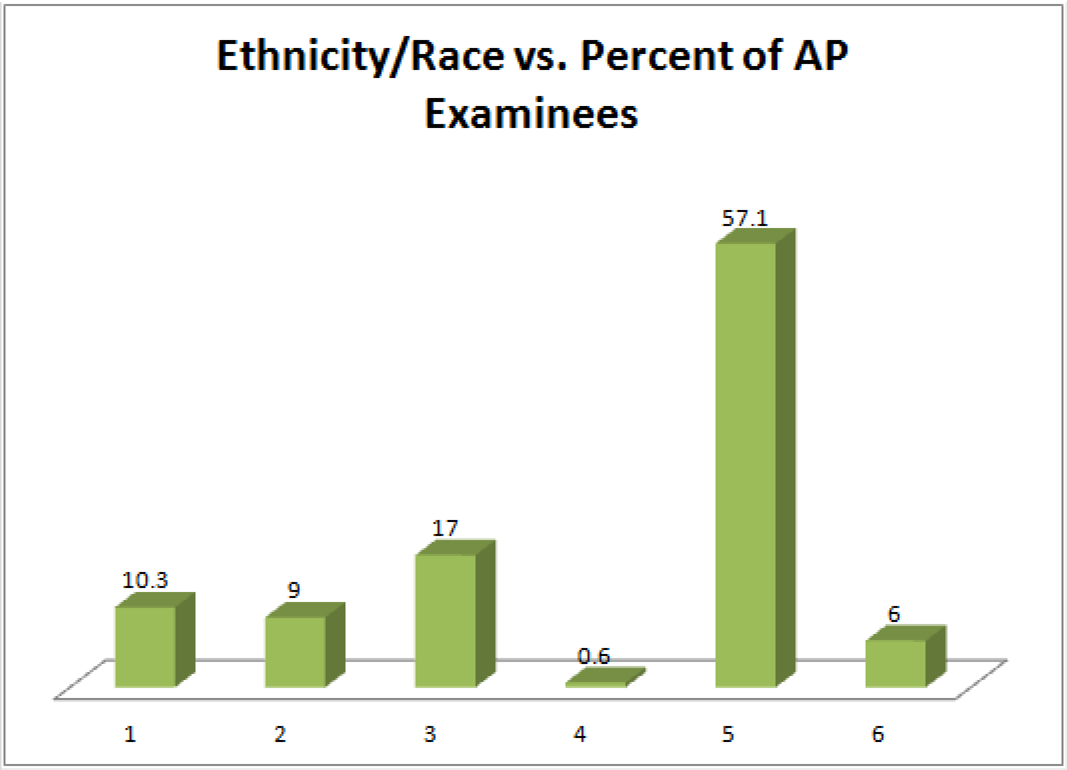
Age groups	**Number of Facebook users**	**Proportion (%) of Facebook users**
13 - 25	65,082,280	45%
26 - 44	53,300,200	36%
45 - 64	27,885,100	19%



Example:

The columns in the table below contain the race/ethnicity of U.S. Public Schools: High School Class of 2011, percentages for the Advanced Placement Examinee Population for that class and percentages for the Overall Student Population. The 3-dimensional graph shows the Race/Ethnicity of U.S. Public Schools (qualitative data) on the **x-axis** and Advanced Placement Examinee Population percentages on the **y-axis**. (**Source: http://www.collegeboard.com** and **Source: http://apreport.collegeboard.org/goals-and-findings/promoting-equity**)

Race/Ethnicity	**AP Examinee Population**	**Overall Student Population**
1 = Asian, Asian American or Pacific Islander	10.3%	5.7%
2 = Black or African American	9.0%	14.7%
3 = Hispanic or Latino	17.0%	17.6%
4 = American Indian or Alaska Native	0.6%	1.1%
5 = White	57.1%	59.2%
6 = Not reported/other	6.0%	1.7%



Go to Outcomes of Education Figure 22 for an example of a bar graph that shows unemployment rates of persons 25 years and older for 2009.

Note: This book contains instructions for constructing a **histogram** and a **box plot** for the TI-83+ and TI-84 calculators. You can find additional instructions for using these calculators on the Texas Instruments (TI) website.

Glossary

Outlier
: An observation that does not fit the rest of the data.

Histograms

This module provides an overview of Descriptive Statistics: Histogram as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

For most of the work you do in this book, you will use a histogram to display the data. One advantage of a histogram is that it can readily display large data sets. A rule of thumb is to use a histogram when the data set consists of 100 values or more.

A **histogram** consists of contiguous boxes. It has both a horizontal axis and a vertical axis. The horizontal axis is labeled with what the data represents (for instance, distance from your home to school). The vertical axis is labeled either **Frequency** or **relative frequency**. The graph will have the same shape with either label. The histogram (like the stemplot) can give you the shape of the data, the center, and the spread of the data. (The next section tells you how to calculate the center and the spread.)

The relative frequency is equal to the frequency for an observed value of the data divided by the total number of data values in the sample. (In the chapter on Sampling and Data, we defined frequency as the number of times an answer occurs.) If:

- f = frequency
- n = total number of data values (or the sum of the individual frequencies), and
- RF = relative frequency,

then:

Equation:

$$\text{RF} = \frac{f}{n}$$

For example, if 3 students in Mr. Ahab's English class of 40 students received from 90% to 100%, then,

$f = 3$, $n = 40$, and $\text{RF} = \frac{f}{n} = \frac{3}{40} = 0.075$

Seven and a half percent of the students received 90% to 100%. Ninety percent to 100 % are quantitative measures.

To construct a histogram, first decide how many **bars** or **intervals**, also called classes, represent the data. Many histograms consist of from 5 to 15 bars or classes for clarity. Choose a starting point for the first interval to be less than the smallest data value. A **convenient starting point** is a lower value carried out to one more decimal place than the value with the most decimal places. For example, if the value with the most decimal places is 6.1 and this is the smallest value, a convenient starting point is 6.05 (6.1 - 0.05 = 6.05). We say that 6.05 has more precision. If the value with the most decimal places is 2.23 and the lowest value is 1.5, a convenient starting point is 1.495 (1.5 - 0.005 = 1.495). If the value with the most decimal places is 3.234 and the lowest value is 1.0, a convenient starting point is 0.9995 (1.0 - .0005 = 0.9995). If all the data happen to be integers and the smallest value is 2, then a convenient starting point is 1.5 (2 - 0.5 = 1.5). Also, when the starting point and other boundaries are carried to one additional decimal place, no data value will fall on a boundary.

Example:
The following data are the heights (in inches to the nearest half inch) of 100 male semiprofessional soccer players. The heights are **continuous** data since height is measured.
60 60.5 61 61 61.5
63.5 63.5 63.5
64 64 64 64 64 64 64 64.5 64.5 64.5 64.5 64.5 64.5 64.5 64.5
66 66 66 66 66 66 66 66 66 66 66.5 66.5 66.5 66.5 66.5 66.5 66.5 66.5
66.5 66.5 66.5 67 67 67 67 67 67 67 67 67 67 67 67 67.5 67.5 67.5 67.5
67.5 67.5 67.5
68 68 69 69 69 69 69 69 69 69 69 69 69.5 69.5 69.5 69.5 69.5
70 70 70 70 70 70 70.5 70.5 70.5 71 71 71
72 72 72 72.5 72.5 73 73.5
74

The smallest data value is 60. Since the data with the most decimal places has one decimal (for instance, 61.5), we want our starting point to have two decimal places. Since the numbers 0.5, 0.05, 0.005, etc. are convenient numbers, use 0.05 and subtract it from 60, the smallest value, for the convenient starting point.
60 - 0.05 = 59.95 which is more precise than, say, 61.5 by one decimal place. The starting point is, then, 59.95.
The largest value is 74. 74+ 0.05 = 74.05 is the ending value.
Next, calculate the width of each bar or class interval. To calculate this width, subtract the starting point from the ending value and divide by the number of bars (you must choose the number of bars you desire). Suppose you choose 8 bars.
Equation:

$$\frac{74.05 - 59.95}{8} = 1.76$$

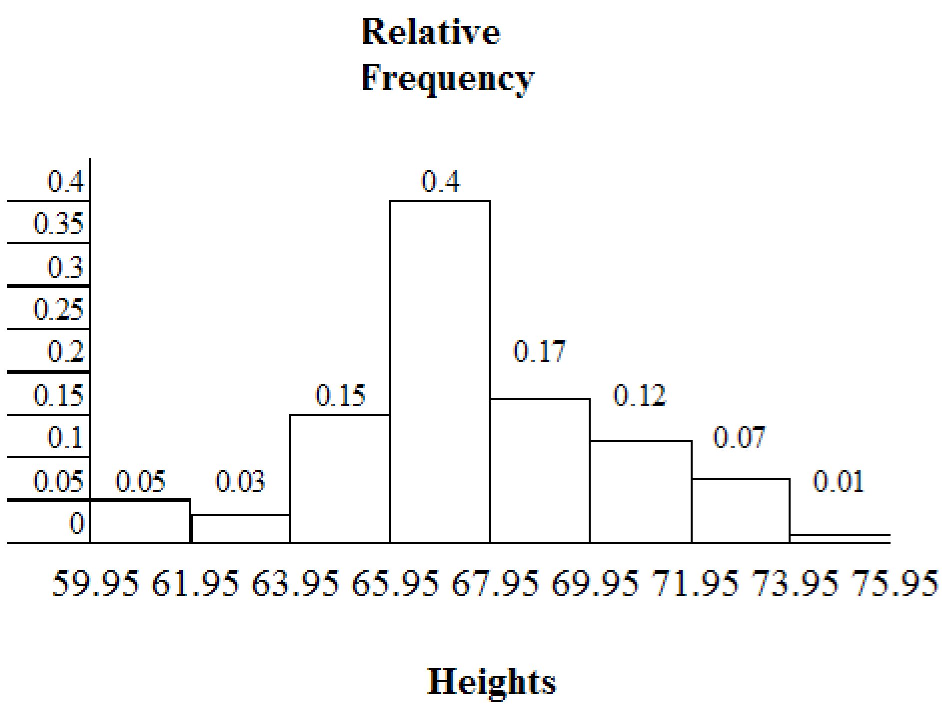
Note: We will round up to 2 and make each bar or class interval 2 units wide. Rounding up to 2 is one way to prevent a value from falling on a boundary. Rounding to the next number is necessary even if it goes against the standard rules of rounding. For this example, using 1.76 as the width would also work.

The boundaries are:

- 59.95
- 59.95 + 2 = 61.95
- 61.95 + 2 = 63.95
- 63.95 + 2 = 65.95
- 65.95 + 2 = 67.95
- 67.95 + 2 = 69.95
- 69.95 + 2 = 71.95
- 71.95 + 2 = 73.95

- 73.95 + 2 = 75.95

The heights 60 through 61.5 inches are in the interval 59.95 - 61.95. The heights that are 63.5 are in the interval 61.95 - 63.95. The heights that are 64 through 64.5 are in the interval 63.95 - 65.95. The heights 66 through 67.5 are in the interval 65.95 - 67.95. The heights 68 through 69.5 are in the interval 67.95 - 69.95. The heights 70 through 71 are in the interval 69.95 - 71.95. The heights 72 through 73.5 are in the interval 71.95 - 73.95. The height 74 is in the interval 73.95 - 75.95.
The following histogram displays the heights on the x-axis and relative frequency on the y-axis.



Example:
The following data are the number of books bought by 50 part-time college students at ABC College. The number of books is discrete data since books

are counted.
1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
4 4 4 4 4 4
5 5 5 5 5
6 6
Eleven students buy 1 book. Ten students buy 2 books. Sixteen students buy 3 books. Six students buy 4 books. Five students buy 5 books. Two students buy 6 books.
Because the data are integers, subtract 0.5 from 1, the smallest data value and add 0.5 to 6, the largest data value. Then the starting point is 0.5 and the ending value is 6.5.
Exercise:

Problem:

Next, calculate the width of each bar or class interval. If the data are discrete and there are not too many different values, a width that places the data values in the middle of the bar or class interval is the most convenient. Since the data consist of the numbers 1, 2, 3, 4, 5, 6 and the starting point is 0.5, a width of one places the 1 in the middle of the interval from 0.5 to 1.5, the 2 in the middle of the interval from 1.5 to 2.5, the 3 in the middle of the interval from 2.5 to 3.5, the 4 in the middle of the interval from _______ to _______, the 5 in the middle of the interval from _______ to _______, and the _______ in the middle of the interval from _______ to _______ .

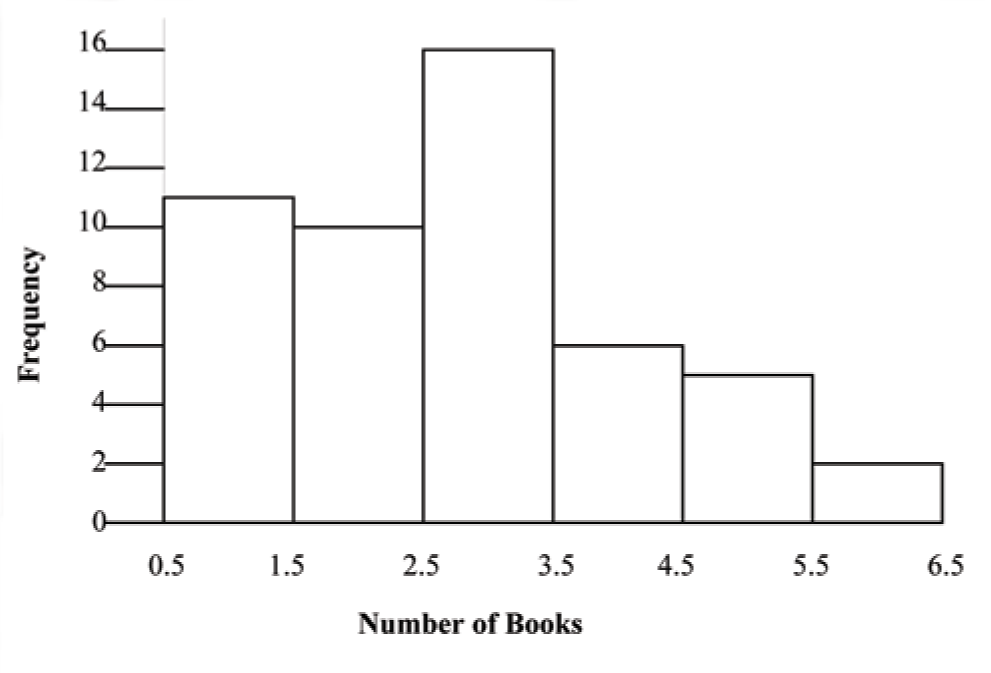
Solution:

- 3.5 to 4.5
- 4.5 to 5.5
- 6
- 5.5 to 6.5

Calculate the number of bars as follows:
Equation:

$$\frac{6.5 - 0.5}{\text{bars}} = 1$$

where 1 is the width of a bar. Therefore, bars $= 6$.
The following histogram displays the number of books on the x-axis and the frequency on the y-axis.



Using the TI-83, 83+, 84, 84+ Calculator Instructions
Go to the Appendix (14:Appendix) in the menu on the left. There are calculator instructions for entering data and for creating a customized histogram. Create the histogram for Example 2.

- Press Y=. Press CLEAR to clear out any equations.
- Press STAT 1:EDIT. If L1 has data in it, arrow up into the name L1, press CLEAR and arrow down. If necessary, do the same for L2.
- Into L1, enter 1, 2, 3, 4, 5, 6
- Into L2, enter 11, 10, 16, 6, 5, 2
- Press WINDOW. Make Xmin = .5, Xmax = 6.5, Xscl = (6.5 - .5)/6, Ymin = -1, Ymax = 20, Yscl = 1, Xres = 1

- Press 2nd Y=. Start by pressing 4:Plotsoff ENTER.
- Press 2nd Y=. Press 1:Plot1. Press ENTER. Arrow down to TYPE. Arrow to the 3rd picture (histogram). Press ENTER.
- Arrow down to Xlist: Enter L1 (2nd 1). Arrow down to Freq. Enter L2 (2nd 2).
- Press GRAPH
- Use the TRACE key and the arrow keys to examine the histogram.

Optional Collaborative Exercise

Count the money (bills and change) in your pocket or purse. Your instructor will record the amounts. As a class, construct a histogram displaying the data. Discuss how many intervals you think is appropriate. You may want to experiment with the number of intervals. Discuss, also, the shape of the histogram.

Record the data, in dollars (for example, 1.25 dollars).

Construct a histogram.

Glossary

Frequency
: The number of times a value of the data occurs.

Relative Frequency
: The ratio of the number of times a value of the data occurs in the set of all outcomes to the number of all outcomes.

Box Plots

Box plots or **box-whisker plots** give a good graphical image of the concentration of the data. They also show how far from most of the data the extreme values are. The box plot is constructed from five values: the smallest value, the first quartile, the median, the third quartile, and the largest value. The median, the first quartile, and the third quartile will be discussed here, and then again in the section on measuring data in this chapter. We use these values to compare how close other data values are to them.

The **median**, a number, is a way of measuring the "center" of the data. You can think of the median as the "middle value," although it does not actually have to be one of the observed values. It is a number that separates ordered data into halves. Half the values are the same number or smaller than the median and half the values are the same number or larger. For example, consider the following data:

1 11.5 6 7.2 4 8 9 10 6.8 8.3 2 2 10 1

Ordered from smallest to largest:

1 1 2 2 4 6 **6.8** **7.2** 8 8.3 9 10 10 11.5

The median is between the 7th value, 6.8, and the 8th value 7.2. To find the median, add the two values together and divide by 2.
Equation:

$$\frac{6.8+7.2}{2}=7$$

The median is 7. Half of the values are smaller than 7 and half of the values are larger than 7.

Quartiles are numbers that separate the data into quarters. Quartiles may or may not be part of the data. To find the quartiles, first find the median or second quartile. The first quartile is the middle value of the lower half of

the data and the third quartile is the middle value of the upper half of the data. To get the idea, consider the same data set shown above:

1 1 2 2 4 6 6.8 7.2 8 8.3 9 10 10 11.5

The median or **second quartile** is 7. The lower half of the data is 1, 1, 2, 2, 4, 6, 6.8. The middle value of the lower half is 2.

1 1 2 **2** 4 6 6.8

The number 2, which is part of the data, is the **first quartile**. One-fourth of the values are the same or less than 2 and three-fourths of the values are more than 2.

The upper half of the data is 7.2, 8, 8.3, 9, 10, 10, 11.5. The middle value of the upper half is 9.

7.2 8 8.3 **9** 10 10 11.5

The number 9, which is part of the data, is the **third quartile**. Three-fourths of the values are less than 9 and one-fourth of the values are more than 9.

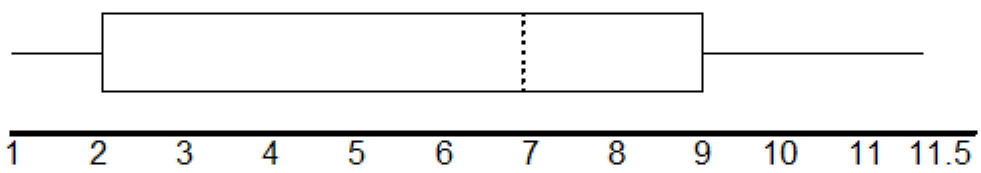
To construct a box plot, use a horizontal number line and a rectangular box. The smallest and largest data values label the endpoints of the axis. The first quartile marks one end of the box and the third quartile marks the other end of the box. **The middle fifty percent of the data fall inside the box.** The "whiskers" extend from the ends of the box to the smallest and largest data values. The box plot gives a good quick picture of the data.

Note: You may encounter box and whisker plots that have dots marking outlier values. In those cases, the whiskers are not extending to the minimum and maximum values.

Consider the following data:

1 1 2 2 4 6 6.8 7.2 8 8.3 9 10 10 11.5

The first quartile is 2, the median is 7, and the third quartile is 9. The smallest value is 1 and the largest value is 11.5. The box plot is constructed as follows (see calculator instructions in the back of this book or on the TI web site):



The two whiskers extend from the first quartile to the smallest value and from the third quartile to the largest value. The median is shown with a dashed line.

Example:
The following data are the heights of 40 students in a statistics class.
59 60 61 62 62 63 63 64 64 64 65 65 65 65 65 65 65 65 65 66 66 67 67 68 68 69 70 70 70 70 70 71 71 72 72 73 74 74 75 77
Construct a box plot:
Using the TI-83, 83+, 84, 84+ Calculator

- Enter data into the list editor (Press STAT 1:EDIT). If you need to clear the list, arrow up to the name L1, press CLEAR, arrow down.
- Put the data values in list L1.
- Press STAT and arrow to CALC. Press 1:1-VarStats. Enter L1.
- Press ENTER
- Use the down and up arrow keys to scroll.

- Smallest value = 59
- Largest value = 77
- Q1: First quartile = 64.5

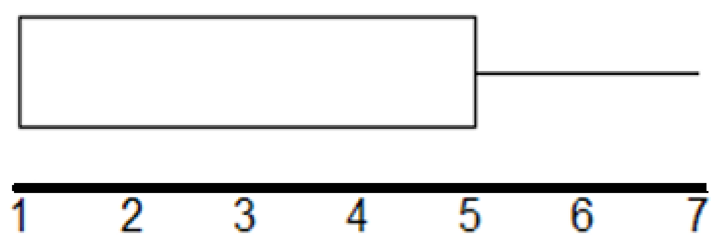
- Q2: Second quartile or median= 66
- Q3: Third quartile = 70

Using the TI-83, 83+, 84, 84+ to Construct the Box Plot
Go to 14:Appendix for Notes for the TI-83, 83+, 84, 84+ Calculator. To create the box plot:

- Press Y=. If there are any equations, press CLEAR to clear them.
- Press 2nd Y=.
- Press 4:Plotsoff. Press ENTER
- Press 2nd Y=
- Press 1:Plot1. Press ENTER.
- Arrow down and then use the right arrow key to go to the 5th picture which is the box plot. Press ENTER.
- Arrow down to Xlist: Press 2nd 1 for L1
- Arrow down to Freq: Press ALPHA. Press 1.
- Press ZOOM. Press 9:ZoomStat.
- Press TRACE and use the arrow keys to examine the box plot.

- **a**Each quarter has 25% of the data.
- **b**The spreads of the four quarters are 64.5 - 59 = 5.5 (first quarter), 66 - 64.5 = 1.5 (second quarter), 70 - 66 = 4 (3rd quarter), and 77 - 70 = 7 (fourth quarter). So, the second quarter has the smallest spread and the fourth quarter has the largest spread.
- **c**Interquartile Range: $IQR = Q3 - Q1 = 70 - 64.5 = 5.5$.
- **d**The interval 59 through 65 has more than 25% of the data so it has more data in it than the interval 66 through 70 which has 25% of the data.
- **e**The middle 50% (middle half) of the data has a range of 5.5 inches.

For some sets of data, some of the largest value, smallest value, first quartile, median, and third quartile may be the same. For instance, you might have a data set in which the median and the third quartile are the same. In this case, the diagram would not have a dotted line inside the box displaying the median. The right side of the box would display both the third quartile and the median. For example, if the smallest value and the first quartile were both 1, the median and the third quartile were both 5, and the largest value was 7, the box plot would look as follows:



Example:
Test scores for a college statistics class held during the day are:
99 56 78 55.5 32 90 80 81 56 59 45 77 84.5 84 70 72 68 32 79 90
Test scores for a college statistics class held during the evening are:
98 78 68 83 81 89 88 76 65 45 98 90 80 84.5 85 79 78 98 90 79 81 25.5
Exercise:

Problem:

- What are the smallest and largest data values for each data set?
- What is the median, the first quartile, and the third quartile for each data set?
- Create a boxplot for each set of data.
- Which boxplot has the widest spread for the middle 50% of the data (the data between the first and third quartiles)? What does this mean for that set of data in comparison to the other set of data?

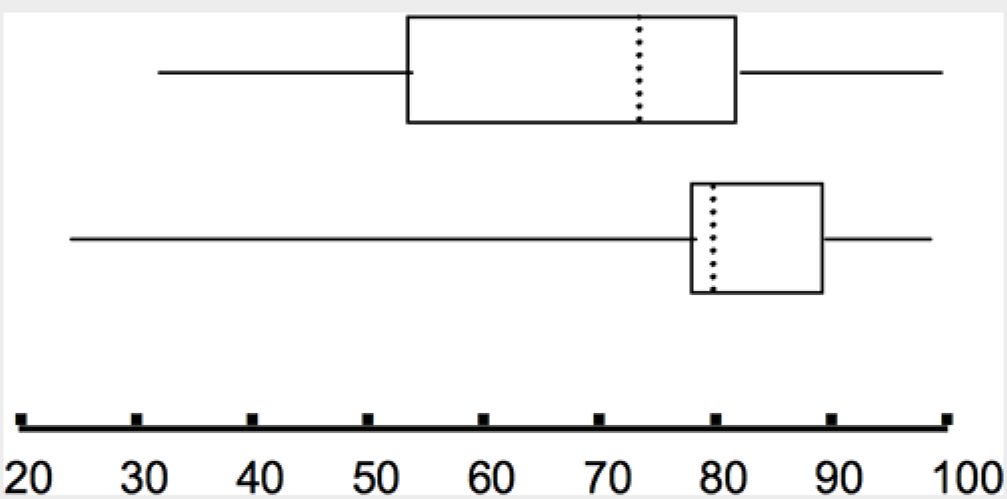
- For each data set, what percent of the data is between the smallest value and the first quartile? (Answer: 25%) the first quartile and the median? (Answer: 25%) the median and the third quartile? the third quartile and the largest value? What percent of the data is between the first quartile and the largest value? (Answer: 75%)

Solution:
First Data Set

- $\text{Xmin} = 32$
- $\text{Q1} = 56$
- $M = 74.5$
- $\text{Q3} = 82.5$
- $\text{Xmax} = 99$

Second Data Set

- $\text{Xmin} = 25.5$
- $\text{Q1} = 78$
- $M = 81$
- $\text{Q3} = 89$
- $\text{Xmax} = 98$



The first data set (the top box plot) has the widest spread for the middle 50% of the data. $\mathrm{IQR} = \mathrm{Q3} - \mathrm{Q1}$ is $82.5 - 56 = 26.5$ for the first data set and $89 - 78 = 11$ for the second data set. So, the first set of data has its middle 50% of scores more spread out.
25% of the data is between M and $\mathrm{Q3}$ and 25% is between $\mathrm{Q3}$ and Xmax.

Glossary

Median
: A number that separates ordered data into halves. Half the values are the same number or smaller than the median and half the values are the same number or larger than the median. The median may or may not be part of the data.

Quartiles
: The numbers that separate the data into quarters. Quartiles may or may not be part of the data. The second quartile is the median of the data.

Measures of the Location of the Data
Descriptive Statistics: Measuring the Location of Data explains percentiles and quartiles and is part of the collection col10555 written by Barbara Illowsky and Susan Dean. Roberta Bloom contributed the section "Interpreting Percentiles, Quartile and the Median."

The common measures of location are **quartiles** and **percentiles** (%iles). Quartiles are special percentiles. The first quartile, Q_1 is the same as the 25th percentile (25th %ile) and the third quartile, Q_3, is the same as the 75th percentile (75th %ile). The median, M, is called both the second quartile and the 50th percentile (50th %ile).

Note: Quartiles are given special attention in the Box Plots module in this chapter.

To calculate quartiles and percentiles, the data must be ordered from smallest to largest. Recall that quartiles divide ordered data into quarters. Percentiles divide ordered data into hundredths. To score in the 90th percentile of an exam does not mean, necessarily, that you received 90% on a test. It means that 90% of test scores are the same or less than your score and 10% of the test scores are the same or greater than your test score.

Percentiles are useful for comparing values. For this reason, universities and colleges use percentiles extensively.

Percentiles are mostly used with very large populations. Therefore, if you were to say that 90% of the test scores are less (and not the same or less) than your score, it would be acceptable because removing one particular data value is not significant.

The **interquartile range** is a number that indicates the spread of the middle half or the middle 50% of the data. It is the difference between the third quartile (Q_3) and the first quartile (Q_1).
Equation:

$$\text{IQR} = Q_3 - Q_1$$

The IQR can help to determine potential **outliers**. **A value is suspected to be a potential outlier if it is less than** $(1.5)(\text{IQR})$ **below the first quartile or more than** $(1.5)(\text{IQR})$ **above the third quartile**. Potential outliers always need further investigation.

Example:
Exercise:

Problem:

For the following 13 real estate prices, calculate the IQR and determine if any prices are outliers. Prices are in dollars. (*Source: San Jose Mercury News*)

389,950 230,500 158,000 479,000 639,000 114,950 5,500,000 387,000 659,000 529,000 575,000 488,800 1,095,000

Solution:

Order the data from smallest to largest.

114,950 158,000 230,500 387,000 389,950 479,000 488,800 529,000 575,000 639,000 659,000 1,095,000 5,500,000

$M = 488{,}800$

$Q_1 = \frac{230500+387000}{2} = 308750$

$Q_3 = \frac{639000+659000}{2} = 649000$

$\text{IQR} = 649000 - 308750 = 340250$

$(1.5)(\text{IQR}) = (1.5)(340250) = 510375$

$Q_1 - (1.5)(\text{IQR}) = 308750 - 510375 = -201625$

$Q_3 + (1.5)(\text{IQR}) = 649000 + 510375 = 1159375$

No house price is less than -201625. However, 5,500,000 is more than 1,159,375. Therefore, 5,500,000 is a potential **outlier**.

Example:
Exercise:

Problem:

For the two data sets in the test scores example, find the following:

- **a**The interquartile range. Compare the two interquartile ranges.
- **b**Any outliers in either set.
- **c**The 30th percentile and the 80th percentile for each set. How much data falls below the 30th percentile? Above the 80th percentile?

Solution:

For the IQRs, see the answer to the test scores example. The first data set has the larger IQR, so the scores between $Q3$ and $Q1$ (middle 50%) for the first data set are more spread out and not clustered about the median.

First Data Set

- $\left(\frac{3}{2}\right) \cdot (\text{IQR}) = \left(\frac{3}{2}\right) \cdot (26.5) = 39.75$
- $\text{Xmax - Q3} = 99 - 82.5 = 16.5$
- $\text{Q1 - Xmin} = 56 - 32 = 24$

$\left(\frac{3}{2}\right) \cdot (\text{IQR}) = 39.75$ is larger than 16.5 and larger than 24, so the first set has no outliers.

Second Data Set

- $\left(\frac{3}{2}\right) \cdot (\text{IQR}) = \left(\frac{3}{2}\right) \cdot (11) = 16.5$
- $\text{Xmax} - \text{Q3} = 98 - 89 = 9$
- $\text{Q1} - \text{Xmin} = 78 - 25.5 = 52.5$

$\left(\frac{3}{2}\right) \cdot (\text{IQR}) = 16.5$ is larger than 9 but smaller than 52.5, so for the second set 45 and 25.5 are outliers.

To find the percentiles, create a frequency, relative frequency, and cumulative relative frequency chart (see "Frequency" from the Sampling and

Data Chapter). Get the percentiles from that chart.

First Data Set

- $30\text{th } \%\text{ile (between the 6th and 7th values)} = \frac{(56 + 59)}{2} = 57.5$
- $80\text{th } \%\text{ile (between the 16th and 17th values)} = \frac{(84 + 84.5)}{2} = 84.25$

Second Data Set

- $30\text{th } \%\text{ile (7th value)} = 78$
- $80\text{th } \%\text{ile (18th value)} = 90$

30% of the data falls below the 30th %ile, and 20% falls above the 80th %ile.

Example:
Finding Quartiles and Percentiles Using a Table
Fifty statistics students were asked how much sleep they get per school night (rounded to the nearest hour). The results were (student data):

AMOUNT OF SLEEP PER SCHOOL NIGHT (HOURS)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
4	2	0.04	0.04
5	5	0.10	0.14

AMOUNT OF SLEEP PER SCHOOL NIGHT (HOURS)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
6	7	0.14	0.28
7	12	0.24	0.52
8	14	0.28	0.80
9	7	0.14	0.94
10	3	0.06	1.00

Find the 28th percentile: Notice the 0.28 in the "cumulative relative frequency" column. 28% of 50 data values = 14. There are 14 values less than the 28th %ile. They include the two 4s, the five 5s, and the seven 6s. The 28th %ile is between the last 6 and the first 7. **The 28th %ile is 6.5.**
Find the median: Look again at the "cumulative relative frequency " column and find 0.52. The median is the 50th %ile or the second quartile. 50% of 50 = 25. There are 25 values less than the median. They include the two 4s, the five 5s, the seven 6s, and eleven of the 7s. The median or 50th %ile is between the 25th (7) and 26th (7) values. **The median is 7.**
Find the third quartile: The third quartile is the same as the 75th percentile. You can "eyeball" this answer. If you look at the "cumulative relative frequency" column, you find 0.52 and 0.80. When you have all the 4s, 5s, 6s and 7s, you have 52% of the data. When you include all the 8s, you have 80% of the data. **The 75th %ile, then, must be an 8** . Another way to look at the problem is to find 75% of 50 (= 37.5) and round up to 38. The third quartile, Q_3, is the 38th value which is an 8. You can check this answer by counting the values. (There are 37 values below the third quartile and 12 values above.)

Example:

Exercise:

Problem: Using the table:

1. Find the 80th percentile.
2. Find the 90th percentile.
3. Find the first quartile.
4. What is another name for the first quartile?

Solution:

1. $\frac{(8+9)}{2} \;=\; 8.5$
   Look where cum. rel. freq. = 0.80. 80% of the data is 8 or less. 80th %ile is between the last 8 and first 9.
2. 9
3. 6
4. First Quartile = 25th %ile

Collaborative Classroom Exercise: Your instructor or a member of the class will ask everyone in class how many sweaters they own. Answer the following questions.

1. How many students were surveyed?
2. What kind of sampling did you do?
3. Construct a table of the data.
4. Construct 2 different histograms. For each, starting value = ______ ending value = _____.
5. Use the table to find the median, first quartile, and third quartile.
6. Construct a box plot.
7. Use the table to find the following:
   - The 10th percentile
   - The 70th percentile
   - The percent of students who own less than 4 sweaters

Interpreting Percentiles, Quartiles, and Median

A percentile indicates the relative standing of a data value when data are sorted into numerical order, from smallest to largest. p% of data values are less than or equal to the pth percentile. For example, 15% of data values are less than or equal to the 15th percentile.

- Low percentiles always correspond to lower data values.
- High percentiles always correspond to higher data values.

A percentile may or may not correspond to a value judgment about whether it is "good" or "bad". The interpretation of whether a certain percentile is good or bad depends on the context of the situation to which the data applies. In some situations, a low percentile would be considered "good'; in other contexts a high percentile might be considered "good". In many situations, there is no value judgment that applies.

Understanding how to properly interpret percentiles is important not only when describing data, but is also important in later chapters of this textbook when calculating probabilities.

Guideline:

When writing the interpretation of a percentile in the context of the given data, the sentence should contain the following information:

- information about the context of the situation being considered,
- the data value (value of the variable) that represents the percentile,
- the percent of individuals or items with data values below the percentile.
- Additionally, you may also choose to state the percent of individuals or items with data values above the percentile.

Example:
On a timed math test, the first quartile for times for finishing the exam was 35 minutes. Interpret the first quartile in the context of this situation.

- 25% of students finished the exam in 35 minutes or less.
- 75% of students finished the exam in 35 minutes or more.
- A low percentile could be considered good, as finishing more quickly on a timed exam is desirable. (If you take too long, you might not be able to finish.)

Example:
On a 20 question math test, the 70th percentile for number of correct answers was 16. Interpret the 70th percentile in the context of this situation.

- 70% of students answered 16 or fewer questions correctly.
- 30% of students answered 16 or more questions correctly.
- Note: A high percentile could be considered good, as answering more questions correctly is desirable.

Example:
At a certain community college, it was found that the 30th percentile of credit units that students are enrolled for is 7 units. Interpret the 30th percentile in the context of this situation.

- 30% of students are enrolled in 7 or fewer credit units
- 70% of students are enrolled in 7 or more credit units
- In this example, there is no "good" or "bad" value judgment associated with a higher or lower percentile. Students attend community college for varied reasons and needs, and their course load varies according to their needs.

Do the following Practice Problems for Interpreting Percentiles
Exercise:

Problem:

- **a** For runners in a race, a low time means a faster run. The winners in a race have the shortest running times. Is it more desirable to have a finish time with a high or a low percentile when running a race?
- **b** The 20th percentile of run times in a particular race is 5.2 minutes. Write a sentence interpreting the 20th percentile in the context of the situation.

- **c** A bicyclist in the 90th percentile of a bicycle race between two towns completed the race in 1 hour and 12 minutes. Is he among the fastest or slowest cyclists in the race? Write a sentence interpreting the 90th percentile in the context of the situation.

Solution:

- **a** For runners in a race it is more desirable to have a low percentile for finish time. A low percentile means a short time, which is faster.
- **b**INTERPRETATION: 20% of runners finished the race in 5.2 minutes or less. 80% of runners finished the race in 5.2 minutes or longer.
- **c**He is among the slowest cyclists (90% of cyclists were faster than him.) INTERPRETATION: 90% of cyclists had a finish time of 1 hour, 12 minutes or less.Only 10% of cyclists had a finish time of 1 hour, 12 minutes or longer

Exercise:

Problem:

- **a** For runners in a race, a higher speed means a faster run. Is it more desirable to have a speed with a high or a low percentile when running a race?
- **b**The 40th percentile of speeds in a particular race is 7.5 miles per hour. Write a sentence interpreting the 40th percentile in the context of the situation.

Solution:

- **a**For runners in a race it is more desirable to have a high percentile for speed. A high percentile means a higher speed, which is faster.
- **b**INTERPRETATION: 40% of runners ran at speeds of 7.5 miles per hour or less (slower). 60% of runners ran at speeds of 7.5 miles per hour or more (faster).

Exercise:

Problem:

On an exam, would it be more desirable to earn a grade with a high or low percentile? Explain.

Solution:

On an exam you would prefer a high percentile; higher percentiles correspond to higher grades on the exam.

Exercise:

Problem:

Mina is waiting in line at the Department of Motor Vehicles (DMV). Her wait time of 32 minutes is the 85th percentile of wait times. Is that good or bad? Write a sentence interpreting the 85th percentile in the context of this situation.

Solution:

When waiting in line at the DMV, the 85th percentile would be a long wait time compared to the other people waiting. 85% of people had shorter wait times than you did. In this context, you would prefer a wait time corresponding to a lower percentile. INTERPRETATION: 85% of people at the DMV waited 32 minutes or less. 15% of people at the DMV waited 32 minutes or longer.

Exercise:

Problem:

In a survey collecting data about the salaries earned by recent college graduates, Li found that her salary was in the 78th percentile. Should Li be pleased or upset by this result? Explain.

Solution:

Li should be pleased. Her salary is relatively high compared to other recent college grads. 78% of recent college graduates earn less than Li does. 22% of recent college graduates earn more than Li does.

Exercise:

Problem:

In a study collecting data about the repair costs of damage to automobiles in a certain type of crash tests, a certain model of car had $1700 in damage and was in the 90th percentile. Should the manufacturer and/or a consumer be pleased or upset by this result? Explain. Write a sentence that interprets the 90th percentile in the context of this problem.

Solution:

The manufacturer and the consumer would be upset. This is a large repair cost for the damages, compared to the other cars in the sample. INTERPRETATION: 90% of the crash tested cars had damage repair costs of $1700 or less; only 10% had damage repair costs of $1700 or more.

Exercise:

Problem:

- The University of California has two criteria used to set admission standards for freshman to be admitted to a college in the UC system:
- a. Students' GPAs and scores on standardized tests (SATs and ACTs) are entered into a formula that calculates an "admissions index" score. The admissions index score is used to set eligibility standards intended to meet the goal of admitting the top 12% of high school students in the state. In this context, what percentile does the top 12% represent?
- b. Students whose GPAs are at or above the 96th percentile of all students at their high school are eligible (called eligible in the local context), even if they are not in the top 12% of all students in the state. What percent of students from each high school are "eligible in the local context"?

Solution:

- **a**The top 12% of students are those who are at or above the **88th percentile** of admissions index scores.
- **b** The **top 4%** of students' GPAs are at or above the 96th percentile, making the top 4% of students "eligible in the local context".

Exercise:

Problem:

Suppose that you are buying a house. You and your realtor have determined that the most expensive house you can afford is the 34th percentile. The 34th percentile of housing prices is $240,000 in the town you want to move to. In this town, can you afford 34% of the houses or 66% of the houses?

Solution:

You can afford 34% of houses. 66% of the houses are too expensive for your budget. INTERPRETATION: 34% of houses cost $240,000 or less. 66% of houses cost $240,000 or more.

**With contributions from Roberta Bloom

Glossary

Interquartile Range (IRQ)
: The distance between the third quartile (Q3) and the first quartile (Q1). IQR = Q3 - Q1.

Outlier
: An observation that does not fit the rest of the data.

Percentile
: A number that divides ordered data into hundredths.

Example:
Let a data set contain 200 ordered observations starting with $\{2.3, 2.7, 2.8, 2.9, 2.9, 3.0...\}$. Then the first percentile is $\frac{(2.7+2.8)}{2} = 2.75$, because 1% of the data is to the left of this point on the number line and 99% of the data is on its right. The second percentile is $\frac{(2.9+2.9)}{2} = 2.9$. Percentiles may or may not be part of the data. In this example, the first percentile is not in the data, but the second percentile is. The median of the data is the second quartile and the 50th percentile. The first and third quartiles are the 25th and the 75th percentiles, respectively.

Quartiles

The numbers that separate the data into quarters. Quartiles may or may not be part of the data. The second quartile is the median of the data.

Measures of the Center of the Data
This chapter discusses measuring descriptive statistical information using the center of the data

The "center" of a data set is also a way of describing location. The two most widely used measures of the "center" of the data are the **mean** (average) and the **median**. To calculate the **mean weight** of 50 people, add the 50 weights together and divide by 50. To find the **median weight** of the 50 people, order the data and find the number that splits the data into two equal parts (previously discussed under box plots in this chapter). The median is generally a better measure of the center when there are extreme values or outliers because it is not affected by the precise numerical values of the outliers. The mean is the most common measure of the center.

Note: The words "mean" and "average" are often used interchangeably. The substitution of one word for the other is common practice. The technical term is "arithmetic mean" and "average" is technically a center location. However, in practice among non-statisticians, "average" is commonly accepted for "arithmetic mean."

The mean can also be calculated by multiplying each distinct value by its frequency and then dividing the sum by the total number of data values. The letter used to represent the sample mean is an x with a bar over it (pronounced "x bar"): $\overline{x}$.

The Greek letter μ (pronounced "mew") represents the population mean. One of the requirements for the sample mean to be a good estimate of the population mean is for the sample taken to be truly random.

To see that both ways of calculating the mean are the same, consider the sample:

11122344444
Equation:

$$\overline{x} = \frac{1+1+1+2+2+3+4+4+4+4+4}{11} = 2.7$$

Equation:

$$\overline{x} = \frac{3 \times 1 + 2 \times 2 + 1 \times 3 + 5 \times 4}{11} = 2.7$$

In the second calculation for the sample mean, the frequencies are 3, 2, 1, and 5.

You can quickly find the location of the median by using the expression $\frac{n+1}{2}$.

The letter n is the total number of data values in the sample. If n is an odd number, the median is the middle value of the ordered data (ordered smallest to largest). If n is an even number, the median is equal to the two middle values added together and divided by 2 after the data has been ordered. For example, if the total number of data values is 97, then $\frac{n+1}{2} = \frac{97+1}{2} = 49$. The median is the 49th value in the ordered data. If the total number of data values is 100, then $\frac{n+1}{2} = \frac{100+1}{2} = 50.5$. The median occurs midway between the 50th and 51st values. The location of the median and the value of the median are **not** the same. The upper case letter M is often used to represent the median. The next example illustrates the location of the median and the value of the median.

Example:
Exercise:

Problem:

AIDS data indicating the number of months an AIDS patient lives after taking a new antibody drug are as follows (smallest to largest):

3488101112131415151616171718212222242425262627272929313233333343435374044447

Calculate the mean and the median.

Solution:

The calculation for the mean is:

$$\overline{x} = \frac{[3+4+(8)(2)+10+11+12+13+14+(15)(2)+(16)(2)+...+35+37+40+(44)(2)+47]}{40} = 23.6$$

To find the median, **M**, first use the formula for the location. The location is:

$$\frac{n+1}{2} = \frac{40+1}{2} = 20.5$$

Starting at the smallest value, the median is located between the 20th and 21st values (the two 24s):

3488101112131415151616171718212222242425262627272929313233333434353740444447

$$M = \frac{24+24}{2} = 24$$

The median is 24.

Using the TI-83,83+,84, 84+ Calculators
Calculator Instructions are located in the menu item 14:Appendix (Notes for the TI-83, 83+, 84, 84+ Calculators).

- Enter data into the list editor. Press STAT 1:EDIT
- Put the data values in list L1.
- Press STAT and arrow to CALC. Press 1:1-VarStats. Press 2nd 1 for L1 and ENTER.
- Press the down and up arrow keys to scroll.

$\overline{x} = 23.6$, $M = 24$

Example:
Exercise:

Problem:

Suppose that, in a small town of 50 people, one person earns $5,000,000 per year and the other 49 each earn $30,000. Which is the better measure of the "center," the mean or the median?

Solution:

$\overline{x} = \frac{5000000+49\times 30000}{50} = 129400$

$M = 30000$

(There are 49 people who earn $30,000 and one person who earns $5,000,000.)

The median is a better measure of the "center" than the mean because 49 of the values are 30,000 and one is 5,000,000. The 5,000,000 is an outlier. The 30,000 gives us a better sense of the middle of the data.

Another measure of the center is the mode. The **mode** is the most frequent value. If a data set has two values that occur the same number of times, then the set is bimodal.

Example:
Statistics exam scores for 20 students are as follows
Statistics exam scores for 20 students are as follows:
50 53 59 59 63 63 72 72 72 72 72 76 78 81 83 84 84 84 90 93
Exercise:

Problem: Find the mode.

Solution:

The most frequent score is 72, which occurs five times. Mode = 72.

Example:
Five real estate exam scores are 430, 430, 480, 480, 495. The data set is bimodal because the scores 430 and 480 each occur twice.

When is the mode the best measure of the "center"? Consider a weight loss program that advertises a mean weight loss of six pounds the first week of the program. The mode might indicate that most people lose two pounds the first week, making the program less appealing.

Note: The mode can be calculated for qualitative data as well as for quantitative data.

Statistical software will easily calculate the mean, the median, and the mode. Some graphing calculators can also make these calculations. In the real world, people make these calculations using software.

The Law of Large Numbers and the Mean

The Law of Large Numbers says that if you take samples of larger and larger size from any population, then the mean $\overline{x}$ of the sample is very likely to get closer and closer to μ. This is discussed in more detail in **The Central Limit Theorem**.

Note: The formula for the mean is located in the Summary of Formulas section course.

Sampling Distributions and Statistic of a Sampling Distribution

You can think of a **sampling distribution** as a **relative frequency distribution** with a great many samples. (See **Sampling and Data** for a review of relative frequency). Suppose thirty randomly selected students were asked the number of movies they watched the previous week. The results are in the **relative frequency table** shown below.

# of movies	Relative Frequency
0	5/30
1	15/30
2	6/30
3	4/30
4	1/30

If you let the number of samples get very large (say, 300 million or more), the relative frequency table becomes a relative frequency distribution.

A **statistic** is a number calculated from a sample. Statistic examples include the mean, the median and the mode as well as others. The sample mean $\overline{x}$ is an example of a statistic which estimates the population mean μ.

Glossary

Mean
: A number that measures the central tendency. A common name for mean is 'average.' The term 'mean' is a shortened form of 'arithmetic mean.' By definition, the mean for a sample (denoted by $\overline{x}$) is $\overline{x} = \frac{\text{Sum of all values in the sample}}{\text{Number of values in the sample}}$, and the mean for a population (denoted by μ) is $\mu = \frac{\text{Sum of all values in the population}}{\text{Number of values in the population}}$.

Median
: A number that separates ordered data into halves. Half the values are the same number or smaller than the median and half the values are the same number or larger than the median. The median may or may not be part of the data.

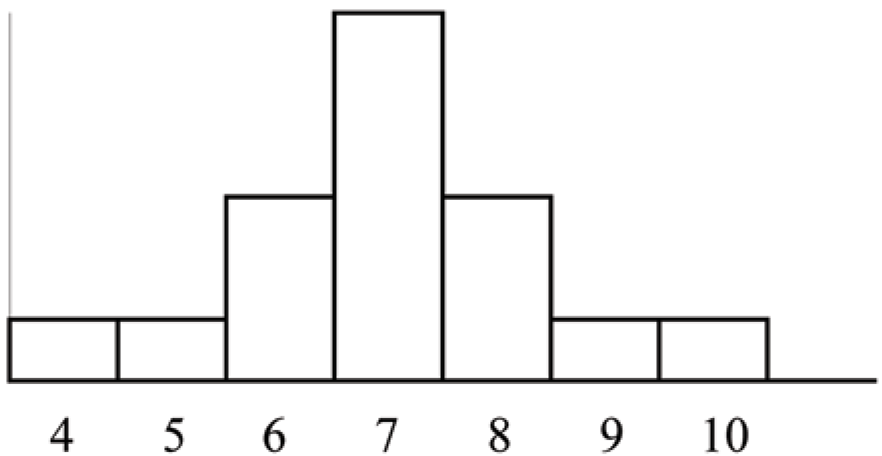
Mode
: The value that appears most frequently in a set of data.

Skewness and the Mean, Median, and Mode

Consider the following data set:

4 5 6 6 6 7 7 7 7 7 7 8 8 8 9 10

This data set produces the histogram shown below. Each interval has width one and each value is located in the middle of an interval.

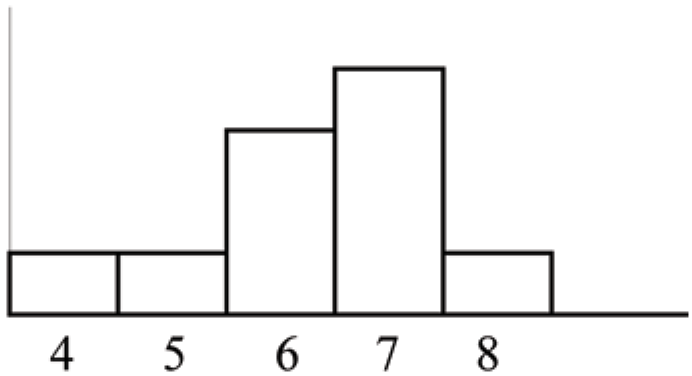


The histogram displays a **symmetrical** distribution of data. A distribution is symmetrical if a vertical line can be drawn at some point in the histogram such that the shape to the left and the right of the vertical line are mirror images of each other. The mean, the median, and the mode are each 7 for these data. **In a perfectly symmetrical distribution, the mean and the median are the same.** This example has one mode (unimodal) and the mode is the same as the mean and median. In a symmetrical distribution that has two modes (bimodal), the two modes would be different from the mean and median.

The histogram for the data:

4 5 6 6 6 7 7 7 7 8

is not symmetrical. The right-hand side seems "chopped off" compared to the left side. The shape distribution is called **skewed to the left** because it is pulled out to the left.

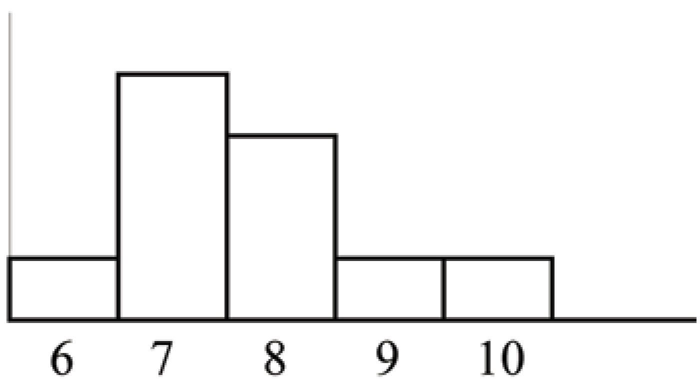


The mean is 6.3, the median is 6.5, and the mode is 7. **Notice that the mean is less than the median and they are both less than the mode.** The mean and the median both reflect the skewing but the mean more so.

The histogram for the data:

6 7 7 7 7 8 8 8 9 10

is also not symmetrical. It is **skewed to the right**.



The mean is 7.7, the median is 7.5, and the mode is 7. Of the three statistics, **the mean is the largest, while the mode is the smallest**. Again, the mean reflects the skewing the most.

To summarize, generally if the distribution of data is skewed to the left, the mean is less than the median, which is often less than the mode. If the distribution of data is skewed to the right, the mode is often less than the median, which is less than the mean.

Skewness and symmetry become important when we discuss probability distributions in later chapters.

Measures of the Spread of the Data
Descriptive Statistics: Measuring the Spread of Data explains standard deviation as a measure of variation in data and is part of the collection col10555 written by Barbara Illowsky and Susan Dean. Roberta Bloom made contributions that helped to clarify the standard deviation and the variance.

An important characteristic of any set of data is the variation in the data. In some data sets, the data values are concentrated closely near the mean; in other data sets, the data values are more widely spread out from the mean. The most common measure of variation, or spread, is the standard deviation.

The **standard deviation** is a number that measures how far data values are from their mean.

The standard deviation

- provides a numerical measure of the overall amount of variation in a data set
- can be used to determine whether a particular data value is close to or far from the mean

The standard deviation provides a measure of the overall variation in a data set
The standard deviation is always positive or 0. The standard deviation is small when the data are all concentrated close to the mean, exhibiting little variation or spread. The standard deviation is larger when the data values are more spread out from the mean, exhibiting more variation.

Suppose that we are studying waiting times at the checkout line for customers at supermarket A and supermarket B; the average wait time at both markets is 5 minutes. At market A, the standard deviation for the waiting time is 2 minutes; at market B the standard deviation for the waiting time is 4 minutes.

Because market B has a higher standard deviation, we know that there is more variation in the waiting times at market B. Overall, wait times at market B are more spread out from the average; wait times at market A are more concentrated near the average.

The standard deviation can be used to determine whether a data value is close to or far from the mean.
Suppose that Rosa and Binh both shop at Market A. Rosa waits for 7 minutes and Binh waits for 1 minute at the checkout counter. At market A, the mean wait time is 5 minutes and the standard deviation is 2 minutes. The standard deviation can be used to determine whether a data value is close to or far from the mean.

Rosa waits for 7 minutes:

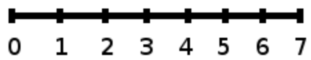
- 7 is 2 minutes longer than the average of 5; 2 minutes is equal to one standard deviation.
- Rosa's wait time of 7 minutes is **2 minutes longer than the average** of 5 minutes.
- Rosa's wait time of 7 minutes is **one standard deviation above the average** of 5 minutes.

Binh waits for 1 minute.

- 1 is 4 minutes less than the average of 5; 4 minutes is equal to two standard deviations.
- Binh's wait time of 1 minute is **4 minutes less than the average** of 5 minutes.
- Binh's wait time of 1 minute is **two standard deviations below the average** of 5 minutes.
- A data value that is two standard deviations from the average is just on the borderline for what many statisticians would consider to be far from the average. Considering data to be far from the mean if it is more than 2 standard deviations away is more of an approximate "rule of thumb" than a rigid rule. In general, the shape of the distribution of the data affects how much of the data is further away than 2 standard deviations. (We will learn more about this in later chapters.)

The number line may help you understand standard deviation. If we were to put 5 and 7 on a number line, 7 is to the right of 5. We say, then, that 7 is **one** standard deviation to the **right** of 5 because
$5 + (1)(2) = 7$.

If 1 were also part of the data set, then 1 is **two** standard deviations to the **left** of 5 because
$5 + (-2)(2) = 1$.



- In general, a **value = mean + (#ofSTDEV)(standard deviation)**
- where #ofSTDEVs = the number of standard deviations
- 7 is **one standard deviation more than the mean** of 5 because: 7=5+**(1)**(2)
- 1 is **two standard deviations less than the mean** of 5 because: 1=5+**(−2)**(2)

The equation **value = mean + (#ofSTDEVs)(standard deviation)** can be expressed for a sample and for a population:

- **sample:** $x = x + (\#\text{ofSTDEV})(\text{s})$
- **Population:** $x = \mu + (\#\text{ofSTDEV})(\sigma)$

The lower case letter s represents the sample standard deviation and the Greek letter σ (sigma, lower case) represents the population standard deviation.

The symbol x is the sample mean and the Greek symbol μ is the population mean.

Calculating the Standard Deviation
If x is a number, then the difference "x - mean" is called its **deviation**. In a data set, there are as many deviations as there are items in the data set. The deviations are used to calculate the standard deviation. If the numbers belong to a population, in symbols a deviation is $x - \mu$. For sample data, in symbols a deviation is $x - x$.

The procedure to calculate the standard deviation depends on whether the numbers are the entire population or are data from a sample. The calculations are similar, but not identical. Therefore the symbol used to represent the standard deviation depends on whether it is calculated from a population or a sample. The lower case letter s represents the sample standard deviation and the Greek letter σ (sigma, lower case) represents the population standard deviation. If the sample has the same characteristics as the population, then s should be a good estimate of σ.

To calculate the standard deviation, we need to calculate the variance first. The **variance** is an **average of the squares of the deviations** (the $x - x$ values for a sample, or the $x - \mu$ values for a population). The symbol σ^2 represents the population variance; the population standard deviation σ is the square root of the population variance. The symbol s^2 represents the sample variance; the sample standard deviation s is the square root of the sample variance. You can think of the standard deviation as a special average of the deviations.

If the numbers come from a census of the entire **population** and not a sample, when we calculate the average of the squared deviations to find the variance, we divide by **N**, the number of items in the population. If the data are from a **sample** rather than a population, when we calculate the average of the squared deviations, we divide by **n-1**, one less than the number of items in the sample. You can see that in the formulas below.

Formulas for the Sample Standard Deviation

- $s = \sqrt{\frac{\Sigma(x-x)^2}{n-1}}$ or $s = \sqrt{\frac{\Sigma f\cdot(x-x)^2}{n-1}}$
- For the sample standard deviation, the denominator is **n-1**, that is the sample size MINUS 1.

Formulas for the Population Standard Deviation

- $\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}}$ or $\sigma = \sqrt{\frac{\Sigma f\cdot(x-\mu)^2}{N}}$
- For the population standard deviation, the denominator is **N**, the number of items in the population.

In these formulas, f represents the frequency with which a value appears. For example, if a value appears once, f is 1. If a value appears three times in the data set or population, f is 3.

Sampling Variability of a Statistic
The statistic of a sampling distribution was discussed in **Descriptive Statistics: Measuring the Center of the Data**. How much the statistic varies from one sample to another is known as the **sampling variability of a statistic**. You typically measure the sampling variability of a statistic by its standard error. The **standard error of the mean** is an example of a standard error. It is a special standard deviation and is known as the standard deviation of the sampling distribution of the mean. You will cover the standard error of the mean in **The Central Limit Theorem** (not now). The notation for the standard error of the mean is $\frac{\sigma}{\sqrt{n}}$ where σ is the standard deviation of the population and n is the size of the sample.

Note: In practice, USE A CALCULATOR OR COMPUTER SOFTWARE TO CALCULATE THE STANDARD DEVIATION. If you are using a TI-83,83+,84+ calculator, you need to select the appropriate standard deviation σ_x or s_x from the summary statistics. We will concentrate on using and interpreting the information that the standard deviation gives us. However you should study the following step-by-step example to help you understand how the standard deviation measures variation from the mean.

Example:
In a fifth grade class, the teacher was interested in the average age and the sample standard deviation of the ages of her students. The following data are the ages for a SAMPLE of $n = 20$ fifth grade students. The ages are rounded to the nearest half year:
9 9.5 9.5 10 10 10 10 10.5 10.5 10.5 10.5 11 11 11 11 11 11 11.5 11.5 11.5
Equation:

$$x = \frac{9 + 9.5 \times 2 + 10 \times 4 + 10.5 \times 4 + 11 \times 6 + 11.5 \times 3}{20} = 10.525$$

The average age is 10.53 years, rounded to 2 places.
The variance may be calculated by using a table. Then the standard deviation is calculated by taking the square root of the variance. We will explain the parts of the table after calculating s.

Data	**Freq.**	**Deviations**	Deviations^2	**(Freq.)**(Deviations^2**)**
x	f	$(x - x)$	$(x - x)^2$	$(f)(x - x)^2$
9	*1*	$9 - 10.525 = -1.525$	$(-1.525)^2 = 2.325625$	$1 \times 2.325625 = 2.325625$
9.5	*2*	$9.5 - 10.525 = -1.025$	$(-1.025)^2 = 1.050625$	$2 \times 1.050625 = 2.101250$
10	*4*	$10 - 10.525 = -0.525$	$(-0.525)^2 = 0.275625$	$4 \times .275625 = 1.1025$
10.5	*4*	$10.5 - 10.525 = -0.025$	$(-0.025)^2 = 0.000625$	$4 \times .000625 = .0025$
11	*6*	$11 - 10.525 = 0.475$	$(0.475)^2 = 0.225625$	$6 \times .225625 = 1.35375$

Data	Freq.	Deviations	Deviations^2	**(Freq.)**(Deviations^2)
11.5	*3*	$11.5 - 10.525 = 0.975$	$(0.975)^2 = 0.950625$	$3 \times .950625 = 2.851875$

The sample variance, s^2, is equal to the sum of the last column (9.7375) divided by the total number of data values minus one (20 - 1):
$s^2 = \frac{9.7375}{20-1} = 0.5125$
The **sample standard deviation** s is equal to the square root of the sample variance:
$s = \sqrt{0.5125} = .0715891$ Rounded to two decimal places, $s = 0.72$
Typically, you do the calculation for the standard deviation on your calculator or computer. The intermediate results are not rounded. This is done for accuracy.
Exercise:

Problem: Verify the mean and standard deviation calculated above on your calculator or computer.

Solution:
Using the TI-83,83+,84+ Calculators

- Enter data into the list editor. Press STAT 1:EDIT. If necessary, clear the lists by arrowing up into the name. Press CLEAR and arrow down.
- Put the data values (9, 9.5, 10, 10.5, 11, 11.5) into list L1 and the frequencies (1, 2, 4, 4, 6, 3) into list L2. Use the arrow keys to move around.
- Press STAT and arrow to CALC. Press 1:1-VarStats and enter L1 (2nd 1), L2 (2nd 2). Do not forget the comma. Press ENTER.
- x=10.525
- Use Sx because this is sample data (not a population): Sx=0.715891

- For the following problems, recall that **value = mean + (#ofSTDEVs)(standard deviation)**
- For a sample: x = x + (#ofSTDEVs)(s)
- For a population: x = μ + (#ofSTDEVs)(σ)
- For this example, use x = x + (#ofSTDEVs)(s) because the data is from a sample

Exercise:

Problem: Find the value that is 1 standard deviation above the mean. Find $(x + 1s)$.

Solution:

$(x + 1s) = 10.53 + (1)(0.72) = 11.25$

Exercise:

Problem: Find the value that is two standard deviations below the mean. Find $(x - 2s)$.

Solution:

$(x - 2s) = 10.53 - (2)(0.72) = 9.09$

Exercise:

Problem: Find the values that are 1.5 standard deviations **from** (below and above) the mean.

Solution:

- $(x - 1.5s) = 10.53 - (1.5)(0.72) = 9.45$
- $(x + 1.5s) = 10.53 + (1.5)(0.72) = 11.61$

Explanation of the standard deviation calculation shown in the table
The deviations show how spread out the data are about the mean. The data value 11.5 is farther from the mean than is the data value 11. The deviations 0.97 and 0.47 indicate that. A positive deviation occurs when the data value is greater than the mean. A negative deviation occurs when the data value is less than the mean; the deviation is -1.525 for the data value 9. **If you add the deviations, the sum is always zero**. (For this example, there are n=20 deviations.) So you cannot simply add the deviations to get the spread of the data. By squaring the deviations, you make them positive numbers, and the sum will also be positive. The variance, then, is the average squared deviation.

The variance is a squared measure and does not have the same units as the data. Taking the square root solves the problem. The standard deviation measures the spread in the same units as the data.

Notice that instead of dividing by n=20, the calculation divided by n-1=20-1=19 because the data is a sample. For the **sample** variance, we divide by the sample size minus one (n-1). Why not divide by n? The answer has to do with the population variance. **The sample variance is an estimate of the population variance.** Based on the theoretical mathematics that lies behind these calculations, dividing by (n-1) gives a better estimate of the population variance.

Note: Your concentration should be on what the standard deviation tells us about the data. The standard deviation is a number which measures how far the data are spread from the mean. Let a calculator or computer do the arithmetic.

The standard deviation, s or σ, is either zero or larger than zero. When the standard deviation is 0, there is no spread; that is, the all the data values are equal to each other. The standard deviation is small when the data are all concentrated close to the mean, and is larger when the data values show more variation from the mean. When the standard deviation is a lot larger than zero, the data values are very spread out about the mean; outliers can make s or σ very large.

The standard deviation, when first presented, can seem unclear. By graphing your data, you can get a better "feel" for the deviations and the standard deviation. You will find that in symmetrical distributions, the standard deviation can be very helpful but in skewed distributions, the standard deviation may not be much help. The reason is that the two sides of a skewed distribution have different spreads. In a skewed distribution, it is better to look at the first quartile, the median, the third quartile, the smallest value, and the largest value. Because numbers can be confusing, **always graph your data**.

Note: The formula for the standard deviation is at the end of the chapter.

Example:
Exercise:

Problem: Use the following data (first exam scores) from Susan Dean's spring pre-calculus class:

3342494953555561 6367686869697273 7478808388888890 929494949496100

- **a**Create a chart containing the data, frequencies, relative frequencies, and cumulative relative frequencies to three decimal places.
- **b**Calculate the following to one decimal place using a TI-83+ or TI-84 calculator:
    - **i**The sample mean
    - **ii**The sample standard deviation
    - **iii**The median
    - **iv**The first quartile
    - **v**The third quartile
    - **vi**IQR
- **c**Construct a box plot and a histogram on the same set of axes. Make comments about the box plot, the histogram, and the chart.

Solution:

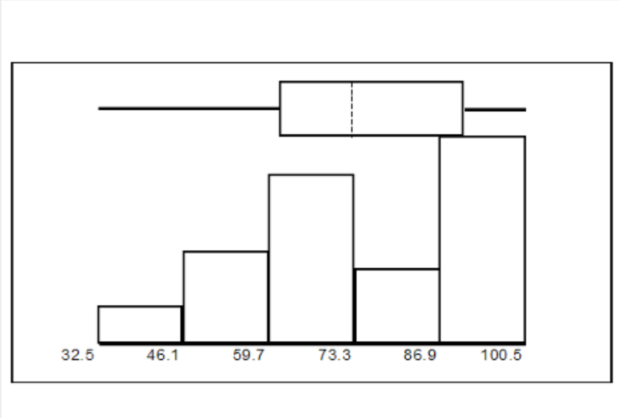
- **a**

Data	**Frequency**	**Relative Frequency**	**Cumulative Relative Frequency**
33	1	0.032	0.032
42	1	0.032	0.064
49	2	0.065	0.129
53	1	0.032	0.161
55	2	0.065	0.226
61	1	0.032	0.258
63	1	0.032	0.29
67	1	0.032	0.322
68	2	0.065	0.387
69	2	0.065	0.452
72	1	0.032	0.484
73	1	0.032	0.516
74	1	0.032	0.548
78	1	0.032	0.580
80	1	0.032	0.612

Data	Frequency	Relative Frequency	Cumulative Relative Frequency
83	1	0.032	0.644
88	3	0.097	0.741
90	1	0.032	0.773
92	1	0.032	0.805
94	4	0.129	0.934
96	1	0.032	0.966
100	1	0.032	**0.998** (Why isn't this value 1?)

- **b**
  - **i**The sample mean = 73.5
  - **ii**The sample standard deviation = 17.9
  - **iii**The median = 73
  - **iv**The first quartile = 61
  - **v**The third quartile = 90
  - **vi**IQR = 90 - 61 = 29

- **c**The x-axis goes from 32.5 to 100.5; y-axis goes from -2.4 to 15 for the histogram; number of intervals is 5 for the histogram so the width of an interval is (100.5 - 32.5) divided by 5 which is equal to 13.6. Endpoints of the intervals: starting point is 32.5, 32.5+13.6 = 46.1, 46.1+13.6 = 59.7, 59.7+13.6 = 73.3, 73.3+13.6 = 86.9, 86.9+13.6 = 100.5 = the ending value; No data values fall on an interval boundary.



The long left whisker in the box plot is reflected in the left side of the histogram. The spread of the exam scores in the lower 50% is greater (73 - 33 = 40) than the spread in the upper 50% (100 - 73 = 27). The histogram, box plot, and chart all reflect this. There are a substantial number of A and B grades (80s, 90s, and 100). The histogram clearly shows this. The box plot shows us that the middle 50% of the exam scores (IQR = 29) are Ds, Cs, and Bs. The box plot also shows us that the lower 25% of the exam scores are Ds and Fs.

Comparing Values from Different Data Sets
The standard deviation is useful when comparing data values that come from different data sets. If the data sets have different means and standard deviations, it can be misleading to compare the data values directly.

- For each data value, calculate how many standard deviations the value is away from its mean.
- Use the formula: value = mean + (#ofSTDEVs)(standard deviation); solve for #ofSTDEVs.
- $\#\text{ofSTDEVs} = \frac{\text{value}-\text{mean}}{\text{standard deviation}}$
- Compare the results of this calculation.

#ofSTDEVs is often called a "z-score"; we can use the symbol z. In symbols, the formulas become:

Sample	x = x + z s	$z = \frac{x-x}{s}$
Population	x = μ + z σ	$z = \frac{x-\mu}{\sigma}$

Example:
Exercise:

Problem:

Two students, John and Ali, from different high schools, wanted to find out who had the highest G.P.A. when compared to his school. Which student had the highest G.P.A. when compared to his school?

Student	GPA	School Mean GPA	School Standard Deviation
John	2.85	3.0	0.7
Ali	77	80	10

Solution:

For each student, determine how many standard deviations (#ofSTDEVs) his GPA is away from the average, for his school. Pay careful attention to signs when comparing and interpreting the answer.

$\#\text{ofSTDEVs} = \frac{\text{value}-\text{mean}}{\text{standard deviation}}$; $z = \frac{x-\mu}{\sigma}$

For John, $z = \#\text{ofSTDEVs} = \frac{2.85-3.0}{0.7} = -0.21$

For Ali, $z = \#\text{ofSTDEVs} = \frac{77-80}{10} = -0.3$

John has the better G.P.A. when compared to his school because his G.P.A. is 0.21 standard deviations **below** his school's mean while Ali's G.P.A. is 0.3 standard deviations **below** his school's mean.

John's z-score of -0.21 is higher than Ali's z-score of -0.3 . For GPA, higher values are better, so we conclude that John has the better GPA when compared to his school.

The following lists give a few facts that provide a little more insight into what the standard deviation tells us about the distribution of the data.

For ANY data set, no matter what the distribution of the data is:

- At least 75% of the data is within 2 standard deviations of the mean.
- At least 89% of the data is within 3 standard deviations of the mean.
- At least 95% of the data is within 4 1/2 standard deviations of the mean.
- This is known as Chebyshev's Rule.

For data having a distribution that is MOUND-SHAPED and SYMMETRIC:

- Approximately 68% of the data is within 1 standard deviation of the mean.
- Approximately 95% of the data is within 2 standard deviations of the mean.
- More than 99% of the data is within 3 standard deviations of the mean.
- This is known as the Empirical Rule.
- It is important to note that this rule only applies when the shape of the distribution of the data is mound-shaped and symmetric. We will learn more about this when studying the "Normal" or "Gaussian" probability distribution in later chapters.

**With contributions from Roberta Bloom

Glossary

Standard Deviation
: A number that is equal to the square root of the variance and measures how far data values are from their mean. Notation: s for sample standard deviation and σ for population standard deviation.

Variance
: Mean of the squared deviations from the mean. Square of the standard deviation. For a set of data, a deviation can be represented as $x - x$ where x is a value of the data and x is the sample mean. The sample variance is equal to the sum of the squares of the deviations divided by the difference of the sample size and 1.

Summary of Formulas

A summary of useful formulas used in examining descriptive statistics

Commonly Used Symbols

- The symbol Σ means to add or to find the sum.
- n = the number of data values in a sample
- N = the number of people, things, etc. in the population
- x = the sample mean
- s = the sample standard deviation
- μ = the population mean
- σ = the population standard deviation
- f = frequency
- x = numerical value

Commonly Used Expressions

- $x*f$ = A value multiplied by its respective frequency
- $\sum x$ = The sum of the values
- $\sum x*f$ = The sum of values multiplied by their respective frequencies
- $(x-x)$ or $(x-\mu)$ = Deviations from the mean (how far a value is from the mean)
- $(x-x)^2$ or $(x-\mu)^2$ = Deviations squared
- $f(x-x)^2$ or $f(x-\mu)^2$ = The deviations squared and multiplied by their frequencies

Mean Formulas:

- $x=\frac{\sum x}{n}$ or $x=\frac{\sum f\cdot x}{n}$
- $\mu=\frac{\sum x}{N}$ or μ= $\frac{\sum f\cdot x}{N}$

Standard Deviation Formulas:

- $s=\sqrt{\frac{\Sigma(x-x)^2}{n-1}}$ or $s=\sqrt{\frac{\Sigma f\cdot(x-x)^2}{n-1}}$
- $\sigma=\sqrt{\frac{\Sigma(x-\mu)^2}{N}}$ or $\sigma=\sqrt{\frac{\Sigma f\cdot(x-\mu)^2}{N}}$

Formulas Relating a Value, the Mean, and the Standard Deviation:

- value = mean + (#ofSTDEVs)(standard deviation), where #ofSTDEVs = the number of standard deviations
- x = x+ (#ofSTDEVs)(s)
- x = μ + (#ofSTDEVs)(σ)

Practice 1: Center of the Data
This module provides students with opportunities to apply concepts related to descriptive statistics. Students are asked to take a set of sample data and calculate a series of statistical values for that data.

Student Learning Outcomes

- The student will calculate and interpret the center, spread, and location of the data.
- The student will construct and interpret histograms an box plots.

Given

Sixty-five randomly selected car salespersons were asked the number of cars they generally sell in one week. Fourteen people answered that they generally sell three cars; nineteen generally sell four cars; twelve generally sell five cars; nine generally sell six cars; eleven generally sell seven cars.

Complete the Table

Data Value (# cars)	**Frequency**	**Relative Frequency**	**Cumulative Relative Frequency**

Discussion Questions

Exercise:

Problem: What does the frequency column sum to? Why?

Solution:

65

Exercise:

Problem: What does the relative frequency column sum to? Why?

Solution:

1

Exercise:

Problem:

What is the difference between relative frequency and frequency for each data value?

Exercise:

Problem:

What is the difference between cumulative relative frequency and relative frequency for each data value?

Enter the Data

Enter your data into your calculator or computer.

Construct a Histogram

Determine appropriate minimum and maximum x and y values and the scaling. Sketch the histogram below. Label the horizontal and vertical axes with words. Include numerical scaling.

Data Statistics

Calculate the following values:

Exercise:

Problem: Sample mean = x =

Solution:

4.75

Exercise:

Problem: Sample standard deviation = s_x =

Solution:

1.39

Exercise:

Problem: Sample size = n =

Solution:

65

Calculations

Use the table in section 2.11.3 to calculate the following values:

Exercise:

Problem: Median =

Solution:

4

Exercise:

Problem: Mode =

Solution:

4

Exercise:

Problem: First quartile =

Solution:

4

Exercise:

Problem: Second quartile = median = 50th percentile =

Solution:

4

Exercise:

Problem: Third quartile =

Solution:

6

Exercise:

Problem: Interquartile range (IQR) = _____ - _____ = _____

Solution:

$6 - 4 = 2$

Exercise:

Problem: 10th percentile =

Solution:

3

Exercise:

Problem: 70th percentile =

Solution:

6

Exercise:

Problem: Find the value that is 3 standard deviations:

- **a**Above the mean
- **b**Below the mean

Solution:

- **a**8.93

- **b**0.58

Box Plot

Construct a box plot below. Use a ruler to measure and scale accurately.

Interpretation

Looking at your box plot, does it appear that the data are concentrated together, spread out evenly, or concentrated in some areas, but not in others? How can you tell?

Practice 2: Spread of the Data
Practice exercise for Descriptive Statistics

Student Learning Outcomes

- The student will calculate measures of the center of the data.
- The student will calculate the spread of the data.

Given

The population parameters below describe the full-time equivalent number of students (FTES) each year at Lake Tahoe Community College from 1976-77 through 2004-2005. (*Source: Graphically Speaking by Bill King, LTCC Institutional Research, December 2005*).

Use these values to answer the following questions:

- μ = 1000 FTES
- Median = 1014 FTES
- σ = 474 FTES
- First quartile = 528.5 FTES
- Third quartile = 1447.5 FTES
- n = 29 years

Calculate the Values

Exercise:

Problem:

A sample of 11 years is taken. About how many are expected to have a FTES of 1014 or above? Explain how you determined your answer.

Solution:

6

Exercise:

Problem: 75% of all years have a FTES:

- **a**At or below:
- **b**At or above:

Solution:

- **a**1447.5
- **b**528.5

Exercise:

Problem: The population standard deviation =

Solution:

474 FTES

Exercise:

Problem:

What percent of the FTES were from 528.5 to 1447.5? How do you know?

Solution:

50%

Exercise:

Problem: What is the IQR? What does the IQR represent?

Solution:

919

Exercise:

Problem:

How many standard deviations away from the mean is the median?

Solution:

0.03

Additional Information: The population FTES for 2005-2006 through 2010-2011 was given in an updated report. (Source: http://www.ltcc.edu/data/ResourcePDF/LTCC_FactBook_2010-11.pdf). The data are reported here.

Year	2005-06	2006-07	2007-08	2008-09	2009-10	2010-11
Total FTES	1585	1690	1735	1935	2021	1890

Exercise:

Problem:

Calculate the mean, median, standard deviation, first quartile, the third quartile and the IQR. Round to one decimal place.

Solution:

mean = 1809.3
median = 1812.5
standard deviation = 151.2
First quartile = 1690

Third quartile = 1935
IQR = 245

Exercise:

Problem:

Construct a boxplot for the FTES for 2005-2006 through 2010-2011 and a boxplot for the FTES for 1976-1977 through 2004-2005.

Exercise:

Problem:

Compare the IQR for the FTES for 1976-77 through 2004-2005 with the IQR for the FTES for 2005-2006 through 2010-2011. Why do you suppose the IQRs are so different?

Solution:

Hint: Think about the number of years covered by each time period and what happened to higher education during those periods.

Exercises
Descriptive Statistics: Homework is part of the collection col10555 written by Barbara Illowsky and Susan Dean and provides homework questions related to lessons about descriptive statistics.

Exercise:

Problem:

Twenty-five randomly selected students were asked the number of movies they watched the previous week. The results are as follows:

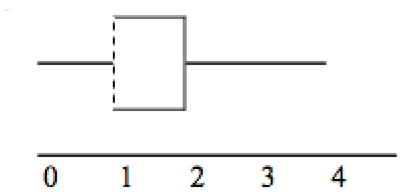
# of movies	**Frequency**	**Relative Frequency**	**Cumulative Relative Frequency**
0	5		
1	9		
2	6		
3	4		
4	1		

- **a**Find the sample mean x
- **b**Find the sample standard deviation, s
- **c**Construct a histogram of the data.
- **d**Complete the columns of the chart.
- **e**Find the first quartile.
- **f**Find the median.
- **g**Find the third quartile.
- **h**Construct a box plot of the data.

- **i**What percent of the students saw fewer than three movies?
- **j**Find the 40th percentile.
- **k**Find the 90th percentile.
- **l**Construct a line graph of the data.
- **m**Construct a stem plot of the data.

Solution:

- **a**1.48
- **b**1.12
- **e**1
- **f**1
- **g**2
- **h**



- **i**80%
- **j**1
- **k**3

Exercise:

Problem:

The median age for U.S. blacks currently is 30.9 years; for U.S. whites it is 42.3 years. ((*Source: http://www.usatoday.com/news/nation/story/2012-05-17/minority-births-census/55029100/1)*)

- **a**Based upon this information, give two reasons why the black median age could be lower than the white median age.
- **b**Does the lower median age for blacks necessarily mean that blacks die younger than whites? Why or why not?

- **c**How might it be possible for blacks and whites to die at approximately the same age, but for the median age for whites to be higher?

Exercise:

Problem:

Forty randomly selected students were asked the number of pairs of sneakers they owned. Let X = the number of pairs of sneakers owned. The results are as follows:

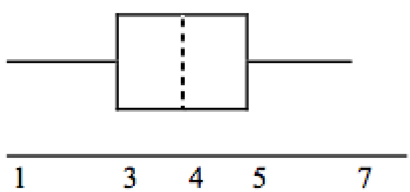
X	Frequency	Relative Frequency	Cumulative Relative Frequency
1	2		
2	5		
3	8		
4	12		
5	12		
7	1		

- **a**Find the sample mean x
- **b**Find the sample standard deviation, s
- **c**Construct a histogram of the data.
- **d**Complete the columns of the chart.
- **e**Find the first quartile.

- **f**Find the median.
- **g**Find the third quartile.
- **h**Construct a box plot of the data.
- **i**What percent of the students owned at least five pairs?
- **j**Find the 40th percentile.
- **k**Find the 90th percentile.
- **l**Construct a line graph of the data
- **m**Construct a stem plot of the data

Solution:

- **a**3.78
- **b**1.29
- **e**3
- **f**4
- **g**5
- **h**



- **i**32.5%
- **j**4
- **k**5

Exercise:

Problem:

600 adult Americans were asked by telephone poll, What do you think constitutes a middle-class income? The results are below. Also, include left endpoint, but not the right endpoint. (*Source: Time magazine; survey by Yankelovich Partners, Inc.*)

Note:"Not sure" answers were omitted from the results.

Salary ($)	Relative Frequency
< 20,000	0.02
20,000 - 25,000	0.09
25,000 - 30,000	0.19
30,000 - 40,000	0.26
40,000 - 50,000	0.18
50,000 - 75,000	0.17
75,000 - 99,999	0.02
100,000+	0.01

- **a**What percent of the survey answered "not sure" ?
- **b**What percent think that middle-class is from $25,000 - $50,000 ?
- **c**Construct a histogram of the data

  1. **i**Should all bars have the same width, based on the data? Why or why not?
  2. **ii**How should the <20,000 and the 100,000+ intervals be handled? Why?

- **d**Find the 40th and 80th percentiles
- **e**Construct a bar graph of the data

Exercise:

Problem:

Following are the published weights (in pounds) of all of the team members of the San Francisco 49ers from a previous year (*Source: San Jose Mercury News*)

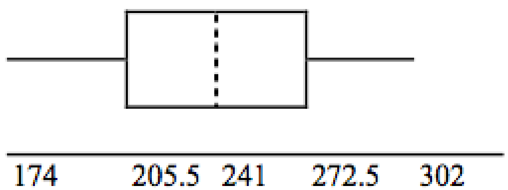
177 205 210 210 232 205 185 185 178 210 206 212 184 174 185 242 188 212 215 247 241 223 220 260 245 259 278 270 280 295 275 285 290 272 273 280 285 286 200 215 185 230 250 241 190 260 250 302 265 290 276 228 265

- **a**Organize the data from smallest to largest value.
- **b**Find the median.
- **c**Find the first quartile.
- **d**Find the third quartile.
- **e**Construct a box plot of the data.
- **f**The middle 50% of the weights are from _______ to _______.
- **g**If our population were all professional football players, would the above data be a sample of weights or the population of weights? Why?
- **h**If our population were the San Francisco 49ers, would the above data be a sample of weights or the population of weights? Why?
- **i**Assume the population was the San Francisco 49ers. Find:
  - **i**the population mean, μ.
  - **ii**the population standard deviation, σ.
  - **iii**the weight that is 2 standard deviations below the mean.
  - **iv**When Steve Young, quarterback, played football, he weighed 205 pounds. How many standard deviations above or below the mean was he?
- **j**That same year, the mean weight for the Dallas Cowboys was 240.08 pounds with a standard deviation of 44.38 pounds. Emmit Smith weighed in at 209 pounds. With respect to his team, who

was lighter, Smith or Young? How did you determine your answer?

Solution:

- **b**241
- **c**205.5
- **d**272.5
- **e**



- **f**205.5, 272.5
- **g**sample
- **h**population
- **i**
  - **i**236.34
  - **ii**37.50
  - **iii**161.34
  - **iv**0.84 std. dev. below the mean
- **j**Young

Exercise:

Problem:

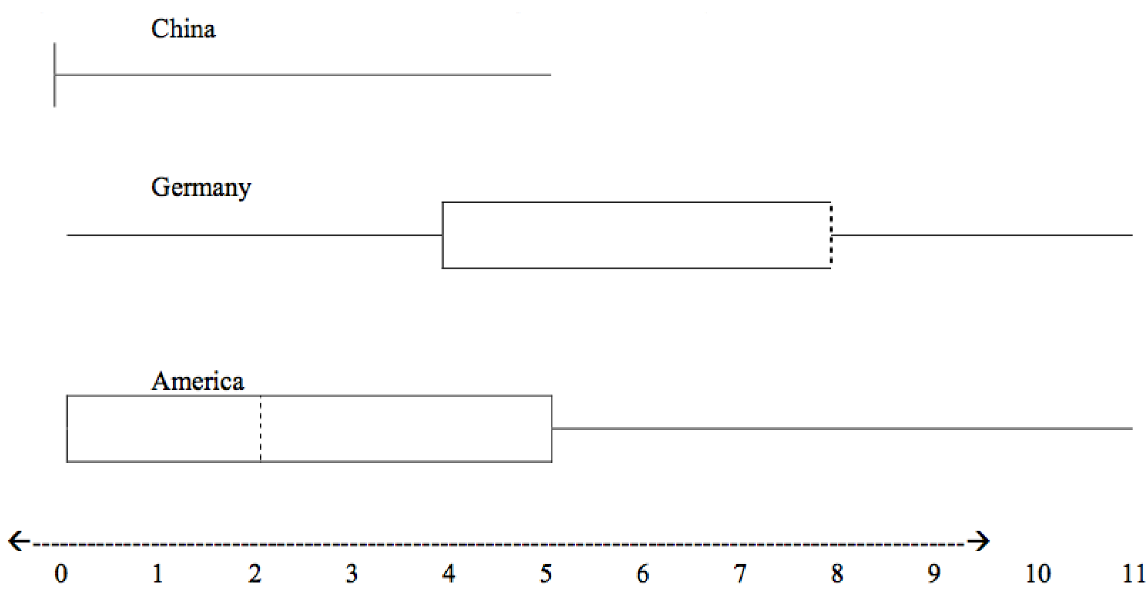
An elementary school class ran 1 mile with a mean of 11 minutes and a standard deviation of 3 minutes. Rachel, a student in the class, ran 1 mile in 8 minutes. A junior high school class ran 1 mile with a mean of 9 minutes and a standard deviation of 2 minutes. Kenji, a student in the class, ran 1 mile in 8.5 minutes. A high school class ran 1 mile with a mean of 7 minutes and a standard deviation of 4 minutes. Nedda, a student in the class, ran 1 mile in 8 minutes.

- **a**Why is Kenji considered a better runner than Nedda, even though Nedda ran faster than he?
- **b**Who is the fastest runner with respect to his or her class? Explain why.

Exercise:

Problem:

In a survey of 20 year olds in China, Germany and America, people were asked the number of foreign countries they had visited in their lifetime. The following box plots display the results.



- **a**In complete sentences, describe what the shape of each box plot implies about the distribution of the data collected.
- **b**Explain how it is possible that more Americans than Germans surveyed have been to over eight foreign countries.
- **c**Compare the three box plots. What do they imply about the foreign travel of twenty year old residents of the three countries when compared to each other?

Exercise:

Problem:

One hundred teachers attended a seminar on mathematical problem solving. The attitudes of a representative sample of 12 of the teachers were measured before and after the seminar. A positive number for change in attitude indicates that a teacher's attitude toward math became more positive. The twelve change scores are as follows:

3 8 -1 2 0 5 -3 1 -1 6 5 -2

- **a**What is the mean change score?
- **b**What is the standard deviation for this population?
- **c**What is the median change score?
- **d**Find the change score that is 2.2 standard deviations below the mean.

Exercise:

Problem:

Three students were applying to the same graduate school. They came from schools with different grading systems. Which student had the best G.P.A. when compared to his school? Explain how you determined your answer.

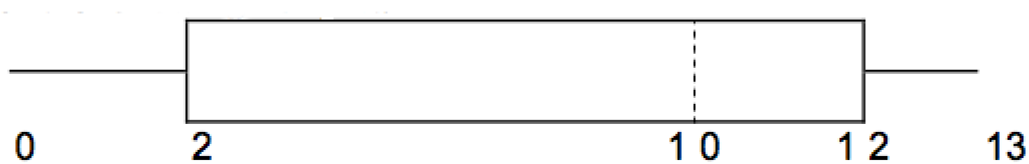
Student	G.P.A.	School Ave. G.P.A.	School Standard Deviation
Thuy	2.7	3.2	0.8
Vichet	87	75	20
Kamala	8.6	8	0.4

Solution:

Kamala

Exercise:

Problem: Given the following box plot:

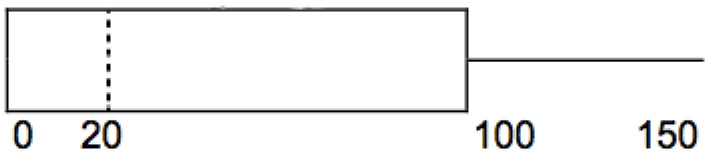


- **a**Which quarter has the smallest spread of data? What is that spread?
- **b**Which quarter has the largest spread of data? What is that spread?
- **c**Find the Inter Quartile Range (IQR).
- **d**Are there more data in the interval 5 - 10 or in the interval 10 - 13? How do you know this?
- **e**Which interval has the fewest data in it? How do you know this?

  - **I** 0-2
  - **II**2-4
  - **III**10-12
  - **IV**12-13

Exercise:

Problem: Given the following box plot:



- **a**Think of an example (in words) where the data might fit into the above box plot. In 2-5 sentences, write down the example.
- **b**What does it mean to have the first and second quartiles so close together, while the second to fourth quartiles are far apart?

Exercise:

Problem:

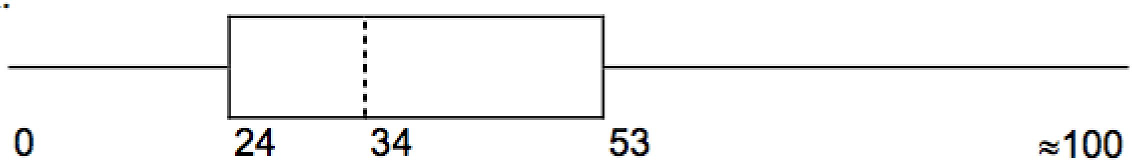
Santa Clara County, CA, has approximately 27,873 Japanese-Americans. Their ages are as follows. (*Source: West magazine*)

Age Group	**Percent of Community**
0-17	18.9
18-24	8.0
25-34	22.8
35-44	15.0
45-54	13.1

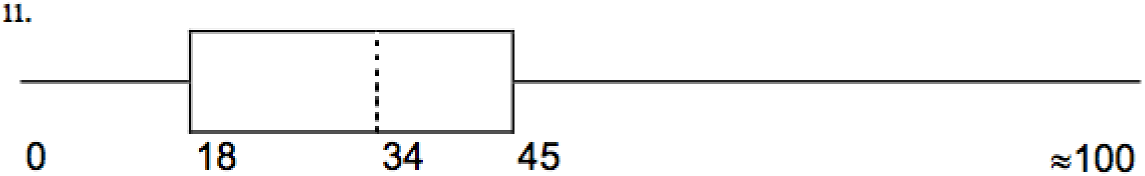
Age Group	Percent of Community
55-64	11.9
65+	10.3

- **a**Construct a histogram of the Japanese-American community in Santa Clara County, CA. The bars will **not** be the same width for this example. Why not?
- **b**What percent of the community is under age 35?
- **c**Which box plot most resembles the information above?

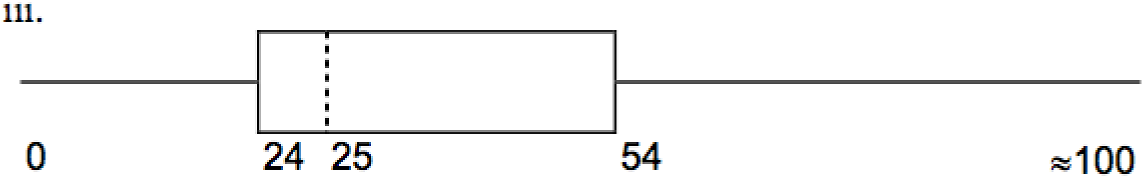
i.



ii.



iii.



Exercise:

Problem:

Suppose that three book publishers were interested in the number of fiction paperbacks adult consumers purchase per month. Each publisher conducted a survey. In the survey, each asked adult consumers the number of fiction paperbacks they had purchased the previous month. The results are below.

# of books	Freq.	Rel. Freq.
0	10	
1	12	
2	16	
3	12	
4	8	
5	6	
6	2	
8	2	

Publisher A

# of books	Freq.	Rel. Freq.
0	18	
1	24	
2	24	
3	22	
4	15	

# of books	Freq.	Rel. Freq.
5	10	
7	5	
9	1	

Publisher B

# of books	Freq.	Rel. Freq.
0-1	20	
2-3	35	
4-5	12	
6-7	2	
8-9	1	

Publisher C

- **a**Find the relative frequencies for each survey. Write them in the charts.
- **b**Using either a graphing calculator, computer, or by hand, use the frequency column to construct a histogram for each publisher's survey. For Publishers A and B, make bar widths of 1. For Publisher C, make bar widths of 2.
- **c**In complete sentences, give two reasons why the graphs for Publishers A and B are not identical.

- **d**Would you have expected the graph for Publisher C to look like the other two graphs? Why or why not?
- **e**Make new histograms for Publisher A and Publisher B. This time, make bar widths of 2.
- **f**Now, compare the graph for Publisher C to the new graphs for Publishers A and B. Are the graphs more similar or more different? Explain your answer.

Exercise:

Problem:

Often, cruise ships conduct all on-board transactions, with the exception of gambling, on a cashless basis. At the end of the cruise, guests pay one bill that covers all on-board transactions. Suppose that 60 single travelers and 70 couples were surveyed as to their on-board bills for a seven-day cruise from Los Angeles to the Mexican Riviera. Below is a summary of the bills for each group.

Amount($)	**Frequency**	**Rel. Frequency**
51-100	5	
101-150	10	
151-200	15	
201-250	15	
251-300	10	
301-350	5	

Singles

Amount($)	**Frequency**	**Rel. Frequency**
100-150	5	
201-250	5	
251-300	5	
301-350	5	
351-400	10	
401-450	10	
451-500	10	
501-550	10	
551-600	5	
601-650	5	

Couples

- **a**Fill in the relative frequency for each group.
- **b**Construct a histogram for the Singles group. Scale the x-axis by $50. widths. Use relative frequency on the y-axis.
- **c**Construct a histogram for the Couples group. Scale the x-axis by $50. Use relative frequency on the y-axis.
- **d**Compare the two graphs:

- **i**List two similarities between the graphs.
- **ii**List two differences between the graphs.
- **iii**Overall, are the graphs more similar or different?

- **e**Construct a new graph for the Couples by hand. Since each couple is paying for two individuals, instead of scaling the x-axis by $50, scale it by $100. Use relative frequency on the y-axis.
- **f**Compare the graph for the Singles with the new graph for the Couples:

  - **i**List two similarities between the graphs.
  - **ii**Overall, are the graphs more similar or different?

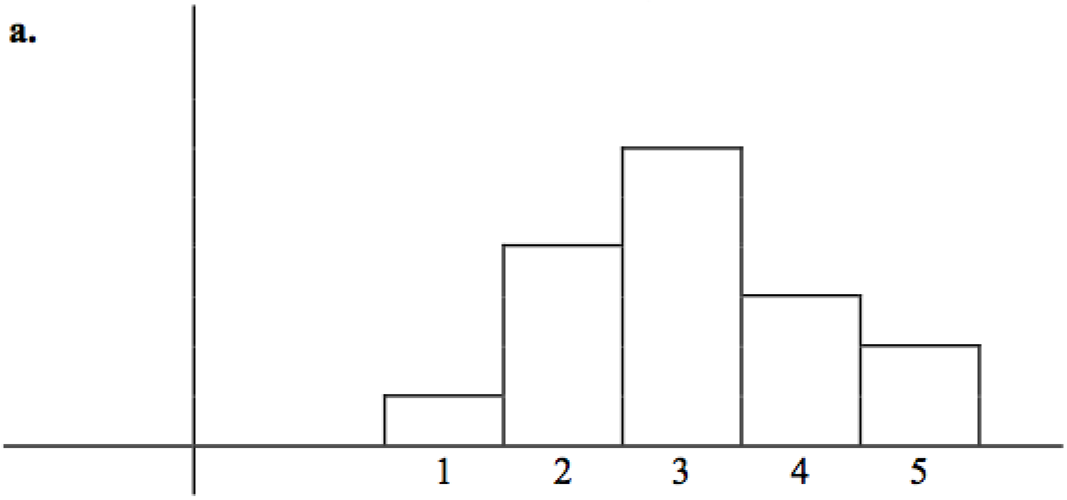
- **i**By scaling the Couples graph differently, how did it change the way you compared it to the Singles?
- **j**Based on the graphs, do you think that individuals spend the same amount, more or less, as singles as they do person by person in a couple? Explain why in one or two complete sentences.

Exercise:

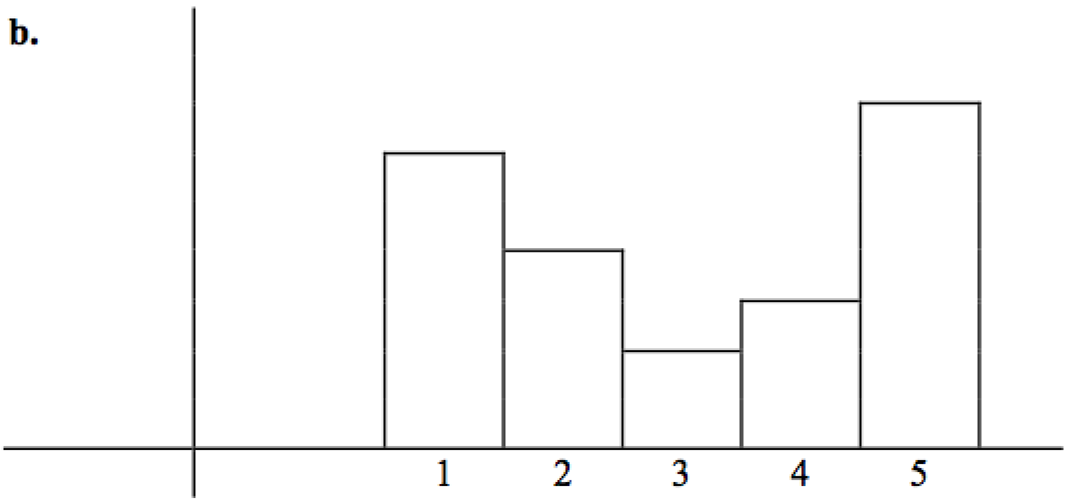
Problem:

Refer to the following histograms and box plot. Determine which of the following are true and which are false. Explain your solution to each part in complete sentences.

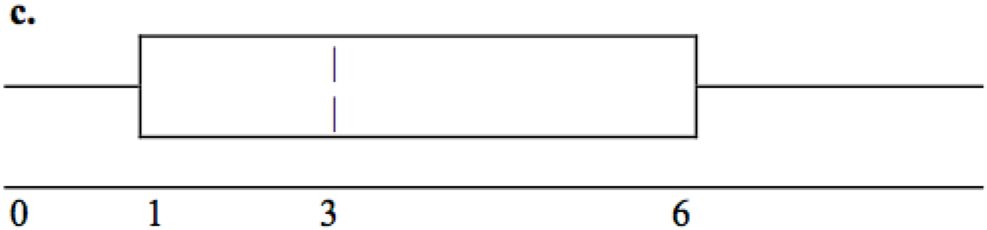
a.



b.



c.



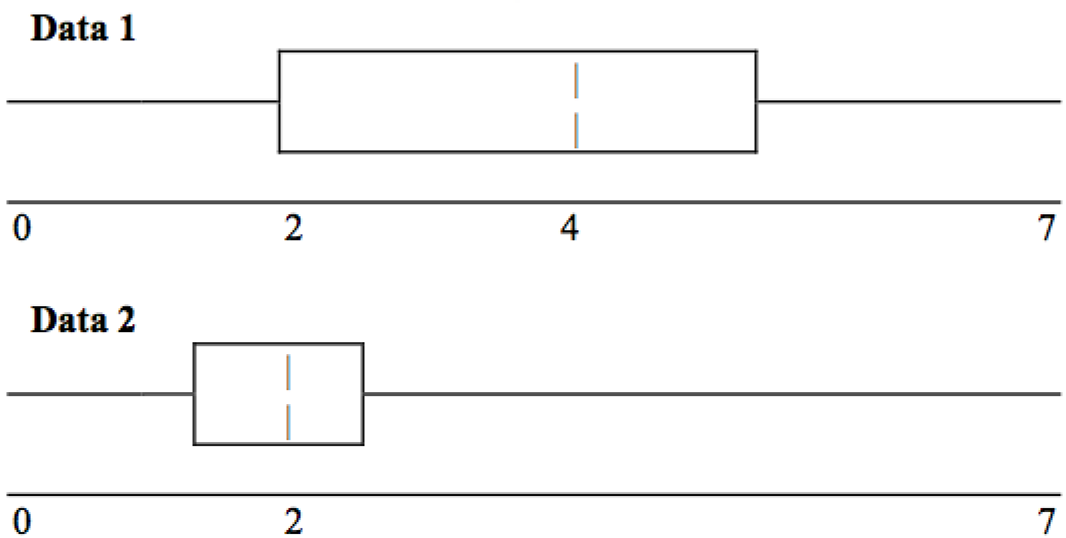
- **a**The medians for all three graphs are the same.
- **b**We cannot determine if any of the means for the three graphs is different.
- **c**The standard deviation for (b) is larger than the standard deviation for (a).
- **d**We cannot determine if any of the third quartiles for the three graphs is different.

Solution:

- **a**True
- **b**True
- **c**True
- **d**False

Exercise:

Problem: Refer to the following box plots.



- **a**In complete sentences, explain why each statement is false.
  - **iData 1** has more data values above 2 than **Data 2** has above 2.
  - **ii**The data sets cannot have the same mode.
  - **iii**For **Data 1**, there are more data values below 4 than there are above 4.
- **b**For which group, Data 1 or Data 2, is the value of “7” more likely to be an outlier? Explain why in complete sentences

Exercise:

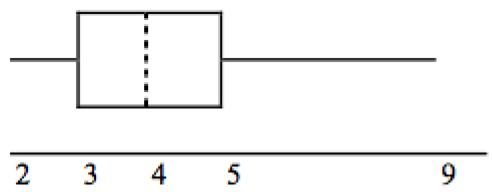
Problem:

In a recent issue of the *IEEE Spectrum*, 84 engineering conferences were announced. Four conferences lasted two days. Thirty-six lasted three days. Eighteen lasted four days. Nineteen lasted five days. Four lasted six days. One lasted seven days. One lasted eight days. One lasted nine days. Let X = the length (in days) of an engineering conference.

- **a**Organize the data in a chart.
- **b**Find the median, the first quartile, and the third quartile.
- **c**Find the 65th percentile.
- **d**Find the 10th percentile.
- **e**Construct a box plot of the data.
- **f**The middle 50% of the conferences last from ________ days to ________ days.
- **g**Calculate the sample mean of days of engineering conferences.
- **h**Calculate the sample standard deviation of days of engineering conferences.
- **i**Find the mode.
- **j**If you were planning an engineering conference, which would you choose as the length of the conference: mean; median; or mode? Explain why you made that choice.
- **k**Give two reasons why you think that 3 - 5 days seem to be popular lengths of engineering conferences.

Solution:

- **b**4,3,5
- **c**4
- **d**3
- **e**



- **f**3,5
- **g**3.94
- **h**1.28
- **i**3
- **j**mode

Exercise:

Problem:

A survey of enrollment at 35 community colleges across the United States yielded the following figures (*source: Microsoft Bookshelf*):

6414 1550 2109 9350 21828 4300 5944 5722 2825 2044 5481 5200 5853 2750 10012 6357 27000 9414 7681 3200 17500 9200 7380 18314 6557 13713 17768 7493 2771 2861 1263 7285 28165 5080 11622

- **a**Organize the data into a chart with five intervals of equal width. Label the two columns "Enrollment" and "Frequency."
- **b**Construct a histogram of the data.
- **c**If you were to build a new community college, which piece of information would be more valuable: the mode or the mean?
- **d**Calculate the sample mean.
- **e**Calculate the sample standard deviation.
- **f**A school with an enrollment of 8000 would be how many standard deviations away from the mean?

Exercise:

Problem:

The median age of the U.S. population in 1980 was 30.0 years. In 1991, the median age was 33.1 years. (*Source: Bureau of the Census*)

- **a**What does it mean for the median age to rise?
- **b**Give two reasons why the median age could rise.
- **c**For the median age to rise, is the actual number of children less in 1991 than it was in 1980? Why or why not?

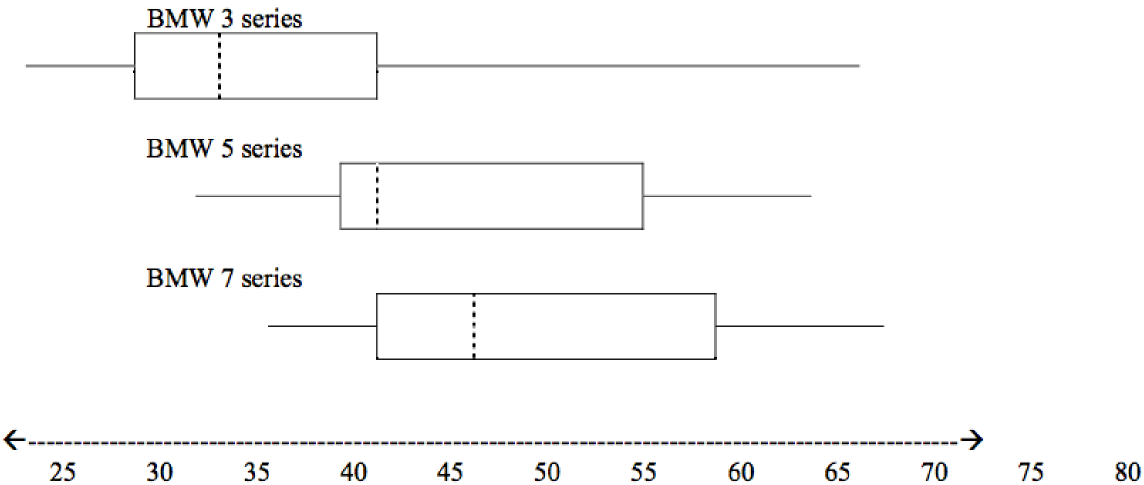
Solution:

- **c**Maybe

Exercise:

Problem:

A survey was conducted of 130 purchasers of new BMW 3 series cars, 130 purchasers of new BMW 5 series cars, and 130 purchasers of new BMW 7 series cars. In it, people were asked the age they were when they purchased their car. The following box plots display the results.



- **a**In complete sentences, describe what the shape of each box plot implies about the distribution of the data collected for that car

series.

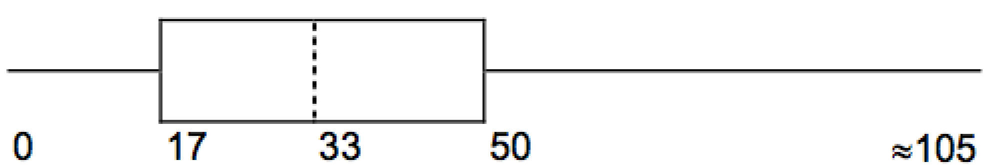
- **b**Which group is most likely to have an outlier? Explain how you determined that.
- **c**Compare the three box plots. What do they imply about the age of purchasing a BMW from the series when compared to each other?
- **d**Look at the BMW 5 series. Which quarter has the smallest spread of data? What is that spread?
- **e**Look at the BMW 5 series. Which quarter has the largest spread of data? What is that spread?
- **f**Look at the BMW 5 series. Estimate the Inter Quartile Range (IQR).
- **g**Look at the BMW 5 series. Are there more data in the interval 31-38 or in the interval 45-55? How do you know this?
- **h**Look at the BMW 5 series. Which interval has the fewest data in it? How do you know this?

    - **i**31-35
    - **ii**38-41
    - **iii**41-64

Exercise:

Problem:

The following box plot shows the U.S. population for 1990, the latest available year. (Source: Bureau of the Census, 1990 Census)



- **a**Are there fewer or more children (age 17 and under) than senior citizens (age 65 and over)? How do you know?
- **b**12.6% are age 65 and over. Approximately what percent of the population are of working age adults (above age 17 to age 65)?

Solution:

- **a**more children
- **b**62.4%

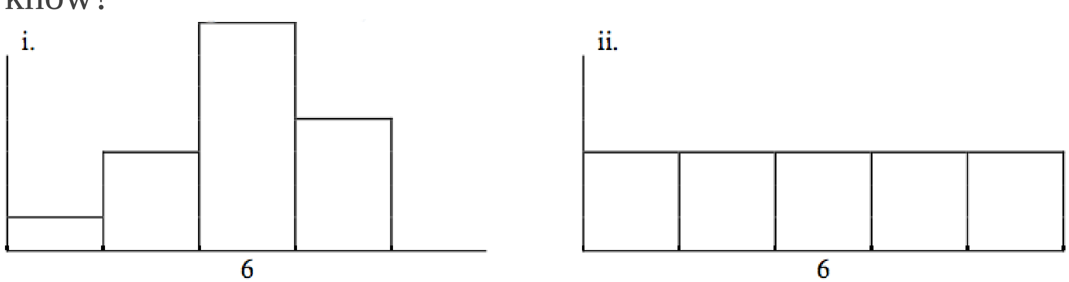
Exercise:

Problem:

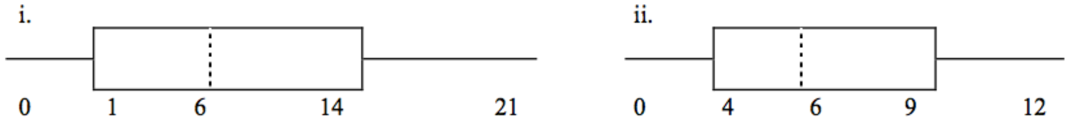
Javier and Ercilia are supervisors at a shopping mall. Each was given the task of estimating the mean distance that shoppers live from the mall. They each randomly surveyed 100 shoppers. The samples yielded the following information:

	Javier	**Ercilla**
x	6.0 miles	6.0 miles
s	4.0 miles	7.0 miles

- **a**How can you determine which survey was correct ?
- **b**Explain what the difference in the results of the surveys implies about the data.
- **c**If the two histograms depict the distribution of values for each supervisor, which one depicts Ercilia's sample? How do you know?



- **d**If the two box plots depict the distribution of values for each supervisor, which one depicts Ercilia's sample? How do you know?



Exercise:

Problem: Student grades on a chemistry exam were:

77, 78, 76, 81, 86, 51, 79, 82, 84, 99

- **a**Construct a stem-and-leaf plot of the data.
- **b**Are there any potential outliers? If so, which scores are they? Why do you consider them outliers?

Solution:

- **b**51,99

Try these multiple choice questions (Exercises 24 - 30).

The next three questions refer to the following information. We are interested in the number of years students in a particular elementary statistics class have lived in California. The information in the following table is from the entire section.

Number of years	Frequency

Number of years	Frequency
7	1
14	3
15	1
18	1
19	4
20	3
22	1
23	1
26	1
40	2
42	2
	Total = 20

Exercise:

Problem: What is the IQR?

- **A**8
- **B**11
- **C**15
- **D**35

Solution:

A

Exercise:

Problem: What is the mode?

- **A**19
- **B**19.5
- **C**14 and 20
- **D**22.65

Solution:

A

Exercise:

Problem: Is this a sample or the entire population?

- **A**sample
- **B**entire population
- **C**neither

Solution:

B

The next two questions refer to the following table. X = the number of days per week that 100 clients use a particular exercise facility.

x	Frequency
0	3
1	12
2	33
3	28
4	11
5	9
6	4

Exercise:

Problem: The 80th percentile is:

- **A**5
- **B**80
- **C**3
- **D**4

Solution:

D

Exercise:

Problem:

The number that is 1.5 standard deviations BELOW the mean is approximately:

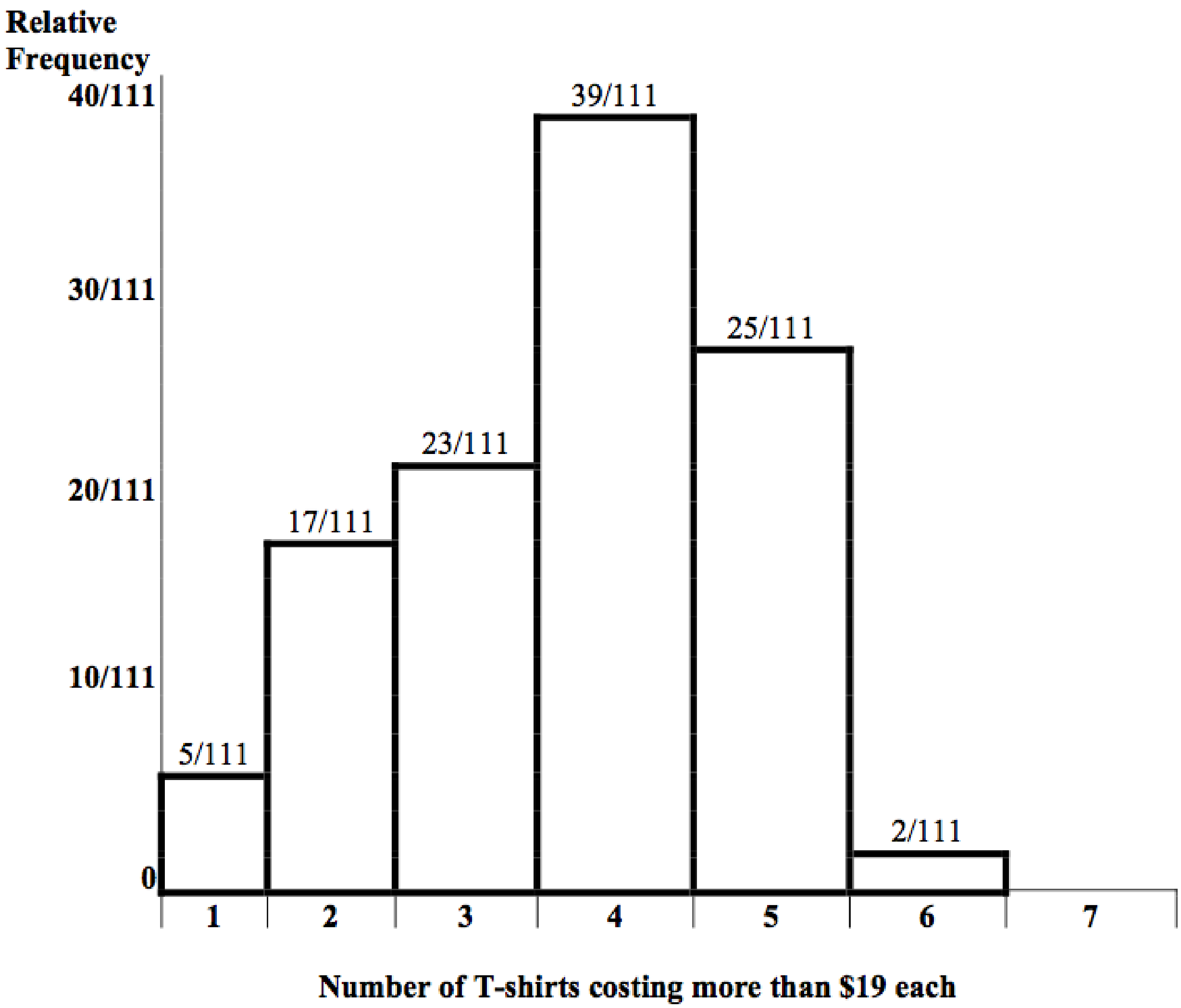
- **A**0.7

- **B**4.8
- **C**-2.8
- **D**Cannot be determined

Solution:

A

The next two questions refer to the following histogram. Suppose one hundred eleven people who shopped in a special T-shirt store were asked the number of T-shirts they own costing more than $19 each.



Exercise:

Problem:

The percent of people that own at most three (3) T-shirts costing more than $19 each is approximately:

- **A**21
- **B**59
- **C**41
- **D**Cannot be determined

Solution:

C

Exercise:

Problem:

If the data were collected by asking the first 111 people who entered the store, then the type of sampling is:

- **A**cluster
- **B**simple random
- **C**stratified
- **D**convenience

Solution:

D

Exercise:

Problem:

Below are the **2010 obesity rates by U.S. states and Washington, DC.**(*Source: http://www.cdc.gov/obesity/data/adult.html)*)

State	Percent (%)	State	Percent (%)
Alabama	32.2	Montana	23.0
Alaska	24.5	Nebraska	26.9
Arizona	24.3	Nevada	22.4
Arkansas	30.1	New Hampshire	25.0
California	24.0	New Jersey	23.8
Colorado	21.0	New Mexico	25.1
Connecticut	22.5	New York	23.9
Delaware	28.0	North Carolina	27.8
Washington, DC	22.2	North Dakota	27.2
Florida	26.6	Ohio	29.2
Georgia	29.6	Oklahoma	30.4
Hawaii	22.7	Oregon	26.8
Idaho	26.5	Pennsylvania	28.6
Illinois	28.2	Rhode Island	25.5
Indiana	29.6	South Carolina	31.5

State	Percent (%)	State	Percent (%)
Iowa	28.4	South Dakota	27.3
Kansas	29.4	Tennessee	30.8
Kentucky	31.3	Texas	31.0
Louisiana	31.0	Utah	22.5
Maine	26.8	Vermont	23.2
Maryland	27.1	Virginia	26.0
Massachusetts	23.0	Washington	25.5
Michigan	30.9	West Virginia	32.5
Minnesota	24.8	Wisconsin	26.3
Mississippi	34.0	Wyoming	25.1
Missouri	30.5		

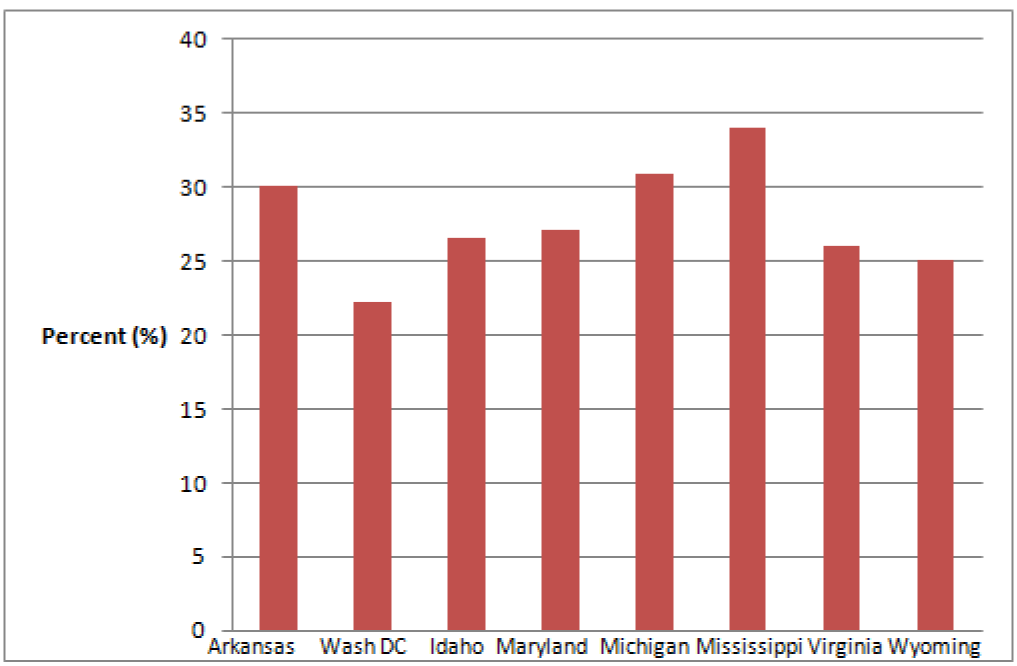
- **a.**Construct a bar graph of obesity rates of your state and the four states closest to your state. Hint: Label the x-axis with the states.
- **b.**Use a random number generator to randomly pick 8 states. Construct a bar graph of the obesity rates of those 8 states.
- **c.**Construct a bar graph for all the states beginning with the letter "A."
- **d.**Construct a bar graph for all the states beginning with the letter "M."

Solution:

Example solution for **b** using the random number generator for the Ti-84 Plus to generate a simple random sample of 8 states. Instructions are below.

- Number the entries in the table 1 - 51 (Includes Washington, DC; Numbered vertically)
- Press MATH
- Arrow over to PRB
- Press 5:randInt(
- Enter 51,1,8)

Eight numbers are generated (use the right arrow key to scroll through the numbers). The numbers correspond to the numbered states (for this example: {47 21 9 23 51 13 25 4}. If any numbers are repeated, generate a different number by using 5:randInt(51,1)). Here, the states (and Washington DC) are {Arkansas, Washington DC, Idaho, Maryland, Michigan, Mississippi, Virginia, Wyoming}. Corresponding percents are {28.7 21.8 24.5 26 28.9 32.8 25 24.6}.



Exercise:

Problem:

A music school has budgeted to purchase 3 musical instruments. They plan to purchase a piano costing $3000, a guitar costing $550, and a drum set costing $600. The mean cost for a piano is $4,000 with a standard deviation of $2,500. The mean cost for a guitar is $500 with a standard deviation of $200. The mean cost for drums is $700 with a standard deviation of $100. Which cost is the lowest, when compared to other instruments of the same type? Which cost is the highest when compared to other instruments of the same type. Justify your answer numerically.

Solution:

For pianos, the cost of the piano is 0.4 standard deviations BELOW the mean. For guitars, the cost of the guitar is 0.25 standard deviations ABOVE the mean. For drums, the cost of the drum set is 1.0 standard deviations BELOW the mean. Of the three, the drums cost the lowest in comparison to the cost of other instruments of the same type. The guitar cost the most in comparison to the cost of other instruments of the same type.

Exercise:

Problem:

Suppose that a publisher conducted a survey asking adult consumers the number of fiction paperback books they had purchased in the previous month. The results are summarized in the table below. (Note that this is the data presented for publisher B in homework exercise 13).

# of books	Freq.	Rel. Freq.

# of books	Freq.	Rel. Freq.
0	18	
1	24	
2	24	
3	22	
4	15	
5	10	
7	5	
9	1	

Publisher B

a. Are there any outliers in the data? Use an appropriate numerical test involving the IQR to identify outliers, if any, and clearly state your conclusion.
b. If a data value is identified as an outlier, what should be done about it?
c. Are any data values further than 2 standard deviations away from the mean? In some situations, statisticians may use this criteria to identify data values that are unusual, compared to the other data values. (Note that this criteria is most appropriate to use for data that is mound-shaped and symmetric, rather than for skewed data.)
d. Do parts (a) and (c) of this problem give the same answer?
e. Examine the shape of the data. Which part, (a) or (c), of this question gives a more appropriate result for this data?
f. Based on the shape of the data which is the most appropriate measure of center for this data: mean, median or mode?

Solution:

- IQR = 4 – 1 = 3 ; Q1 – 1.5*IQR = 1 – 1.5(3) = -3.5 ; Q3 + 1.5*IQR = 4 + 1.5(3) = 8.5 ;The data value of 9 is larger than 8.5. The purchase of 9 books in one month is an outlier.
- The outlier should be investigated to see if there is an error or some other problem in the data; then a decision whether to include or exclude it should be made based on the particular situation. If it was a correct value then the data value should remain in the data set. If there is a problem with this data value, then it should be corrected or removed from the data. For example: If the data was recorded incorrectly (perhaps a 9 was miscoded and the correct value was 6) then the data should be corrected. If it was an error but the correct value is not known it should be removed from the data set.
- xbar – 2s = 2.45 – 2*1.88 = -1.31 ; xbar + 2s = 2.45 + 2*1.88 = 6.21 ; Using this method, the five data values of 7 books purchased and the one data value of 9 books purchased would be considered unusual.
- No: part (a) identifies only the value of 9 to be an outlier but part (c) identifies both 7 and 9.
- The data is skewed (to the right). It would be more appropriate to use the method involving the IQR in part (a), identifying only the one value of 9 books purchased as an outlier. Note that part (c) remarks that identifying unusual data values by using the criteria of being further than 2 standard deviations away from the mean is most appropriate when the data are mound-shaped and symmetric.
- The data are skewed to the right. For skewed data it is more appropriate to use the median as a measure of center.

**Exercises 32 and 33 contributed by Roberta Bloom

Introduction
This module provides an introduction of Linear Regression and Correlation as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Discuss basic ideas of linear regression and correlation.
- Create and interpret a line of best fit.
- Calculate and interpret the correlation coefficient.
- Calculate and interpret outliers.

Introduction

Professionals often want to know how two or more numeric variables are related. For example, is there a relationship between the grade on the second math exam a student takes and the grade on the final exam? If there is a relationship, what is it and how strong is the relationship?

In another example, your income may be determined by your education, your profession, your years of experience, and your ability. The amount you pay a repair person for labor is often determined by an initial amount plus an hourly fee. These are all examples in which regression can be used.

The type of data described in the examples is **bivariate** data - "bi" for two variables. In reality, statisticians use **multivariate** data, meaning many variables.

In this chapter, you will be studying the simplest form of regression, "linear regression" with one independent variable (x). This involves data that fits a line in two dimensions. You will also study correlation which measures how strong the relationship is.

Linear Regression and Correlation: Linear Equations

Linear regression for two variables is based on a linear equation with one independent variable. It has the form:

Equation:

$$y=a+\text{bx}$$

where a and b are constant numbers.

x is the independent variable, and y is the dependent variable. Typically, you choose a value to substitute for the independent variable and then solve for the dependent variable.

Example:

The following examples are linear equations.

Equation:

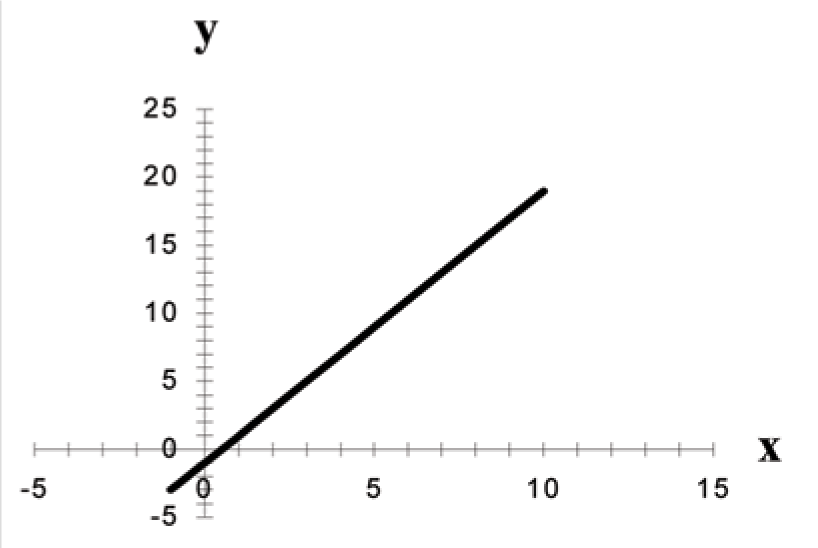
$$y=3+2\text{x}$$

Equation:

$$y=-0.01+1.2\text{x}$$

The graph of a linear equation of the form $y=a+\text{bx}$ is a **straight line**. Any line that is not vertical can be described by this equation.

Example:



Graph of the equation $y = -1 + 2x$.

Linear equations of this form occur in applications of life sciences, social sciences, psychology, business, economics, physical sciences, mathematics, and other areas.

Example:
Aaron's Word Processing Service (AWPS) does word processing. Its rate is \$32 per hour plus a \$31.50 one-time charge. The total cost to a customer depends on the number of hours it takes to do the word processing job.

Exercise:

Problem:

Find the equation that expresses the **total cost** in terms of the **number of hours** required to finish the word processing job.

Solution:

Let x = the number of hours it takes to get the job done.

Let y = the total cost to the customer.

The $31.50 is a fixed cost. If it takes x hours to complete the job, then $(32)(x)$ is the cost of the word processing only. The total cost is:

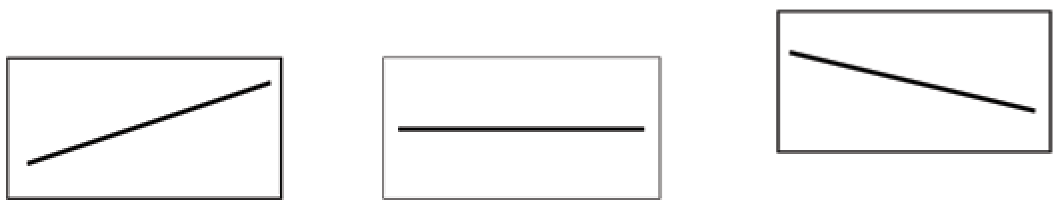
$y = 31.50 + 32\text{x}$

Linear Regression and Correlation: Slope and Y-Intercept of a Linear Equation

For the linear equation $y = a + \mathrm{bx}$, b = slope and a = y-intercept.

From algebra recall that the slope is a number that describes the steepness of a line and the y-intercept is the y coordinate of the point $(0, a)$ where the line crosses the y-axis.

If $b > 0$, the line slopes upward to the right.	If $b = 0$, the line is horizontal.	If $b < 0$, the line slopes downward to the right.



Three possible graphs of $y = a + \mathrm{bx}$.

Example:
Svetlana tutors to make extra money for college. For each tutoring session, she charges a one time fee of $25 plus $15 per hour of tutoring. A linear equation that expresses the total amount of money Svetlana earns for each session she tutors is $y = 25 + 15\mathrm{x}$.

Exercise:

Problem:

What are the independent and dependent variables? What is the y-intercept and what is the slope? Interpret them using complete sentences.

Solution:

The independent variable (x) is the number of hours Svetlana tutors each session. The dependent variable (y) is the amount, in dollars, Svetlana earns for each session.

The y-intercept is 25 ($a = 25$). At the start of the tutoring session, Svetlana charges a one-time fee of $25 (this is when $x = 0$). The slope is 15 ($b = 15$). For each session, Svetlana earns $15 for each hour she tutors.

Scatter Plots
This module provides an overview of Linear Regression and Correlation: Scatter Plots as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

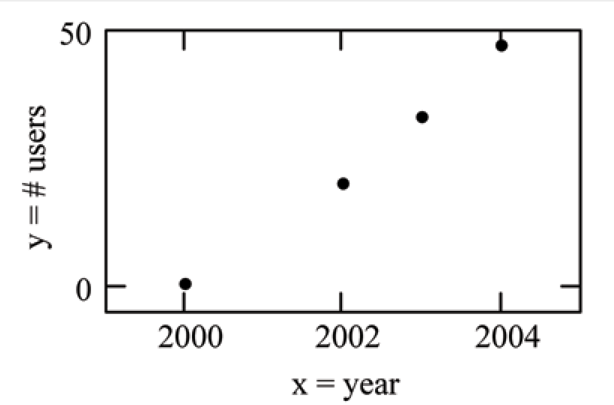
Before we take up the discussion of linear regression and correlation, we need to examine a way to display the relation between two variables x and y. The most common and easiest way is a **scatter plot**. The following example illustrates a scatter plot.

Example:
From an article in the *Wall Street Journal*: In Europe and Asia, m-commerce is popular. M-commerce users have special mobile phones that work like electronic wallets as well as provide phone and Internet services. Users can do everything from paying for parking to buying a TV set or soda from a machine to banking to checking sports scores on the Internet. For the years 2000 through 2004, was there a relationship between the year and the number of m-commerce users? Construct a scatter plot. Let x = the year and let y = the number of m-commerce users, in millions.

Table showing the number of m-commerce users (in millions) by year.

x **(year)**	y **(# of users)**
2000	0.5
2002	20.0

Scatter plot showing the number of m-commerce users (in millions) by year.



x (year)	y (# of users)
2003	33.0
2004	47.0

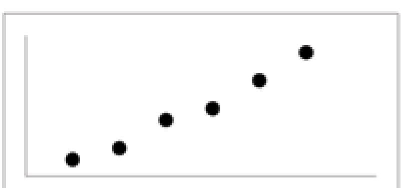
A scatter plot shows the **direction** and **strength** of a relationship between the variables. A clear direction happens when there is either:

- High values of one variable occurring with high values of the other variable or low values of one variable occurring with low values of the other variable.
- High values of one variable occurring with low values of the other variable.

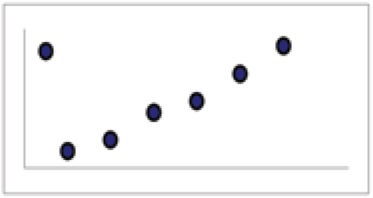
You can determine the strength of the relationship by looking at the scatter plot and seeing how close the points are to a line, a power function, an exponential function, or to some other type of function.

When you look at a scatterplot, you want to notice the **overall pattern** and any **deviations** from the pattern. The following scatterplot examples illustrate these concepts.

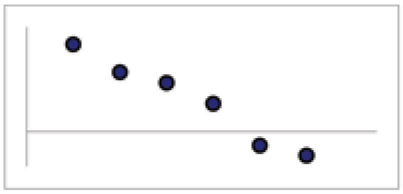
Positive Linear Pattern (Strong)



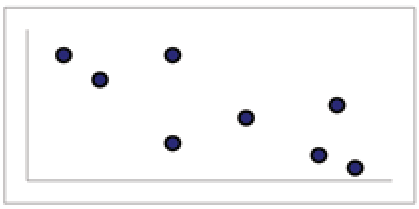
Linear Pattern w/ One Deviation



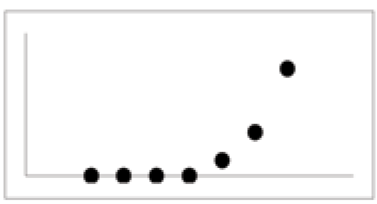
Negative Linear Pattern (Strong)



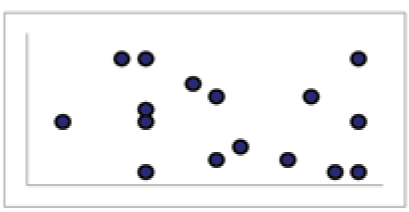
Negative Linear Pattern (Weak)



Exponential Growth Pattern



No Pattern



In this chapter, we are interested in scatter plots that show a linear pattern. Linear patterns are quite common. The linear relationship is strong if the points are close to a straight line. If we think that the points show a linear relationship, we would like to draw a line on the scatter plot. This line can be calculated through a process called **linear regression**. However, we only calculate a regression line if one of the variables helps to explain or predict

the other variable. If x is the independent variable and y the dependent variable, then we can use a regression line to predict y for a given value of x.

The Regression Equation
Linear Regression and Correlation: The Regression Equation is a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean. Contributions from Roberta Bloom include instructions for finding and graphing the regression equation and scatterplot using the LinRegTTest on the TI-83,83+,84+ calculators.

Data rarely fit a straight line exactly. Usually, you must be satisfied with rough predictions. Typically, you have a set of data whose scatter plot appears to **"fit"** a straight line. This is called a **Line of Best Fit or Least Squares Line**.

Optional Collaborative Classroom Activity

If you know a person's pinky (smallest) finger length, do you think you could predict that person's height? Collect data from your class (pinky finger length, in inches). The independent variable, x, is pinky finger length and the dependent variable, y, is height.

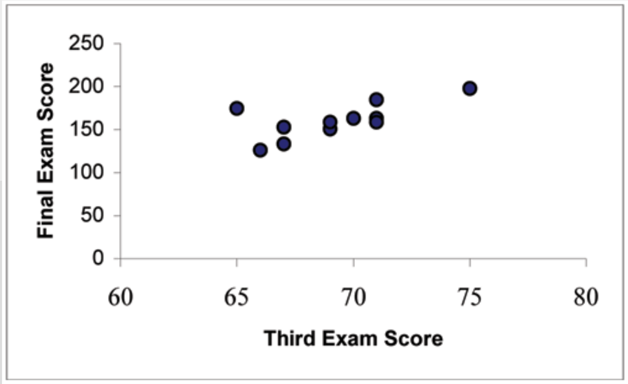
For each set of data, plot the points on graph paper. Make your graph big enough and **use a ruler**. Then "by eye" draw a line that appears to "fit" the data. For your line, pick two convenient points and use them to find the slope of the line. Find the y-intercept of the line by extending your lines so they cross the y-axis. Using the slopes and the y-intercepts, write your equation of "best fit". Do you think everyone will have the same equation? Why or why not?

Using your equation, what is the predicted height for a pinky length of 2.5 inches?

Example:
A random sample of 11 statistics students produced the following data where x is the third exam score, out of 80, and y is the final exam score, out of 200. Can you predict the final exam score of a random student if you know the third exam score?

Table showing the scores on the final exam based on scores from the third exam.

Scatter plot showing the scores on the final exam based on scores from the third exam.

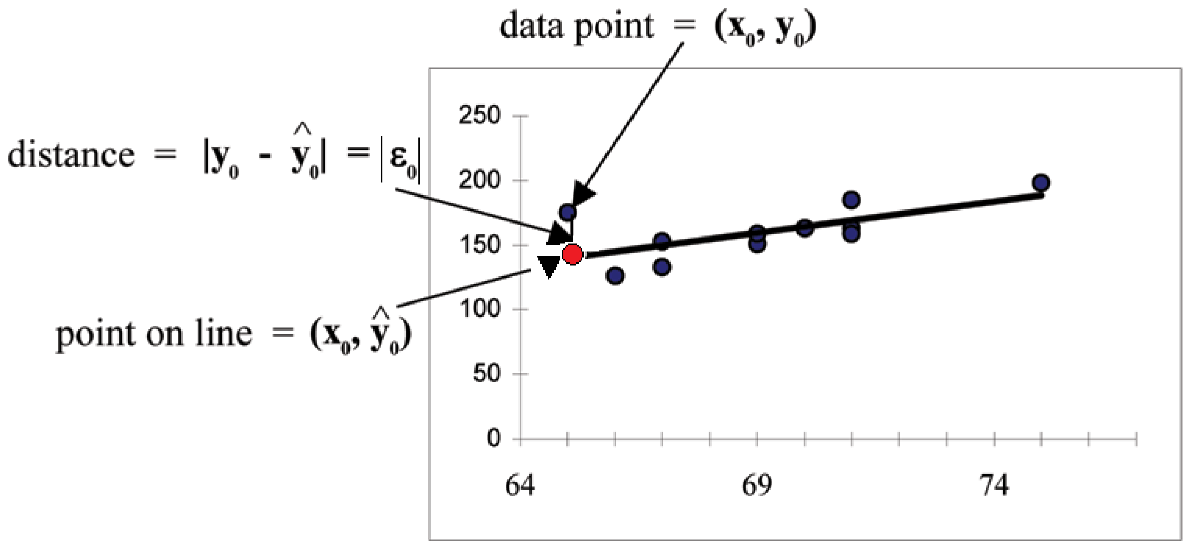


x (third exam score)	y (final exam score)
65	175
67	133
71	185
71	163
66	126
75	198
67	153
70	163
71	159
69	151
69	159

The third exam score, x, is the independent variable and the final exam score, y, is the dependent variable. We will plot a regression line that best "fits" the data. If each of you were to fit a line "by eye", you would draw different lines. We can use what is called a **least-squares regression line** to obtain the best fit line.

Consider the following diagram. Each point of data is of the the form (x, y) and each point of the line of best fit using least-squares linear regression has the form $(x, \hat{y})$.

The $\hat{y}$ is read **"y hat"** and is the **estimated value of** y. It is the value of y obtained using the regression line. It is not generally equal to y from data.



The term $y_0 - \hat{y}_0 = \varepsilon_0$ is called the **"error" or residual**. It is not an error in the sense of a mistake. The **absolute value of a residual** measures the vertical distance between the actual value of y and the estimated value of y. In other words, it measures the vertical distance between the actual data point and the predicted point on the line.

If the observed data point lies above the line, the residual is positive, and the line underestimates the actual data value for y. If the observed data point lies below the line, the residual is negative, and the line overestimates that actual data value for y.

In the diagram above, $y_0 - \hat{y}_0 = \varepsilon_0$ is the residual for the point shown. Here the point lies above the line and the residual is positive.

ε = the Greek letter **epsilon**

For each data point, you can calculate the residuals or errors, $y_i - \hat{y}_i = \varepsilon_i$ for $i = 1, 2, 3, ..., 11$.

Each $|\varepsilon|$ is a vertical distance.

For the example about the third exam scores and the final exam scores for the 11 statistics students, there are 11 data points. Therefore, there are 11 ε values. If you square each ε and add, you get

$$\left(\varepsilon_1\right)^2 + \left(\varepsilon_2\right)^2 + ... + \left(\varepsilon_{11}\right)^2 = \sum_{\mathrm{i}=1}^{11} \varepsilon^2$$

This is called the **Sum of Squared Errors (SSE)**.

Using calculus, you can determine the values of a and b that make the **SSE** a minimum. When you make the **SSE** a minimum, you have determined the points that are on the line of best fit. It turns out that the line of best fit has the equation:

Equation:

$$\hat{y} = a + \mathrm{bx}$$

where $a = \bar{y} - b \cdot \bar{x}$ and $b = \frac{\Sigma(x-\bar{x})\cdot(y-\bar{y})}{\Sigma(x-\bar{x})^2}$.

$\bar{x}$ and $\bar{y}$ are the sample means of the x values and the y values, respectively. The best fit line always passes through the point $(\bar{x}, \bar{y})$.

The slope b can be written as $b = r \cdot \left(\frac{s_y}{s_x}\right)$ where s_y = the standard deviation of the y values and s_x = the standard deviation of the x values. r is the correlation coefficient which is discussed in the next section.

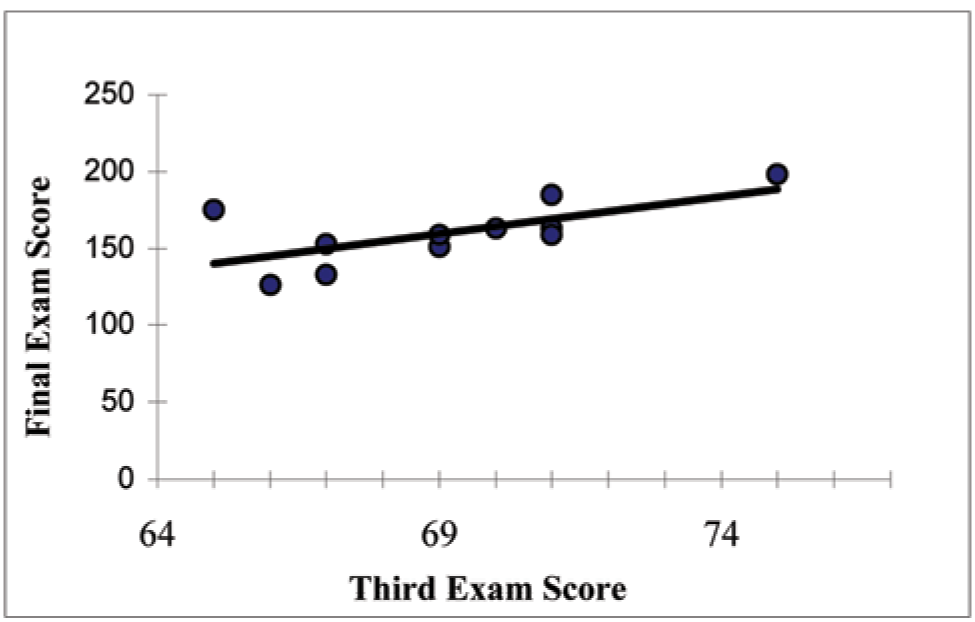
Least Squares Criteria for Best Fit

The process of fitting the best fit line is called **linear regression**. The idea behind finding the best fit line is based on the assumption that the data are scattered about a straight line. The criteria for the best fit line is that the sum of the squared errors (SSE) is minimized, that is made as small as possible. Any other line you might choose would have a higher SSE than the best fit line. This best fit line is called the **least squares regression line**
.

Note: Computer spreadsheets, statistical software, and many calculators can quickly calculate the best fit line and create the graphs. The calculations tend to be tedious if done by hand. Instructions to use the TI-83, TI-83+, and TI-84+ calculators to find the best fit line and create a scatterplot are shown at the end of this section.

THIRD EXAM vs FINAL EXAM EXAMPLE:

The graph of the line of best fit for the third exam/final exam example is shown below:



The least squares regression line (best fit line) for the third exam/final exam example has the equation:

Equation:

$$\hat{y} = -173.51 + 4.83\text{x}$$

Note:

- Remember, it is always important to plot a scatter diagram first. If the scatter plot indicates that there is a linear relationship between the variables, then it is reasonable to use a best fit line to make predictions for y given x within the domain of x-values in the sample data, **but not necessarily for x-values outside that domain.**
- You could use the line to predict the final exam score for a student who earned a grade of 73 on the third exam.
- You should NOT use the line to predict the final exam score for a student who earned a grade of 50 on the third exam, because 50 is not within the domain of the x-values in the sample data, which are between 65 and 75.

UNDERSTANDING SLOPE

The slope of the line, b, describes how changes in the variables are related. It is important to interpret the slope of the line in the context of the situation represented by the data. You should be able to write a sentence interpreting the slope in plain English.

INTERPRETATION OF THE SLOPE: The slope of the best fit line tells us how the dependent variable (y) changes for every one unit increase in the independent (x) variable, on average.

THIRD EXAM vs FINAL EXAM EXAMPLE

- Slope: The slope of the line is b = 4.83.

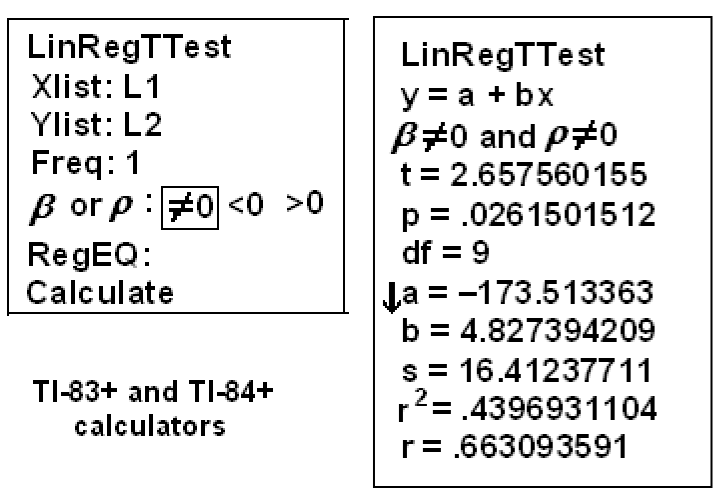
- Interpretation: For a one point increase in the score on the third exam, the final exam score increases by 4.83 points, on average.

Using the TI-83+ and TI-84+ Calculators

Using the Linear Regression T Test: LinRegTTest

In the STAT list editor, enter the X data in list L1 and the Y data in list L2, paired so that the corresponding (x,y) values are next to each other in the lists. (If a particular pair of values is repeated, enter it as many times as it appears in the data.)
On the STAT TESTS menu, scroll down with the cursor to select the LinRegTTest. (Be careful to select LinRegTTest as some calculators may also have a different item called LinRegTInt.)
On the LinRegTTest input screen enter: Xlist: L1 ; Ylist: L2 ; Freq: 1
On the next line, at the prompt β or ρ, highlight "≠ 0" and press ENTER
Leave the line for "RegEq:" blank
Highlight Calculate and press ENTER.

LinRegTTest Input Screen and Output Screen



The output screen contains a lot of information. For now we will focus on a few items from the output, and will return later to the other items.

- The second line says y=a+bx. Scroll down to find the values a=-173.513, and b=4.8273 ; the equation of the best fit line is $\hat{y} = -173.51 + 4.83x$
- The two items at the bottom are r^2 = .43969 and r=.663. For now, just note where to find these values; we will discuss them in the next two sections.

Graphing the Scatterplot and Regression Line

We are assuming your X data is already entered in list L1 and your Y data is in list L2
Press 2nd STATPLOT ENTER to use Plot 1
On the input screen for PLOT 1, highlight**On**and press ENTER
For TYPE: highlight the very first icon which is the scatterplot and press ENTER
Indicate Xlist: L1 and Ylist: L2
For Mark: it does not matter which symbol you highlight.
Press the ZOOM key and then the number 9 (for menu item "ZoomStat") ; the calculator will fit the window to the data
To graph the best fit line, press the "Y=" key and type the equation -173.5+4.83X into equation Y1. (The X key is immediately left of the STAT key). Press ZOOM 9 again to graph it.
Optional: If you want to change the viewing window, press the WINDOW key. Enter your desired window using Xmin, Xmax, Ymin, Ymax

**With contributions from Roberta Bloom

The Correlation Coefficient
Linear Regression and Correlation: The Correlation Coefficient and Coefficient of Determination is a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean with contributions from Roberta Bloom. The name has been changed from Correlation Coefficient.

The Correlation Coefficient r

Besides looking at the scatter plot and seeing that a line seems reasonable, how can you tell if the line is a good predictor? Use the correlation coefficient as another indicator (besides the scatterplot) of the strength of the relationship between x and y.

The **correlation coefficient, r,** developed by Karl Pearson in the early 1900s, is a numerical measure of the strength of association between the independent variable x and the dependent variable y.

The correlation coefficient is calculated as
Equation:

$$r = \frac{n \cdot \Sigma x \cdot y - (\Sigma x) \cdot (\Sigma y)}{\sqrt{[n \cdot \Sigma x^2 - (\Sigma x)^2] \cdot [n \cdot \Sigma y^2 - (\Sigma y)^2]}}$$

where n = the number of data points.

If you suspect a linear relationship between x and y, then r can measure how strong the linear relationship is.
What the VALUE of r tells us:

- The value of r is always between -1 and +1: $-1 \leq r \leq 1$.
- The size of the correlation r indicates the strength of the linear relationship between x and y. Values of r close to -1 or to +1 indicate a stronger linear relationship between x and y.
- If r=0 there is absolutely no linear relationship between x and y **(no linear correlation)**.

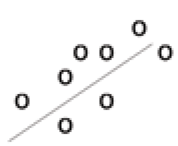
- If $r = 1$, there is perfect positive correlation. If $r = -1$, there is perfect negative correlation. In both these cases, all of the original data points lie on a straight line. Of course, in the real world, this will not generally happen.

What the SIGN of r tells us

- A positive value of r means that when x increases, y tends to increase and when x decreases, y tends to decrease **(positive correlation)**.
- A negative value of r means that when x increases, y tends to decrease and when x decreases, y tends to increase **(negative correlation)**.
- The sign of r is the same as the sign of the slope, b, of the best fit line.

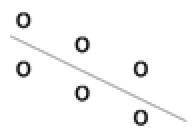
Note: Strong correlation does not suggest that x causes y or y causes x. We say **"correlation does not imply causation."** For example, every person who learned math in the 17th century is dead. However, learning math does not necessarily cause death!

Positive Correlation



A scatter plot showing data with a positive correlation. $0 < \text{r} < 1$

Negative Correlation



A scatter plot showing data with a negative correlation. $-1 < r < 0$

Zero Correlation

A scatter plot showing data with zero correlation. r =0

The formula for r looks formidable. However, computer spreadsheets, statistical software, and many calculators can quickly calculate r. The correlation coefficient r is the bottom item in the output screens for the LinRegTTest on the TI-83, TI-83+, or TI-84+ calculator (see previous section for instructions).

The Coefficient of Determination

r^2 is called the coefficient of determination. r^2 is the square of the correlation coefficient , but is usually stated as a percent, rather than in decimal form. r^2 has an interpretation in the context of the data:

- r^2, when expressed as a percent, represents the percent of variation in the dependent variable y that can be explained by variation in the independent variable x using the regression (best fit) line.
- 1-r^2, when expressed as a percent, represents the percent of variation in y that is NOT explained by variation in x using the regression line. This can be seen as the scattering of the observed data points about the regression line.

Consider the third exam/final exam example introduced in the previous section

- The line of best fit is: $\hat{y} = -173.51 + 4.83$x
- The correlation coefficient is $r = 0.6631$
- The coefficient of determination is r^2 = 0.6631^2 = 0.4397
- **Interpretation of r^2 in the context of this example:**
- Approximately 44% of the variation (0.4397 is approximately 0.44) in the final exam grades can be explained by the variation in the grades on the third exam, using the best fit regression line.
- Therefore approximately 56% of the variation (1 - 0.44 = 0.56) in the final exam grades can NOT be explained by the variation in the grades on the third exam, using the best fit regression line. (This is seen as the scattering of the points about the line.)

**With contributions from Roberta Bloom.

Glossary

Coefficient of Correlation
: A measure developed by Karl Pearson (early 1900s) that gives the strength of association between the independent variable and the dependent variable. The formula is:

Equation:

$$r = \frac{n \sum \text{xy} - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}},$$

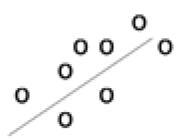
where n is the number of data points. The coefficient cannot be more then 1 and less then -1. The closer the coefficient is to ± 1, the stronger the evidence of a significant linear relationship between x and y.

Facts About the Correlation Coefficient for Linear Regression

This module provides an overview of Facts About the Correlation Coefficient for Linear Regression as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

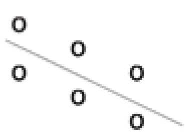
- A positive r means that when x increases, y increases and when x decreases, y decreases **(positive correlation)**.
- A negative r means that when x increases, y decreases and when x decreases, y increases **(negative correlation)**.
- An r of zero means there is absolutely no linear relationship between x and y **(no correlation)**.
- High correlation does not suggest that x causes y or y causes x. We say **"correlation does not imply causation."** For example, every person who learned math in the 17th century is dead. However, learning math does not necessarily cause death!

Positive Correlation



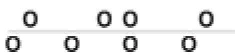
A scatter plot showing data with a positive correlation.

Negative Correlation



A scatter plot showing data with a negative correlation.

Zero Correlation



A scatter plot showing data with zero correlation.

If $r = -1$ or $r = +1$, then all the data points lie exactly on a straight line.

If the linear correlation is strong, then the line can be used to predict a y value.

Prediction
Linear Regression and Correlation: Prediction is a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean with contributions from Roberta Bloom.

Recall the third exam/final exam example.

We examined the scatterplot and showed that the correlation coefficient is significant. We found the equation of the best fit line for the final exam grade as a function of the grade on the third exam. We can now use the least squares regression line for prediction.

Suppose you want to estimate, or predict, the final exam score of statistics students who received 73 on the third exam. The exam scores **(x-values)** range from 65 to 75. **Since 73 is between the x-values 65 and 75**, substitute $x = 73$ into the equation. Then:
Equation:

$$\hat{y} = -173.51 + 4.83(73) = 179.08$$

We predict that statistic students who earn a grade of 73 on the third exam will earn a grade of 179.08 on the final exam, on average.

Example:
Recall the third exam/final exam example.
Exercise:

Problem:

What would you predict the final exam score to be for a student who scored a 66 on the third exam?

Solution:

145.27

Exercise:

Problem:

What would you predict the final exam score to be for a student who scored a 90 on the third exam?

Solution:

The x values in the data are between 65 and 75. 90 is outside of the domain of the observed x values in the data (independent variable), so you cannot reliably predict the final exam score for this student. (Even though it is possible to enter x into the equation and calculate a y value, you should not do so!)

To really understand how unreliable the prediction can be outside of the observed x values in the data, make the substitution x = 90 into the equation.

$$\hat{y} = -173.51 + 4.83(90) = 261.19$$

The final exam score is predicted to be 261.19. The largest the final exam score can be is 200.

Note: The process of predicting inside of the observed x values in the data is called **interpolation**. The process of predicting outside of the observed x values in the data is called **extrapolation**.

**With contributions from Roberta Bloom

Outliers
Linear Regression and Correlation: Outliers is a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean. The module has been modified to include a graphical method for identifying outliers contributed by Roberta Bloom.

In some data sets, there are values **(observed data points)** called **outliers**. **Outliers are observed data points that are far from the least squares line.** They have large "errors", where the "error" or residual is the vertical distance from the line to the point.

Outliers need to be examined closely. Sometimes, for some reason or another, they should not be included in the analysis of the data. It is possible that an outlier is a result of erroneous data. Other times, an outlier may hold valuable information about the population under study and should remain included in the data. The key is to carefully examine what causes a data point to be an outlier.

Besides outliers, a sample may contain one or a few points that are called **influential points**. Influential points are observed data points that are far from the other observed data points in the horizontal direction. These points may have a big effect on the slope of the regression line. To begin to identify an influential point, you can remove it from the data set and see if the slope of the regression line is changed significantly.

Computers and many calculators can be used to identify outliers from the data. Computer output for regression analysis will often identify both outliers and influential points so that you can examine them.

Identifying Outliers
We could guess at outliers by looking at a graph of the scatterplot and best fit line. However we would like some guideline as to how far away a point needs to be in order to be considered an outlier. **As a rough rule of thumb, we can flag any point that is located further than two standard deviations above or below the best fit line as an outlier**. The standard deviation used is the standard deviation of the residuals or errors.

We can do this visually in the scatterplot by drawing an extra pair of lines that are two standard deviations above and below the best fit line. Any data points that are outside this extra pair of lines are flagged as potential outliers. Or we can do this numerically by calculating each residual and comparing it to twice the standard deviation. On the TI-83, 83+, or 84+, the graphical approach is easier. The graphical procedure is shown first, followed by the numerical calculations. You would generally only need to use one of these methods.

Example:
Exercise:

Problem:

In the third exam/final exam example, you can determine if there is an outlier or not. If there is an outlier, as an exercise, delete it and fit the remaining data to a new line. For this example, the new line ought to fit the remaining data better. This means the **SSE** should be smaller and the correlation coefficient ought to be closer to 1 or -1.

Solution:

Graphical Identification of Outliers
With the TI-83,83+,84+ graphing calculators, it is easy to identify the outlier graphically and visually. If we were to measure the vertical distance from any data point to the corresponding point on the line of best fit and that distance was equal to $2s$ or farther, then we would consider the data point to be "too far" from the line of best fit. We need to find and graph the lines that are two standard deviations below and above the regression line. Any points that are outside these two lines are outliers. We will call these lines Y2 and Y3:

As we did with the equation of the regression line and the correlation coefficient, we will use technology to calculate this standard deviation for us. Using the **LinRegTTest** with this data, scroll down through the output screens to find **s=16.412**

Line Y2=-173.5+4.83x-2(16.4) and line Y3=-173.5+4.83x+2(16.4)

where $\hat{y}$=-173.5+4.83x is the line of best fit. Y2 and Y3 have the same slope as the line of best fit.

Graph the scatterplot with the best fit line in equation Y1, then enter the two extra lines as Y2 and Y3 in the "Y="equation editor and press ZOOM 9. You will find that the only data point that is not between lines Y2 and Y3 is the point x=65, y=175. On the calculator screen it is just barely outside these lines. The outlier is the student who had a grade of 65 on the third exam and 175 on the final exam; this point is further than 2 standard deviations away from the best fit line.

Sometimes a point is so close to the lines used to flag outliers on the graph that it is difficult to tell if the point is between or outside the lines. On a computer, enlarging the graph may help; on a small calculator screen, zooming in may make the graph clearer. Note that when the graph does not give a clear enough picture, you can use the numerical comparisons to identify outliers.
[missing_resource: linrgoutlier.gif]

Numerical Identification of Outliers
In the table below, the first two columns are the third exam and final exam data. The third column shows the predicted $\hat{y}$ values calculated from the line of best fit: $\hat{y}$=-173.5+4.83x. The residuals, or errors, have been calculated in the fourth column of the table:
$\text{observed y value} - \text{predicted y value} = y - \hat{y}$.

s is the standard deviation of all the $y - \hat{y} = \varepsilon$ values where n = the total number of data points. If each residual is calculated and squared, and the results are added, we get the SSE. The standard deviation of the residuals is calculated from the SSE as:

$$s = \sqrt{\frac{\text{SSE}}{n-2}}$$

Rather than calculate the value of s ourselves, we can find s using the computer or calculator. For this example, the calculator function

LinRegTTest found s = 16.4 as the standard deviation of the residuals 35 -17 16 -6 -19 9 3 -1 -10 -9 -1 .

x	y	$\hat{y}$	$y - \hat{y}$
65	175	140	$175 - 140 = 35$
67	133	150	$133 - 150 = -17$
71	185	169	$185 - 169 = 16$
71	163	169	$163 - 169 = -6$
66	126	145	$126 - 145 = -19$
75	198	189	$198 - 189 = 9$
67	153	150	$153 - 150 = 3$
70	163	164	$163 - 164 = -1$
71	159	169	$159 - 169 = -10$
69	151	160	$151 - 160 = -9$
69	159	160	$159 - 160 = -1$

We are looking for all data points for which the residual is greater than 2s=2(16.4)=32.8 or less than -32.8. Compare these values to the residuals in column 4 of the table. The only such data point is the student who had a grade of 65 on the third exam and 175 on the final exam; the residual for this student is 35.

How does the outlier affect the best fit line?
Numerically and graphically, we have identified the point (65,175) as an outlier. We should re-examine the data for this point to see if there are any problems with the data. If there is an error we should fix the error if possible, or delete the data. If the data is correct, we would leave it in the data set. **For this problem, we will suppose that we examined the data and found that this outlier data was an error. Therefore we will continue on and delete the outlier, so that we can explore how it affects the results, as a learning experience.**

Compute a new best-fit line and correlation coefficient using the 10 remaining points:
On the TI-83, TI-83+, TI-84+ calculators, delete the outlier from L1 and L2. Using the LinRegTTest, the new line of best fit and the correlation coefficient are:

$\hat{y} = -355.19 + 7.39$x and $r = 0.9121$

The new line with $r = 0.9121$ is a stronger correlation than the original (r=0.6631) because $r = 0.9121$ is closer to 1. This means that the new line is a better fit to the 10 remaining data values. The line can better predict the final exam score given the third exam score.

Numerical Identification of Outliers: Calculating s and Finding Outliers Manually

If you do not have the function LinRegTTest, then you can calculate the outlier in the first example by doing the following.

First, **square each** $|y - \hat{y}|$ (See the TABLE above):

The squares are 35^2 17^2 16^2 6^2 19^2 9^2 3^2 1^2 10^2 9^2 1^2

Then, add (sum) all the $|y - \hat{y}|$ **squared terms** using the formula

$$\sum_{i=1}^{11}\left(\left|y_i - \hat{y}_i\right|\right)^2 = \sum_{i=1}^{11}{\varepsilon_i}^2$$ (Recall that $y_i - \hat{y}_i = \varepsilon_i$.)

$= 35^2 + 17^2 + 16^2 + 6^2 + 19^2 + 9^2 + 3^2 + 1^2 + 10^2 + 9^2 + 1^2$

$= 2440 =$ **SSE**. The result, **SSE** is the Sum of Squared Errors.

Next, calculate s, the standard deviation of all the $y - \hat{y} = \varepsilon$ values where n = the total number of data points.

The calculation is $s = \sqrt{\frac{\text{SSE}}{n-2}}$

For the third exam/final exam problem, $s = \sqrt{\frac{2440}{11-2}} = 16.47$

Next, multiply s by 1.9:
$(1.9) \cdot (16.47) = 31.29$
31.29 is almost 2 standard deviations away from the mean of the $y - \hat{y}$ values.

If we were to measure the vertical distance from any data point to the corresponding point on the line of best fit and that distance is at least $1.9s$, then we would consider the data point to be "too far" from the line of best fit. We call that point a **potential outlier**.

For the example, if any of the $\left|y - \hat{y}\right|$ values are **at least** 31.29, the corresponding (x, y) data point is a potential outlier.

For the third exam/final exam problem, all the $\left|y - \hat{y}\right|$'s are less than 31.29 except for the first one which is 35.

$35 > 31.29$ That is, $\left|y - \hat{y}\right| \geq (1.9) \cdot (s)$

The point which corresponds to $\left|y - \hat{y}\right| = 35$ is $(65, 175)$. **Therefore, the data point $(65, 175)$ is a potential outlier.** For this example, we will delete it. (Remember, we do not always delete an outlier.)

The next step is to compute a new best-fit line using the 10 remaining points. The new line of best fit and the correlation coefficient are:

$\hat{y} = -355.19 + 7.39x$ and $r = 0.9121$

Example:
Exercise:

Problem:

Using this new line of best fit (based on the remaining 10 data points), what would a student who receives a 73 on the third exam expect to receive on the final exam? Is this the same as the prediction made using the original line?

Solution:

Using the new line of best fit, $\hat{y} = -355.19 + 7.39(73) = 184.28$. A student who scored 73 points on the third exam would expect to earn 184 points on the final exam.

The original line predicted $\hat{y} = -173.51 + 4.83(73) = 179.08$ so the prediction using the new line with the outlier eliminated differs from the original prediction.

Example:
(*From The Consumer Price Indexes Web site*) The Consumer Price Index (CPI) measures the average change over time in the prices paid by urban consumers for consumer goods and services. The CPI affects nearly all Americans because of the many ways it is used. One of its biggest uses is as a measure of inflation. By providing information about price changes in the Nation's economy to government, business, and labor, the CPI helps them to make economic decisions. The President, Congress, and the

Federal Reserve Board use the CPI's trends to formulate monetary and fiscal policies. In the following table, x is the year and y is the CPI.

x	y
1915	10.1
1926	17.7
1935	13.7
1940	14.7
1947	24.1
1952	26.5
1964	31.0
1969	36.7
1975	49.3
1979	72.6
1980	82.4
1986	109.6
1991	130.7
1999	166.6

Data:

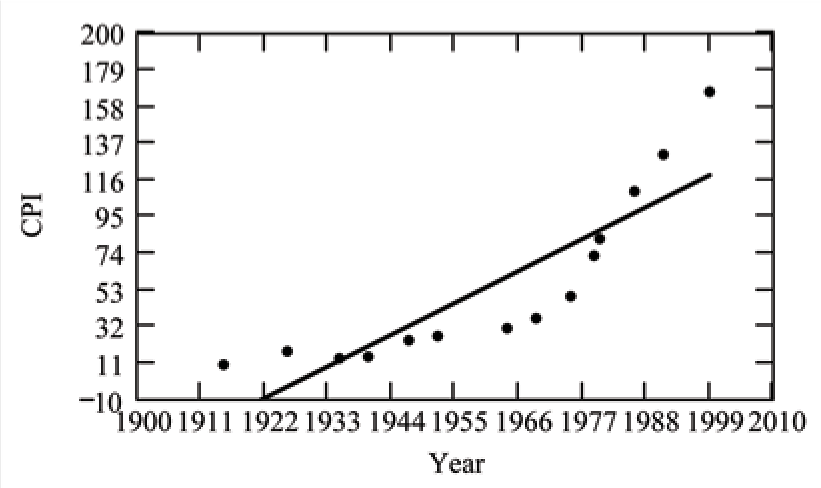
Exercise:

Problem:

- Make a scatterplot of the data.
- Calculate the least squares line. Write the equation in the form $\hat{y} = a + \mathrm{b}x$.
- Draw the line on the scatterplot.
- Find the correlation coefficient. Is it significant?
- What is the average CPI for the year 1990?

Solution:

- Scatter plot and line of best fit.
- $\hat{y} = -3204 + 1.662x$ is the equation of the line of best fit.
- $r = 0.8694$
- The number of data points is $n = 14$. Use the 95% Critical Values of the Sample Correlation Coefficient table at the end of Chapter 12. $n - 2 = 12$. The corresponding critical value is 0.532. Since $0.8694 > 0.532$, r is significant.
- $\hat{y} = -3204 + 1.662(1990) = 103.4$ CPI
- Using the calculator LinRegTTest, we find that s = 25.4 ; graphing the lines Y2=-3204+1.662X-2(25.4) and Y3=-3204+1.662X+2(25.4) shows that no data values are outside those lines, identifying no outliers. (Note that the year 1999 was very close to the upper line, but still inside it.)



Note: In the example, notice the pattern of the points compared to the line. Although the correlation coefficient is significant, the pattern in the scatterplot indicates that a curve would be a more appropriate model to use than a line. In this example, a statistician should prefer to use other methods to fit a curve to this data, rather than model the data with the line we found. In addition to doing the calculations, it is always important to look at the scatterplot when deciding whether a linear model is appropriate.

If you are interested in seeing more years of data, visit the Bureau of Labor Statistics CPI website ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt ; our data is taken from the column entitled "Annual Avg." (third column from the right). For example you could add more current years of data. Try adding the more recent years 2004 : CPI=188.9, 2008 : CPI=215.3 and 2011: CPI=224.9. See how it affects the model. (Check: $\hat{y} = -4436 + 2.295x$. r $= 0.9018$. Is r significant? Is the fit better with the addition of the new points?)

**With contributions from Roberta Bloom

Glossary

Outlier
: An observation that does not fit the rest of the data.

Linear Regression and Correlation: Summary

Bivariate Data: Each data point has two values. The form is (x, y).

Line of Best Fit or Least Squares Line (LSL): $\hat{y} = a + \text{bx}$

x = independent variable; y = dependent variable

Residual: $\text{Actual y value} - \text{predicted y value} = y - \hat{y}$
Correlation Coefficient r:

1. Used to determine whether a line of best fit is good for prediction.
2. Between -1 and 1 inclusive. The closer r is to 1 or -1, the closer the original points are to a straight line.
3. If r is negative, the slope is negative. If r is positive, the slope is positive.
4. If $r = 0$, then the line is horizontal.

Sum of Squared Errors (SSE): The smaller the **SSE**, the better the original set of points fits the line of best fit.

Outlier: A point that does not seem to fit the rest of the data.

Practice: Linear Regression
This module provides a practice of Linear Regression and Correlation as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcomes

- The student will evaluate bivariate data and determine if a line is an appropriate fit to the data.

Given

Below are real data for the first two decades of AIDS reporting. (*Source: Centers for Disease Control and Prevention, National Center for HIV, STD, and TB Prevention*)

Year	# AIDS cases diagnosed	# AIDS deaths
Pre-1981	91	29
1981	319	121
1982	1,170	453
1983	3,076	1,482
1984	6,240	3,466
1985	11,776	6,878
1986	19,032	11,987

1987	28,564	16,162
1988	35,447	20,868
1989	42,674	27,591
1990	48,634	31,335
1991	59,660	36,560
1992	78,530	41,055
1993	78,834	44,730
1994	71,874	49,095
1995	68,505	49,456
1996	59,347	38,510
1997	47,149	20,736
1998	38,393	19,005
1999	25,174	18,454
2000	25,522	17,347
2001	25,643	17,402
2002	26,464	16,371
Total	**802,118**	**489,093**

Adults and Adolescents only, United States

Note: We will use the columns “year” and “# AIDS cases diagnosed” for all questions unless otherwise stated.

Graphing

Graph “year” vs. “# AIDS cases diagnosed.” **Plot the points on the graph located below in the section titled "Plot"** . Do not include pre-1981. Label both axes with words. Scale both axes.

Data

Exercise:

Problem:

Enter your data into your calculator or computer. The pre-1981 data should not be included. Why is that so?

Linear Equation

Write the linear equation below, rounding to 4 decimal places:

Note: For any prediction questions, the answers are calculated using the least squares (best fit) line equation cited in the solution.

Exercise:

Problem: Calculate the following:

- **a** $a =$
- **b** $b =$

- **c** corr. $=$
- **d** n =(# of pairs)

Solution:

- **a** $a =$ -3,448,225
- **b** $b = 1750$
- **c** corr. $= 0.4526$
- **d** $n = 22$

Exercise:

Problem: equation: $\hat{y} =$

Solution:

$\hat{y} =$ -3,448,225 $+1750x$

Solve

Exercise:

Problem: Solve.

- **a** When $x = 1985$, $\hat{y} =$
- **b** When $x = 1990$, $\hat{y} =$

Solution:

- **a** 25,525
- **b** 34,275

Plot

Plot the 2 above points on the graph below. Then, connect the 2 points to form the regression line.

Obtain the graph on your calculator or computer.

Discussion Questions

Look at the graph above.
Exercise:

Problem: Does the line seem to fit the data? Why or why not?

Exercise:

Problem: Do you think a linear fit is best? Why or why not?

Exercise:

Problem:

Hand draw a smooth curve on the graph above that shows the flow of the data.

Exercise:

Problem:

What does the correlation imply about the relationship between time (years) and the number of diagnosed AIDS cases reported in the U.S.?

Exercise:

Problem:

Why is “year” the independent variable and “# AIDS cases diagnosed.” the dependent variable (instead of the reverse)?

Exercise:

Problem: Solve.

- **a**When $x = 1970$, $\hat{y} =$:
- **b**Why doesn’t this answer make sense?

Solution:

- **a** -725

Exercises

This module provides a homework for Linear Regression and Correlation as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Exercise:

Problem:

For each situation below, state the independent variable and the dependent variable.

- **a**A study is done to determine if elderly drivers are involved in more motor vehicle fatalities than all other drivers. The number of fatalities per 100,000 drivers is compared to the age of drivers.
- **b**A study is done to determine if the weekly grocery bill changes based on the number of family members.
- **c**Insurance companies base life insurance premiums partially on the age of the applicant.
- **d**Utility bills vary according to power consumption.
- **e**A study is done to determine if a higher education reduces the crime rate in a population.

Solution:

- **a**Independent: Age; Dependent: Fatalities
- **d**Independent: Power Consumption; Dependent: Utility

Exercise:

Problem:

In 1990 the number of driver deaths per 100,000 for the different age groups was as follows (Source: *The National Highway Traffic Safety Administration's National Center for Statistics and Analysis*):

Age	Number of Driver Deaths per 100,000
15-24	28
25-39	15
40-69	10
70-79	15
80+	25

- **a**For each age group, pick the midpoint of the interval for the x value. (For the 80+ group, use 85.)
- **b**Using "ages" as the independent variable and "Number of driver deaths per 100,000" as the dependent variable, make a scatter plot of the data.
- **c**Calculate the least squares (best–fit) line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **d**Find the correlation coefficient.
- **e**Pick two ages and find the estimated fatality rates.
- **f**Use the two points in (e) to plot the least squares line on your graph from (b).
- **g**Based on the above data, is there a linear relationship between age of a driver and driver fatality rate?
- **h**What is the slope of the least squares (best-fit) line? Interpret the slope.

Exercise:

Problem:

The average number of people in a family that received welfare for various years is given below. (Source: *House Ways and Means Committee, Health and Human Services Department*)

Year	Welfare family size
1969	4.0
1973	3.6
1975	3.2
1979	3.0
1983	3.0
1988	3.0
1991	2.9

- **a**Using “year” as the independent variable and “welfare family size” as the dependent variable, make a scatter plot of the data.
- **b**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **c**Find the correlation coefficient.
- **d**Pick two years between 1969 and 1991 and find the estimated welfare family sizes.
- **e**Use the two points in (d) to plot the least squares line on your graph from (b).
- **f**Based on the above data, is there a linear relationship between the year and the average number of people in a welfare family?
- **g**Using the least squares line, estimate the welfare family sizes for 1960 and 1995. Does the least squares line give an accurate estimate for those years? Explain why or why not.
- **h**Are there any outliers in the above data?
- **i**What is the estimated average welfare family size for 1986? Does the least squares line give an accurate estimate for that year? Explain why or why not.
- **j**What is the slope of the least squares (best-fit) line? Interpret the slope.

Solution:

- **b** $\hat{y} = 88.7206 - 0.0432x$
- **c**-0.8533
- **g**No
- **h**No.
- **i**2.97, Yes
- **j**slope = -0.0432. As the year increases by one, the welfare family size decreases by 0.0432 people.

Exercise:

Problem:

Use the AIDS data from the practice for this section, but this time use the columns “year #” and “# new AIDS deaths in U.S.” Answer all of the questions from the practice again, using the new columns.

Exercise:

Problem:

The height (sidewalk to roof) of notable tall buildings in America is compared to the number of stories of the building (beginning at street level). (Source: *Microsoft Bookshelf*)

Height (in feet)	Stories
1050	57
428	28

Height (in feet)	Stories
362	26
529	40
790	60
401	22
380	38
1454	110
1127	100
700	46

- **a**Using “stories” as the independent variable and “height” as the dependent variable, make a scatter plot of the data.
- **b**Does it appear from inspection that there is a relationship between the variables?
- **c**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **d**Find the correlation coefficient.
- **e**Find the estimated heights for 32 stories and for 94 stories.
- **f**Use the two points in (e) to plot the least squares line on your graph from (b).
- **g**Based on the above data, is there a linear relationship between the number of stories in tall buildings and the height of the buildings?
- **h**Are there any outliers in the above data? If so, which point(s)?
- **i**What is the estimated height of a building with 6 stories? Does the least squares line give an accurate estimate of height? Explain why or why not.

- **j**Based on the least squares line, adding an extra story adds about how many feet to a building?
- **k**What is the slope of the least squares (best-fit) line? Interpret the slope.

Solution:

- **b**Yes
- **c** $\hat{y} = 102.4287 + 11.7585x$
- **d**0.9436
- **e**478.70 feet; 1207.73 feet
- **g**Yes
- **h**Yes; $(57{,}1050)$
- **i**172.98; No
- **j**11.7585 feet
- **k**slope = 11.7585. As the number of stories increases by one, the height of the building increases by 11.7585 feet.

Exercise:

Problem:

Below is the life expectancy for an individual born in the United States in certain years. (Source: *National Center for Health Statistics*)

Year of Birth	**Life Expectancy**
1930	59.7
1940	62.9

Year of Birth	Life Expectancy
1950	70.2
1965	69.7
1973	71.4
1982	74.5
1987	75
1992	75.7

- **a**Decide which variable should be the independent variable and which should be the dependent variable.
- **b**Draw a scatter plot of the ordered pairs.
- **c**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **d**Find the correlation coefficient.
- **e**Find the estimated life expectancy for an individual born in 1950 and for one born in 1982.
- **f**Why aren't the answers to part (e) the values on the above chart that correspond to those years?
- **g**Use the two points in (e) to plot the least squares line on your graph from (b).
- **h**Based on the above data, is there a linear relationship between the year of birth and life expectancy?
- **i**Are there any outliers in the above data?
- **j**Using the least squares line, find the estimated life expectancy for an individual born in 1850. Does the least squares line give an accurate estimate for that year? Explain why or why not.
- **k**What is the slope of the least squares (best-fit) line? Interpret the slope.

Exercise:

Problem:

The percent of female wage and salary workers who are paid hourly rates is given below for the years 1979 - 1992. (Source: *Bureau of Labor Statistics, U.S. Dept. of Labor*)

Year	**Percent of workers paid hourly rates**
1979	61.2
1980	60.7
1981	61.3
1982	61.3
1983	61.8
1984	61.7
1985	61.8
1986	62.0
1987	62.7
1990	62.8
1992	62.9

- **a**Using “year” as the independent variable and “percent” as the dependent variable, make a scatter plot of the data.

- **b**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **c**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \mathrm{bx}$
- **d**Find the correlation coefficient.
- **e**Find the estimated percents for 1991 and 1988.
- **f**Use the two points in (e) to plot the least squares line on your graph from (b).
- **g**Based on the above data, is there a linear relationship between the year and the percent of female wage and salary earners who are paid hourly rates?
- **h**Are there any outliers in the above data?
- **i**What is the estimated percent for the year 2050? Does the least squares line give an accurate estimate for that year? Explain why or why not?
- **j**What is the slope of the least squares (best-fit) line? Interpret the slope.

Solution:

- **b**Yes
- **c** $\hat{y} = -266.8863 + 0.1656x$
- **d**0.9448
- **e**62.9206; 62.4237
- **h**No
- **i**72.639; No
- **j**slope = 0.1656. As the year increases by one, the percent of workers paid hourly rates increases by 0.1565.

Exercise:

Problem:

The maximum discount value of the Entertainment® card for the “Fine Dining” section, Edition 10, for various pages is given below.

Page number	Maximum value ($)
4	16
14	19
25	15
32	17
43	19
57	15
72	16
85	15
90	17

- **a**Decide which variable should be the independent variable and which should be the dependent variable.
- **b**Draw a scatter plot of the ordered pairs.
- **c**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **d**Find the correlation coefficient.
- **e**Find the estimated maximum values for the restaurants on page 10 and on page 70.
- **f**Use the two points in (e) to plot the least squares line on your graph from (b).
- **g**Does it appear that the restaurants giving the maximum value are placed in the beginning of the “Fine Dining” section? How did you arrive at your answer?
- **h**Suppose that there were 200 pages of restaurants. What do you estimate to be the maximum value for a restaurant listed on page 200?

- **i**Is the least squares line valid for page 200? Why or why not?
- **j**What is the slope of the least squares (best-fit) line? Interpret the slope.

The next two questions refer to the following data: The cost of a leading liquid laundry detergent in different sizes is given below.

Size (ounces)	Cost ($)	Cost per ounce
16	3.99	
32	4.99	
64	5.99	
200	10.99	

Exercise:

Problem:

- **a**Using “size” as the independent variable and “cost” as the dependent variable, make a scatter plot.
- **b**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **c**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **d**Find the correlation coefficient.
- **e**If the laundry detergent were sold in a 40 ounce size, find the estimated cost.

- **f**If the laundry detergent were sold in a 90 ounce size, find the estimated cost.
- **g**Use the two points in (e) and (f) to plot the least squares line on your graph from (a).
- **h**Does it appear that a line is the best way to fit the data? Why or why not?
- **i**Are there any outliers in the above data?
- **j**Is the least squares line valid for predicting what a 300 ounce size of the laundry detergent would cost? Why or why not?
- **k**What is the slope of the least squares (best-fit) line? Interpret the slope.

Solution:

- **b**Yes
- **c** $\hat{y} = 3.5984 + 0.0371x$
- **d**0.9986
- **e**$5.08
- **f**$6.93
- **i**No
- **j**Not valid
- **k**slope = 0.0371. As the number of ounces increases by one, the cost of the liquid detergent increases by $0.0371 (or about 4 cents).

Exercise:

Problem:

- **a**Complete the above table for the cost per ounce of the different sizes.
- **b**Using “Size” as the independent variable and “Cost per ounce” as the dependent variable, make a scatter plot of the data.
- **c**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$

- **e**Find the correlation coefficient.
- **f**If the laundry detergent were sold in a 40 ounce size, find the estimated cost per ounce.
- **g**If the laundry detergent were sold in a 90 ounce size, find the estimated cost per ounce.
- **h**Use the two points in (f) and (g) to plot the least squares line on your graph from (b).
- **i**Does it appear that a line is the best way to fit the data? Why or why not?
- **j**Are there any outliers in the above data?
- **k**Is the least squares line valid for predicting what a 300 ounce size of the laundry detergent would cost per ounce? Why or why not?
- **l**What is the slope of the least squares (best-fit) line? Interpret the slope.

Exercise:

Problem:

According to flyer by a Prudential Insurance Company representative, the costs of approximate probate fees and taxes for selected net taxable estates are as follows:

Net Taxable Estate ($)	Approximate Probate Fees and Taxes ($)
600,000	30,000
750,000	92,500
1,000,000	203,000

Net Taxable Estate ($)	Approximate Probate Fees and Taxes ($)
1,500,000	438,000
2,000,000	688,000
2,500,000	1,037,000
3,000,000	1,350,000

- **a**Decide which variable should be the independent variable and which should be the dependent variable.
- **b**Make a scatter plot of the data.
- **c**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient.
- **f**Find the estimated total cost for a net taxable estate of $1,000,000. Find the cost for $2,500,000.
- **g**Use the two points in (f) to plot the least squares line on your graph from (b).
- **h**Does it appear that a line is the best way to fit the data? Why or why not?
- **i**Are there any outliers in the above data?
- **j**Based on the above, what would be the probate fees and taxes for an estate that does not have any assets?
- **k**What is the slope of the least squares (best-fit) line? Interpret the slope.

Solution:

- **c**Yes
- **d** $\hat{y} = -337{,}424.6478 + 0.5463x$

- **e**0.9964
- **f**$208,872.49; $1,028,318.20
- **h**Yes
- **i**No
- **k**slope = 0.5463. As the net taxable estate increases by one dollar, the approximate probate fees and taxes increases by 0.5463 dollars (about 55 cents).

Exercise:

Problem:

The following are advertised sale prices of color televisions at Anderson’s.

Size (inches)	**Sale Price ($)**
9	147
20	197
27	297
31	447
35	1177
40	2177
60	2497

- **a**Decide which variable should be the independent variable and which should be the dependent variable.
- **b**Make a scatter plot of the data.
- **c**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient.
- **f**Find the estimated sale price for a 32 inch television. Find the cost for a 50 inch television.
- **g**Use the two points in (f) to plot the least squares line on your graph from (b).
- **h**Does it appear that a line is the best way to fit the data? Why or why not?
- **i**Are there any outliers in the above data?
- **j**What is the slope of the least squares (best-fit) line? Interpret the slope.

Exercise:

Problem:

Below are the average heights for American boys. (Source: *Physician's Handbook, 1990*)

Age (years)	**Height (cm)**
birth	50.8
2	83.8
3	91.4

Age (years)	Height (cm)
5	106.6
7	119.3
10	137.1
14	157.5

- **a**Decide which variable should be the independent variable and which should be the dependent variable.
- **b**Make a scatter plot of the data.
- **c**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient.
- **f**Find the estimated average height for a one year–old. Find the estimated average height for an eleven year–old.
- **g**Use the two points in (f) to plot the least squares line on your graph from (b).
- **h**Does it appear that a line is the best way to fit the data? Why or why not?
- **i**Are there any outliers in the above data?
- **j**Use the least squares line to estimate the average height for a sixty–two year–old man. Do you think that your answer is reasonable? Why or why not?
- **k**What is the slope of the least squares (best-fit) line? Interpret the slope.

Solution:

- **c**Yes
- **d** $\hat{y} = 65.0876 + 7.0948x$

- **e**0.9761
- **f**72.2 cm; 143.13 cm
- **h**Yes
- **i**No
- **j**505.0 cm; No
- **k**slope = 7.0948. As the age of an American boy increases by one year, the average height increases by 7.0948 cm.

Exercise:

Problem:

The following chart gives the gold medal times for every other Summer Olympics for the women's 100 meter freestyle (swimming).

Year	Time (seconds)
1912	82.2
1924	72.4
1932	66.8
1952	66.8
1960	61.2
1968	60.0
1976	55.65
1984	55.92

Year	Time (seconds)
1992	54.64

- **a**Decide which variable should be the independent variable and which should be the dependent variable.
- **b**Make a scatter plot of the data.
- **c**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient.
- **f**Find the estimated gold medal time for 1932. Find the estimated time for 1984.
- **g**Why are the answers from (f) different from the chart values?
- **h**Use the two points in (f) to plot the least squares line on your graph from (b).
- **i**Does it appear that a line is the best way to fit the data? Why or why not?
- **j**Use the least squares line to estimate the gold medal time for the next Summer Olympics. Do you think that your answer is reasonable? Why or why not?

The next three questions use the following state information.

State	# letters in name	Year entered the Union	Rank for entering the Union	Area (square miles)

State	# letters in name	Year entered the Union	Rank for entering the Union	Area (square miles)
Alabama	7	1819	22	52,423
Colorado		1876	38	104,100
Hawaii		1959	50	10,932
Iowa		1846	29	56,276
Maryland		1788	7	12,407
Missouri		1821	24	69,709
New Jersey		1787	3	8,722
Ohio		1803	17	44,828
South Carolina	13	1788	8	32,008
Utah		1896	45	84,904
Wisconsin		1848	30	65,499

Exercise:

Problem:

We are interested in whether or not the number of letters in a state name depends upon the year the state entered the Union.

- **a**Decide which variable should be the independent variable and which should be the dependent variable.
- **b**Make a scatter plot of the data.
- **c**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient. What does it imply about the significance of the relationship?
- **f**Find the estimated number of letters (to the nearest integer) a state would have if it entered the Union in 1900. Find the estimated number of letters a state would have if it entered the Union in 1940.
- **g**Use the two points in (f) to plot the least squares line on your graph from (b).
- **h**Does it appear that a line is the best way to fit the data? Why or why not?
- **i**Use the least squares line to estimate the number of letters a new state that enters the Union this year would have. Can the least squares line be used to predict it? Why or why not?

Solution:

- **c**No
- **d** $\hat{y} = 47.03 - 0.216x$
- **e**-0.4280
- **f**6; 5

Exercise:

Problem:

We are interested in whether there is a relationship between the ranking of a state and the area of the state.

- **a**Let rank be the independent variable and area be the dependent variable.

- **b**What do you think the scatter plot will look like? Make a scatter plot of the data.
- **c**Does it appear from inspection that there is a relationship between the variables? Why or why not?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient. What does it imply about the significance of the relationship?
- **f**Find the estimated areas for Alabama and for Colorado. Are they close to the actual areas?
- **g**Use the two points in (f) to plot the least squares line on your graph from (b).
- **h**Does it appear that a line is the best way to fit the data? Why or why not?
- **i**Are there any outliers?
- **j**Use the least squares line to estimate the area of a new state that enters the Union. Can the least squares line be used to predict it? Why or why not?
- **k**Delete "Hawaii" and substitute "Alaska" for it. Alaska is the fortieth state with an area of 656,424 square miles.
- **l**Calculate the new least squares line.
- **m**Find the estimated area for Alabama. Is it closer to the actual area with this new least squares line or with the previous one that included Hawaii? Why do you think that's the case?
- **n**Do you think that, in general, newer states are larger than the original states?

Exercise:

Problem:

We are interested in whether there is a relationship between the rank of a state and the year it entered the Union.

- **a**Let year be the independent variable and rank be the dependent variable.
- **b**What do you think the scatter plot will look like? Make a scatter plot of the data.

- **c**Why must the relationship be positive between the variables?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient. What does it imply about the significance of the relationship?
- **f**Let's say a fifty-first state entered the union. Based upon the least squares line, when should that have occurred?
- **g**Using the least squares line, how many states do we currently have?
- **h**Why isn't the least squares line a good estimator for this year?

Solution:

- **d** $\hat{y} = -480.5845 + 0.2748x$
- **e**0.9553
- **f**1934

Exercise:

Problem:

Below are the percents of the U.S. labor force (excluding self-employed and unemployed) that are members of a union. We are interested in whether the decrease is significant. (Source: *Bureau of Labor Statistics, U.S. Dept. of Labor*)

Year	Percent
1945	35.5
1950	31.5

Year	Percent
1960	31.4
1970	27.3
1980	21.9
1986	17.5
1993	15.8

- **a**Let year be the independent variable and percent be the dependent variable.
- **b**What do you think the scatter plot will look like? Make a scatter plot of the data.
- **c**Why will the relationship between the variables be negative?
- **d**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$
- **e**Find the correlation coefficient. What does it imply about the significance of the relationship?
- **f**Based on your answer to (e), do you think that the relationship can be said to be decreasing?
- **g**If the trend continues, when will there no longer be any union members? Do you think that will happen?

The next two questions refer to the following information: The data below reflects the 1991-92 Reunion Class Giving. (Source: *SUNY Albany alumni magazine*)

Class Year	Average Gift	Total Giving
1922	41.67	125
1927	60.75	1,215
1932	83.82	3,772
1937	87.84	5,710
1947	88.27	6,003
1952	76.14	5,254
1957	52.29	4,393
1962	57.80	4,451
1972	42.68	18,093
1976	49.39	22,473
1981	46.87	20,997
1986	37.03	12,590

Exercise:

Problem:

We will use the columns "class year" and "total giving" for all questions, unless otherwise stated.

- **a**What do you think the scatter plot will look like? Make a scatter plot of the data.
- **b**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a + \text{bx}$

- **c**Find the correlation coefficient. What does it imply about the significance of the relationship?
- **d**For the class of 1930, predict the total class gift.
- **e**For the class of 1964, predict the total class gift.
- **f**For the class of 1850, predict the total class gift. Why doesn't this value make any sense?

Solution:

- **b** $\hat{y} = -569{,}770.2796 + 296.0351$
- **c**0.8302
- **d**$1577.48
- **e**$11,642.68
- **f**-$22,105.33

Exercise:

Problem:

We will use the columns "class year" and "average gift" for all questions, unless otherwise stated.

- **a**What do you think the scatter plot will look like? Make a scatter plot of the data.
- **b**Calculate the least squares line. Put the equation in the form of: $\hat{y} = a +$ bx
- **c**Find the correlation coefficient. What does it imply about the significance of the relationship?
- **d**For the class of 1930, predict the average class gift.
- **e**For the class of 1964, predict the average class gift.
- **f**For the class of 2010, predict the average class gift. Why doesn't this value make any sense?

Try these multiple choice questions

Exercise:

Problem:

A correlation coefficient of -0.95 means there is a _____________ between the two variables.

- **A**Strong positive correlation
- **B**Weak negative correlation
- **C**Strong negative correlation
- **D**No Correlation

Solution:

C

Exercise:

Problem:

According to the data reported by the New York State Department of Health regarding West Nile Virus for the years 2000-2004, the least squares line equation for the number of reported dead birds (x) versus the number of human West Nile virus cases (y) is $\hat{y} = -10.2638 + 0.0491x$. If the number of dead birds reported in a year is 732, how many human cases of West Nile virus can be expected?

- **A**25.7
- **B**46.2
- **C**-25.7
- **D**7513

Solution:

A

The next three questions refer to the following data: (showing the number of hurricanes by category to directly strike the mainland U.S. each decade) obtained from *www.nhc.noaa.gov/gifs/table6.gif* A major hurricane is one with a strength rating of 3, 4 or 5.

Decade	**Total Number of Hurricanes**	**Number of Major Hurricanes**
1941-1950	24	10
1951-1960	17	8
1961-1970	14	6
1971-1980	12	4
1981-1990	15	5
1991-2000	14	5
2001 – 2004	9	3

Exercise:

Problem:

Using only completed decades (1941 – 2000), calculate the least squares line for the number of major hurricanes expected based upon the total number of hurricanes.

- **A** $\hat{y} = -1.67x + 0.5$
- **B** $\hat{y} = 0.5\text{x} - 1.67$
- **C** $\hat{y} = 0.94x - 1.67$
- **D** $\hat{y} = -2\text{x} + 1$

Solution:

A

Exercise:

Problem:

The data for 2001-2004 show 9 hurricanes have hit the mainland United States. The line of best fit predicts 2.83 major hurricanes to hit mainland U.S. Can the least squares line be used to make this prediction?

- **A**No, because 9 lies outside the independent variable values
- **B**Yes, because, in fact, there have been 3 major hurricanes this decade
- **C**No, because 2.83 lies outside the dependent variable values
- **D**Yes, because how else could we predict what is going to happen this decade.

Solution:

A

Introduction
This module introduces the concept of Probability, the chance of an event occurring.

Student Learning Objectives

By the end of this chapter, the student should be able to:

- Understand and use the terminology of probability.
- Determine whether two events are mutually exclusive and whether two events are independent.
- Construct and interpret Contingency Tables.

Introduction

It is often necessary to "guess" about the outcome of an event in order to make a decision. Politicians study polls to guess their likelihood of winning an election. Teachers choose a particular course of study based on what they think students can comprehend. Doctors choose the treatments needed for various diseases based on their assessment of likely results. You may have visited a casino where people play games chosen because of the belief that the likelihood of winning is good. You may have chosen your course of study based on the probable availability of jobs.

You have, more than likely, used probability. In fact, you probably have an intuitive sense of probability. Probability deals with the chance of an event occurring. Whenever you weigh the odds of whether or not to do your homework or to study for an exam, you are using probability. In this chapter, you will learn to solve probability problems using a systematic approach.

Optional Collaborative Classroom Exercise

Your instructor will survey your class. Count the number of students in the class today.

- Raise your hand if you have any change in your pocket or purse. Record the number of raised hands.
- Raise your hand if you rode a bus within the past month. Record the number of raised hands.
- Raise your hand if you answered "yes" to BOTH of the first two questions. Record the number of raised hands.

Use the class data as estimates of the following probabilities. $\mathrm{P(change)}$ means the probability that a randomly chosen person in your class has change in his/her pocket or purse. $\mathrm{P(bus)}$ means the probability that a randomly chosen person in your class rode a bus within the last month and so on. Discuss your answers.

- Find $\mathrm{P(change)}$.
- Find $\mathrm{P(bus)}$.
- Find $\mathrm{P(change\ and\ bus)}$ Find the probability that a randomly chosen student in your class has change in his/her pocket or purse and rode a bus within the last month.
- Find $\mathrm{P(change|\ bus)}$ Find the probability that a randomly chosen student has change given that he/she rode a bus within the last month. Count all the students that rode a bus. From the group of students who rode a bus, count those who have change. The probability is equal to those who have change and rode a bus divided by those who rode a bus.

Terminology
Probability: Terminology is part of the collection col10555 written by Barbara Illowsky and Susan Dean defines key terms related to Probability and has contributions from Roberta Bloom.

Probability is a measure that is associated with how certain we are of outcomes of a particular experiment or activity. An **experiment** is a planned operation carried out under controlled conditions. If the result is not predetermined, then the experiment is said to be a **chance** experiment. Flipping one fair coin twice is an example of an experiment.

The result of an experiment is called an **outcome**. A **sample space** is a set of all possible outcomes. Three ways to represent a sample space are to list the possible outcomes, to create a tree diagram, or to create a Venn diagram. The uppercase letter S is used to denote the sample space. For example, if you flip one fair coin, $\mathrm{S} = \{\mathrm{H}, \mathrm{T}\}$ where H = heads and T = tails are the outcomes.

An **event** is any combination of outcomes. Upper case letters like A and B represent events. For example, if the experiment is to flip one fair coin, event A might be getting at most one head. The probability of an event A is written $\mathrm{P}(\mathrm{A})$.

The **probability** of any outcome is the **long-term relative frequency** of that outcome. **Probabilities are between 0 and 1, inclusive** (includes 0 and 1 and all numbers between these values). $\mathrm{P}(\mathrm{A}) = 0$ means the event A can never happen. $\mathrm{P}(\mathrm{A}) = 1$ means the event A always happens. $\mathrm{P}(\mathrm{A}) = 0.5$ means the event A is equally likely to occur or not to occur. For example, if you flip one fair coin repeatedly (from 20 to 2,000 to 20,000 times) the relative fequency of heads approaches 0.5 (the probability of heads).

Equally likely means that each outcome of an experiment occurs with equal probability. For example, if you toss a **fair**, six-sided die, each face (1, 2, 3, 4, 5, or 6) is as likely to occur as any other face. If you toss a fair coin, a Head(H) and a Tail(T) are equally likely to occur. If you randomly guess the answer to a true/false question on an exam, you are equally likely to select a correct answer or an incorrect answer.

To calculate the probability of an event A when all outcomes in the sample space are equally likely, count the number of outcomes for event A and divide by the total number of outcomes in the sample space. For example, if you toss a fair dime and a fair nickel, the sample space is $\{\mathrm{HH},\ \mathrm{TH},\ \mathrm{HT},\ \mathrm{TT}\}$ where T = tails and H = heads. The sample space has four outcomes. A = getting one head. There are two outcomes $\{\mathrm{HT},\ \mathrm{TH}\}$. $\mathrm{P(A)} = \frac{2}{4}$.

Suppose you roll one fair six-sided die, with the numbers {1,2,3,4,5,6} on its faces. Let event E = rolling a number that is at least 5. There are two outcomes $\{5,\ 6\}$. $\mathrm{P(E)} = \frac{2}{6}$. If you were to roll the die only a few times, you would not be surprised if your observed results did not match the probability. If you were to roll the die a very large number of times, you would expect that, overall, 2/6 of the rolls would result in an outcome of "at least 5". You would not expect exactly 2/6. The long-term relative frequency of obtaining this result would approach the theoretical probability of 2/6 as the number of repetitions grows larger and larger.

This important characteristic of probability experiments is the known as the **Law of Large Numbers**: as the number of repetitions of an experiment is increased, the relative frequency obtained in the experiment tends to become closer and closer to the theoretical probability. Even though the outcomes don't happen according to any set pattern or order, overall, the long-term observed relative frequency will approach the theoretical probability. (The word **empirical** is often used instead of the word observed.) The Law of Large Numbers will be discussed again in Chapter 7.

It is important to realize that in many situations, the outcomes are not equally likely. A coin or die may be **unfair**, or **biased** . Two math professors in Europe had their statistics students test the Belgian 1 Euro coin and discovered that in 250 trials, a head was obtained 56% of the time and a tail was obtained 44% of the time. The data seem to show that the coin is not a fair coin; more repetitions would be helpful to draw a more accurate conclusion about such bias. Some dice may be biased. Look at the dice in a game you have at home; the spots on each face are usually small holes carved out and then painted to make the spots visible. Your dice may or may not be biased; it is possible that the outcomes may be affected by the

slight weight differences due to the different numbers of holes in the faces. Gambling casinos have a lot of money depending on outcomes from rolling dice, so casino dice are made differently to eliminate bias. Casino dice have flat faces; the holes are completely filled with paint having the same density as the material that the dice are made out of so that each face is equally likely to occur. Later in this chapter we will learn techniques to use to work with probabilities for events that are not equally likely.

"OR" Event:
An outcome is in the event $A\ \text{ OR }\ B$ if the outcome is in A or is in B or is in both A and B. For example, let $\text{A} = \{1, 2, 3, 4, 5\}$ and $\text{B} = \{4, 5, 6, 7, 8\}$. $A\ \text{ OR }\ \text{B}\ =\ \{1, 2, 3, 4, 5, 6, 7, 8\}$. Notice that 4 and 5 are NOT listed twice.

"AND" Event:
An outcome is in the event A AND B if the outcome is in both A and B at the same time. For example, let A and B be $\{1, 2, 3, 4, 5\}$ and $\{4, 5, 6, 7, 8\}$, respectively. Then $\text{A AND B} = \{4, 5\}$.

The **complement** of event A is denoted A' (read "A prime"). A' consists of all outcomes that are **NOT** in A. Notice that $\text{P(A)} + \text{P(A')} = 1$. For example, let $\text{S} = \{1, 2, 3, 4, 5, 6\}$ and let $\text{A} = \{1, 2, 3, 4\}$. Then, $\text{A'} = \{5, 6\}$. $\text{P(A)} = \frac{4}{6}$, $\text{P(A')} = \frac{2}{6}$, and $\text{P(A)} + \text{P(A')} = \frac{4}{6} + \frac{2}{6} = 1$

The **conditional probability** of A given B is written P(A|B). P(A|B) is the probability that event A will occur given that the event B has already occurred. **A conditional reduces the sample space**. We calculate the probability of A from the reduced sample space B. The formula to calculate P(A|B) is

$$\text{P(A|B)} = \frac{\text{P(A AND B)}}{\text{P(B)}}$$

where P(B) is greater than 0.

For example, suppose we toss one fair, six-sided die. The sample space $\text{S} = \{1, 2, 3, 4, 5, 6\}$. Let A = face is 2 or 3 and B = face is even (2, 4, 6). To calculate P(A|B), we count the number of outcomes 2 or 3 in the

sample space $\mathrm{B} = \{2, 4, 6\}$. Then we divide that by the number of outcomes in B (and not S).

We get the same result by using the formula. Remember that S has 6 outcomes.

$$\mathrm{P(A|B)} = \frac{\mathrm{P(A\ and\ B)}}{\mathrm{P(B)}} = \frac{\text{(the number of outcomes that are 2 or 3 and even in S) / 6}}{\text{(the number of outcomes that are even in S) / 6}} = \frac{1/6}{3/6} = \frac{1}{3}$$

Understanding Terminology and Symbols
It is important to read each problem carefully to think about and understand what the events are. Understanding the wording is the first very important step in solving probability problems. Reread the problem several times if necessary. Clearly identify the event of interest. Determine whether there is a condition stated in the wording that would indicate that the probability is conditional; carefully identify the condition, if any.

Exercise:

Problem:

In a particular college class, there are male and female students. Some students have long hair and some students have short hair. Write the **symbols** for the probabilities of the events for parts (a) through (j) below. (Note that you can't find numerical answers here. You were not given enough information to find any probability values yet; concentrate on understanding the symbols.)

- Let F be the event that a student is female.
- Let M be the event that a student is male.
- Let S be the event that a student has short hair.
- Let L be the event that a student has long hair.

- **a** The probability that a student does not have long hair.
- **b** The probability that a student is male or has short hair.
- **c** The probability that a student is a female and has long hair.
- **d** The probability that a student is male, given that the student has long hair.

- **e** The probability that a student has long hair, given that the student is male.
- **f** Of all the female students, the probability that a student has short hair.
- **g** Of all students with long hair, the probability that a student is female.
- **h** The probability that a student is female or has long hair.
- **i** The probability that a randomly selected student is a male student with short hair.
- **j** The probability that a student is female.

Solution:

- **a** P(L')=P(S)
- **b** P(M or S)
- **c** P(F and L)
- **d** P(M|L)
- **e** P(L|M)
- **f** P(S|F)
- **g** P(F|L)
- **h** P(F or L)
- **i** P(M and S)
- **j** P(F)

**With contributions from Roberta Bloom

Glossary

Conditional Probability
: The likelihood that an event will occur given that another event has already occurred.

Equally Likely
: Each outcome of an experiment has the same probability.

Experiment

A planned activity carried out under controlled conditions.

Event

A subset in the set of all outcomes of an experiment. The set of all outcomes of an experiment is called a **sample space** and denoted usually by S. An event is any arbitrary subset in **S**. It can contain one outcome, two outcomes, no outcomes (empty subset), the entire sample space, etc. Standard notations for events are capital letters such as A, B, C, etc.

Outcome (observation)

A particular result of an experiment.

Probability

A number between 0 and 1, inclusive, that gives the likelihood that a specific event will occur. The foundation of statistics is given by the following 3 axioms (by A. N. Kolmogorov, 1930's): Let S denote the sample space and A and B are two events in S . Then:

- $0 \leq P(A) \leq 1;$.
- If A and B are any two mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.
- $P(S) = 1$.

Sample Space

The set of all possible outcomes of an experiment.

Independent and Mutually Exclusive Events
Probability: Independent and Mutually Exclusive Events is part of the collection col10555 written by Barbara Illowsky and Susan Dean and explains the concept of independent events, where the probability of event A does not have any effect on the probability of event B, and mutually exclusive events, where events A and B cannot occur at the same time. The module has contributions from Roberta Bloom.

Independent and mutually exclusive do **not** mean the same thing.

Independent Events

Two events are independent if the following are true:

- $\mathrm{P(A|B) = P(A)}$
- $\mathrm{P(B|A) = P(B)}$
- $\mathrm{P(A\ AND\ B) = P(A) \cdot P(B)}$

Two events A and B are **independent** if the knowledge that one occurred does not affect the chance the other occurs. For example, the outcomes of two roles of a fair die are independent events. The outcome of the first roll does not change the probability for the outcome of the second roll. To show two events are independent, you must show **only one** of the above conditions. If two events are NOT independent, then we say that they are **dependent**.

Sampling may be done **with replacement** or **without replacement**.

- **With replacement**: If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once. When sampling is done with replacement, then events are considered to be independent, meaning the result of the first pick will not change the probabilities for the second pick.
- **Without replacement:**: When sampling is done without replacement, then each member of a population may be chosen only once. In this case, the probabilities for the second pick are affected by the result of

the first pick. The events are considered to be dependent or not independent.

If it is not known whether A and B are independent or dependent, **assume they are dependent until you can show otherwise**.

Mutually Exclusive Events

A and B are **mutually exclusive** events if they cannot occur at the same time. This means that A and B do not share any outcomes and $\mathrm{P(A\ AND\ B) = 0}$.

For example, suppose the sample space $\mathrm{S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}}$. Let $\mathrm{A = \{1, 2, 3, 4, 5\}}$, $\mathrm{B = \{4, 5, 6, 7, 8\}}$, and $\mathrm{C = \{7, 9\}}$. $\mathrm{A\ AND\ B = \{4, 5\}}$. $\mathrm{P(A\ AND\ B)} = \frac{2}{10}$ and is not equal to zero. Therefore, A and B are not mutually exclusive. A and C do not have any numbers in common so $\mathrm{P(A\ AND\ C) = 0}$. Therefore, A and C are mutually exclusive.

If it is not known whether A and B are mutually exclusive, **assume they are not until you can show otherwise**.

The following examples illustrate these definitions and terms.

Example:
Flip two fair coins. (This is an experiment.)
The sample space is $\{\mathrm{HH, HT, TH, TT}\}$ where T = tails and H = heads. The outcomes are HH, HT, TH, and TT. The outcomes HT and TH are different. The HT means that the first coin showed heads and the second coin showed tails. The TH means that the first coin showed tails and the second coin showed heads.

- Let A = the event of getting **at most one tail**. (At most one tail means 0 or 1 tail.) Then A can be written as $\{\mathrm{HH, HT, TH}\}$. The outcome HH shows 0 tails. HT and TH each show 1 tail.

- Let B = the event of getting all tails. B can be written as $\{\text{TT}\}$. B is the **complement** of A. So, $\text{B} = \text{A'}$. Also, $\text{P(A)} + \text{P(B)} = \text{P(A)} + \text{P(A')} = 1$.
- The probabilities for A and for B are $\text{P(A)} = \frac{3}{4}$ and $\text{P(B)} = \frac{1}{4}$.
- Let C = the event of getting all heads. $\text{C} = \{\text{HH}\}$. Since $\text{B} = \{\text{TT}\}$, $\text{P(B AND C)} = 0$. B and C are mutually exclusive. (B and C have no members in common because you cannot have all tails and all heads at the same time.)
- Let D = event of getting **more than one** tail. $D = \{\text{TT}\}$. $\text{P(D)} = \frac{1}{4}$.
- Let E = event of getting a head on the first roll. (This implies you can get either a head or tail on the second roll.) $E = \{\text{HT}, \text{HH}\}$. $\text{P(E)} = \frac{2}{4}$.
- Find the probability of getting **at least one** (1 or 2) tail in two flips. Let F = event of getting at least one tail in two flips. $F = \{\text{HT}, \text{TH}, \text{TT}\}$. $\text{P(F)} = \frac{3}{4}$

Example:

Roll one fair 6-sided die. The sample space is $\{1, 2, 3, 4, 5, 6\}$. Let event A = a face is odd. Then $\text{A} = \{1, 3, 5\}$. Let event B = a face is even. Then $\text{B} = \{2, 4, 6\}$.

- Find the complement of A, A'. The complement of A, A', is B because A and B together make up the sample space. $\text{P(A)} + \text{P(B)} = \text{P(A)} + \text{P(A')} = 1$. Also, $\text{P(A)} = \frac{3}{6}$ and $\text{P(B)} = \frac{3}{6}$
- Let event C = odd faces larger than 2. Then $C = \{3, 5\}$. Let event D = all even faces smaller than 5. Then $D = \{2, 4\}$. $\text{P(C and D)} = 0$ because you cannot have an odd and even face at the same time. Therefore, C and D are mutually exclusive events.
- Let event E = all faces less than 5. $E = \{1, 2, 3, 4\}$.
  Exercise:

Problem:

Are C and E mutually exclusive events? (Answer yes or no.) Why or why not?

Solution:

No. C = $\{3, 5\}$ and E = $\{1, 2, 3, 4\}$. $\mathrm{P(C\ AND\ E)} = \frac{1}{6}$. To be mutually exclusive, $\mathrm{P(C\ AND\ E)}$ must be 0.

- Find $\mathrm{P(C|A)}$. This is a conditional. Recall that the event C is $\{3, 5\}$ and event A is $\{1, 3, 5\}$. To find $\mathrm{P(C|A)}$, find the probability of C using the sample space A. You have reduced the sample space from the original sample space $\{1, 2, 3, 4, 5, 6\}$ to $\{1, 3, 5\}$. So, $\mathrm{P(C|A)} = \frac{2}{3}$

Example:
Let event G = taking a math class. Let event H = taking a science class. Then, $\mathrm{G\ AND\ H}$ = taking a math class and a science class. Suppose $\mathrm{P(G)} = 0.6$, $\mathrm{P(H)} = 0.5$, and $\mathrm{P(G\ AND\ H)} = 0.3$. Are G and H independent?
If G and H are independent, then you must show **ONE** of the following:

- $\mathrm{P(G|H)} = \mathrm{P(G)}$
- $\mathrm{P(H|G)} = \mathrm{P(H)}$
- $\mathrm{P(G\ AND\ H)} = \mathrm{P(G)} \cdot \mathrm{P(H)}$

Note:The choice you make depends on the information you have. You could choose any of the methods here because you have the necessary information.

Exercise:

Problem: Show that $\mathrm{P(G|H)} = \mathrm{P(G)}$.

Solution:

$$\mathrm{P(G|H)} = \frac{\mathrm{P(G\ AND\ H)}}{\mathrm{P(H)}} = \frac{0.3}{0.5} = 0.6 = \mathrm{P(G)}$$

Exercise:

Problem: Show $\mathrm{P(G\ AND\ H)} = \mathrm{P(G)} \cdot \mathrm{P(H)}$.

Solution:

$$\mathrm{P(G)} \cdot \mathrm{P(H)}\ =\ 0.6 \cdot 0.5\ =\ 0.3\ =\ \mathrm{P(G\ AND\ H)}$$

Since G and H are independent, then, knowing that a person is taking a science class does not change the chance that he/she is taking math. If the two events had not been independent (that is, they are dependent) then knowing that a person is taking a science class would change the chance he/she is taking math. For practice, show that $\mathrm{P(H|G)} = \mathrm{P(H)}$ to show that G and H are independent events.

Example:

In a box there are 3 red cards and 5 blue cards. The red cards are marked with the numbers 1, 2, and 3, and the blue cards are marked with the numbers 1, 2, 3, 4, and 5. The cards are well-shuffled. You reach into the box (you cannot see into it) and draw one card.

Let R = red card is drawn, B = blue card is drawn, E = even-numbered card is drawn.

The sample space $S\ =\ \mathrm{R1, R2, R3, B1, B2, B3, B4, B5}$. S has 8 outcomes.

- $\mathrm{P(R)} = \frac{3}{8}$. $\mathrm{P(B)} = \frac{5}{8}$. $\mathrm{P(R\ AND\ B)} = 0$. (You cannot draw one card that is both red and blue.)

- $\mathrm{P(E)} = \frac{3}{8}$. (There are 3 even-numbered cards, R2, B2, and B4.)
- $\mathrm{P(E|B)} = \frac{2}{5}$. (There are 5 blue cards: B1, B2, B3, B4, and B5. Out of the blue cards, there are 2 even cards: B2 and B4.)
- $\mathrm{P(B|E)} = \frac{2}{3}$. (There are 3 even-numbered cards: R2, B2, and B4. Out of the even-numbered cards, 2 are blue: B2 and B4.)
- The events R and B are mutually exclusive because $\mathrm{P(R\ AND\ B)} = 0$.
- Let G = card with a number greater than 3. $G = \{\mathrm{B4}, \mathrm{B5}\}$. $\mathrm{P(G)} = \frac{2}{8}$. Let H = blue card numbered between 1 and 4, inclusive. $H = \{\mathrm{B1}, \mathrm{B2}, \mathrm{B3}, \mathrm{B4}\}$. $\mathrm{P(G|H)} = \frac{1}{4}$. (The only card in H that has a number greater than 3 is B4.) Since $\frac{2}{8} = \frac{1}{4}$, $\mathrm{P(G)} = \mathrm{P(G|H)}$ which means that G and H are independent.

Example:

In a particular college class, 60% of the students are female. 50 % of all students in the class have long hair. 45% of the students are female and have long hair. Of the female students, 75% have long hair. Let F be the event that the student is female. Let L be the event that the student has long hair. One student is picked randomly. Are the events of being female and having long hair independent?

- The following probabilities are given in this example:
- $\mathrm{P(F\ )} = 0.60$; $\mathrm{P(L\ )} = 0.50$
- $\mathrm{P(F\ AND\ L)} = 0.45$
- $\mathrm{P(L|F)} = 0.75$

Note:The choice you make depends on the information you have. You could use the first or last condition on the list for this example. You do not know P(F|L) yet, so you can not use the second condition.

Solution 1
Check whether P(F and L) = P(F)P(L): We are given that P(F and L) = 0.45 ; but P(F)P(L) = (0.60)(0.50)= 0.30 The events of being female and having long hair are not independent because P(F and L) does not equal P(F)P(L).
Solution 2
check whether P(L|F) equals P(L): We are given that P(L|F) = 0.75 but P(L) = 0.50; they are not equal. The events of being female and having long hair are not independent.
Interpretation of Results
The events of being female and having long hair are not independent; knowing that a student is female changes the probability that a student has long hair.

**Example 5 contributed by Roberta Bloom

Glossary

Independent Events
: The occurrence of one event has no effect on the probability of the occurrence of any other event. Events A and B are independent if one of the following is true: (1). $P(A|B) = P(A)$; (2) $P(B|A) = P(B)$; (3) $P(A \text{ and } B) = P(A)P(B)$.

Mutually Exclusive
: An observation cannot fall into more than one class (category). Being in more than one category prevents being in a mutually exclusive category.

Contingency Tables
This module introduces the contingency table as a way of determining conditional probabilities.

A **contingency table** provides a way of portraying data that can facilitate calculating probabilities. The table helps in determining conditional probabilities quite easily. The table displays sample values in relation to two different variables that may be dependent or contingent on one another. Later on, we will use contingency tables again, but in another manner. Contingincy tables provide a way of portraying data that can facilitate calculating probabilities.

Example:
Suppose a study of speeding violations and drivers who use car phones produced the following fictional data:

	Speeding violation in the last year	**No speeding violation in the last year**	**Total**
Car phone user	25	280	305
Not a car phone user	45	405	450
Total	70	685	755

The total number of people in the sample is 755. The row totals are 305 and 450. The column totals are 70 and 685. Notice that $305 + 450 = 755$ and $70 + 685 = 755$.
Calculate the following probabilities using the table
Exercise:

Problem: $\mathrm{P}(\text{person is a car phone user}) =$

Solution:

$$\frac{\text{number of car phone users}}{\text{total number in study}} = \frac{305}{755}$$

Exercise:

Problem: $\text{P(person had no violation in the last year)} =$

Solution:

$\frac{\text{number that had no violation}}{\text{total number in study}} = \frac{685}{755}$

Exercise:

Problem:

$\text{P(person had no violation in the last year AND was a car phone user)} =$

Solution:

$\frac{280}{755}$

Exercise:

Problem:

$\text{P(person is a car phone user OR person had no violation in the last year)} =$

Solution:

$\left(\frac{305}{755} + \frac{685}{755}\right) - \frac{280}{755} = \frac{710}{755}$

Exercise:

Problem:

$\text{P(person is a car phone user GIVEN person had a violation in the last year)} =$

Solution:

$\frac{25}{70}$ (The sample space is reduced to the number of persons who had a violation.)

Exercise:

Problem:

$\text{P(person had no violation last year GIVEN person was not a car phone user)} =$

Solution:

$\frac{405}{450}$ (The sample space is reduced to the number of persons who were not car phone users.)

Example:
The following table shows a random sample of 100 hikers and the areas of hiking preferred:

Sex	The Coastline	Near Lakes and Streams	On Mountain Peaks	Total
Female	18	16	___	45
Male	___	___	14	55
Total	___	41	___	___

Hiking Area Preference

Exercise:

Problem: Complete the table.

Solution:

Sex	The Coastline	Near Lakes and Streams	On Mountain Peaks	Total
Female	18	16	**11**	45
Male	**16**	**25**	14	55
Total	**34**	41	**25**	**100**

Hiking Area Preference

Exercise:

Problem:

Are the events "being female" and "preferring the coastline" independent events?

Let F = being female and let C = preferring the coastline.

- **a**$\mathrm{P(F\ AND\ C)}$ =
- **b**$\mathrm{P(F)} \cdot \mathrm{P(C)}$ =

Are these two numbers the same? If they are, then F and C are independent. If they are not, then F and C are not independent.

Solution:

- **a** $\mathrm{P(F\ AND\ C)} = \frac{18}{100} = 0.18$
- **b** $\mathrm{P(F)} \cdot \mathrm{P(C)} = \frac{45}{100} \cdot \frac{34}{100} = 0.45 \cdot 0.34 = 0.153$

$\mathrm{P(F\ AND\ C)} \neq \mathrm{P(F)} \cdot \mathrm{P(C)}$, so the events F and C are not independent.

Exercise:

Problem:

Find the probability that a person is male given that the person prefers hiking near lakes and streams. Let M = being male and let L = prefers hiking near lakes and streams.

- **a**What word tells you this is a conditional?
- **b**Fill in the blanks and calculate the probability: $\mathrm{P}(\_\_\_|\_\_\_) = \_\_\_$.
- **c**Is the sample space for this problem all 100 hikers? If not, what is it?

Solution:

- **a**The word 'given' tells you that this is a conditional.
- **b**$\mathrm{P(M|L)} = \frac{25}{41}$
- **c**No, the sample space for this problem is 41.

Exercise:

Problem:

Find the probability that a person is female or prefers hiking on mountain peaks. Let F = being female and let P = prefers mountain peaks.

- **a**$\mathrm{P(F)} =$
- **b**$\mathrm{P(P)} =$
- **c**$\mathrm{P(F\ AND\ P)} =$
- **d**Therefore, $\mathrm{P(F\ OR\ P)} =$

Solution:

- **a**$\mathrm{P(F)} = \frac{45}{100}$
- **b**$\mathrm{P(P)} = \frac{25}{100}$
- **c**$\mathrm{P(F\ AND\ P)} = \frac{11}{100}$
- **d**$\mathrm{P(F\ OR\ P)} = \frac{45}{100} + \frac{25}{100} - \frac{11}{100} = \frac{59}{100}$

Example:
Muddy Mouse lives in a cage with 3 doors. If Muddy goes out the first door, the probability that he gets caught by Alissa the cat is $\frac{1}{5}$ and the probability he is not caught is $\frac{4}{5}$. If he goes out the second door, the probability he gets caught by Alissa is $\frac{1}{4}$ and the probability he is not caught is $\frac{3}{4}$. The probability that Alissa catches Muddy coming out of the third door is $\frac{1}{2}$ and the probability she does not catch Muddy is $\frac{1}{2}$. It is equally likely that Muddy will choose any of the three doors so the probability of choosing each door is $\frac{1}{3}$.

Caught or Not	Door One	Door Two	Door Three	Total
Caught	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{6}$	____
Not Caught	$\frac{4}{15}$	$\frac{3}{12}$	$\frac{1}{6}$	____

Caught or Not	Door One	Door Two	Door Three	Total
Total	____	____	____	1

Door Choice

- The first entry $\frac{1}{15} = \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$ is P(Door One AND Caught).
- The entry $\frac{4}{15} = \left(\frac{4}{5}\right)\left(\frac{1}{3}\right)$ is P(Door One AND Not Caught).

Verify the remaining entries.

Exercise:

Problem:

Complete the probability contingency table. Calculate the entries for the totals. Verify that the lower-right corner entry is 1.

Solution:

Caught or Not	Door One	Door Two	Door Three	Total
Caught	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{19}{60}$
Not Caught	$\frac{4}{15}$	$\frac{3}{12}$	$\frac{1}{6}$	$\frac{41}{60}$
Total	$\frac{5}{15}$	$\frac{4}{12}$	$\frac{2}{6}$	1

Door Choice

Exercise:

Problem: What is the probability that Alissa does not catch Muddy?

Solution:

$\frac{41}{60}$

Exercise:

Problem:

What is the probability that Muddy chooses Door One **OR** Door Two given that Muddy is caught by Alissa?

Solution:

$\frac{9}{19}$

Note: You could also do this problem by using a probability tree. See the Tree Diagrams (Optional) section of this chapter for examples.

Glossary

Contingency Table
: The method of displaying a frequency distribution as a table with rows and columns to show how two variables may be dependent (contingent) upon each other. The table provides an easy way to calculate conditional probabilities.

Probability Topics: Summary of Formulas
This module provides a review of the probability formulas, including the definitions of independent, complementary, and mutually exclusive events as well as the addition and multiplication rules.

Formula

Compliment

If A and A' are complements then $\text{P(A)} + \text{P(A')} = 1$

Formula

Mutually Exclusive

If A and B are mutually exclusive then $\text{P(A AND B)} = 0$; so $\text{P(A OR B)} = \text{P(A)} + \text{P(B)}$.

If A and B are NOT mutually exclusive then $\text{P(A OR B)} = \text{P(A)} + \text{P(B)} - \text{P(A AND B)}$

Formula

Independence

If A and B are independent then:

- $\text{P(A|B)} = \text{P(A)}$
- $\text{P(B|A)} = \text{P(B)}$
- $\text{P(A AND B)} = \text{P(A)P(B)}$

Practice: Contingency Tables
This module provides the opportunity for students to apply what they've learned about probability to solve a series of problems given a set of data. Students will practice constructing and interpreting contingency tables.

Student Learning Outcomes

- The student will construct and interpret contingency tables.

Given

An article in the *New England Journal of Medicine* , reported about a study of smokers in California and Hawaii. In one part of the report, the self-reported ethnicity and smoking levels per day were given. Of the people smoking at most 10 cigarettes per day, there were 9886 African Americans, 2745 Native Hawaiians, 12,831 Latinos, 8378 Japanese Americans, and 7650 Whites. Of the people smoking 11-20 cigarettes per day, there were 6514 African Americans, 3062 Native Hawaiians, 4932 Latinos, 10,680 Japanese Americans, and 9877 Whites. Of the people smoking 21-30 cigarettes per day, there were 1671 African Americans, 1419 Native Hawaiians, 1406 Latinos, 4715 Japanese Americans, and 6062 Whites. Of the people smoking at least 31 cigarettes per day, there were 759 African Americans, 788 Native Hawaiians, 800 Latinos, 2305 Japanese Americans, and 3970 Whites. (*(Source: http://www.nejm.org/doi/full/10.1056/NEJMoa033250)*)

Complete the Table

Complete the table below using the data provided.

Smoking Level	**African American**	**Native Hawaiian**	**Latino**	**Japanese Americans**	**White**	**TOTALS**
1-10						
11-20						
21-30						
31+						
TOTALS						

Smoking Levels by Ethnicity

Analyze the Data

Suppose that one person from the study is randomly selected.

Exercise:

Problem: Find the probability that person smoked 11-20 cigarettes per day.

Solution:

$\frac{35{,}065}{100{,}450}$

Exercise:

Problem: Find the probability that person was Latino.

Solution:

$\frac{19{,}969}{100{,}450}$

Discussion Questions

Exercise:

Problem:

In words, explain what it means to pick one person from the study and that person is "Japanese American **AND** smokes 21-30 cigarettes per day." Also, find the probability.

Solution:

$\frac{4{,}715}{100{,}450}$

Exercise:

Problem:

In words, explain what it means to pick one person from the study and that person is "Japanese American **OR** smokes 21-30 cigarettes per day." Also, find the probability.

Solution:

$\frac{36{,}636}{100{,}450}$

Exercise:

Problem:

In words, explain what it means to pick one person from the study and that person is "Japanese American **GIVEN** that person smokes 21-30 cigarettes per day." Also, find the probability.

Solution:

$\frac{4715}{15{,}273}$

Exercise:

Problem: Prove that smoking level/day and ethnicity are dependent events.

Exercises

This module provides a number of homework exercises related to Probability.

Exercise:

Problem:

Suppose that you have 8 cards. 5 are green and 3 are yellow. The 5 green cards are numbered 1, 2, 3, 4, and 5. The 3 yellow cards are numbered 1, 2, and 3. The cards are well shuffled. You randomly draw one card.

- G = card drawn is green
- E = card drawn is even-numbered

- **a**List the sample space.
- **b**$\text{P(G)} =$
- **c**$\text{P(G|E)} =$
- **d**$\text{P(G AND E)} =$
- **e**$\text{P(G OR E)} =$
- **f**Are G and E mutually exclusive? Justify your answer numerically.

Solution:

- **a**$\{\text{G1, G2, G3, G4, G5, Y1, Y2, Y3}\}$
- **b**$\frac{5}{8}$
- **c**$\frac{2}{3}$
- **d** $\frac{2}{8}$
- **e** $\frac{6}{8}$
- **f**No

Exercise:

Problem:

Refer to the previous problem. Suppose that this time you randomly draw two cards, one at a time, and **with replacement**.

- $G_1 =$ first card is green
- $G_2 =$ second card is green

- **a**Draw a tree diagram of the situation.
- **b** $P(G_1 \text{ AND } G_2) =$
- **c** $P(\text{at least one green}) =$
- **d** $P(G_2 \mid G_1) =$
- **e**Are G_2 and G_1 independent events? Explain why or why not.

Exercise:

Problem:

Refer to the previous problems. Suppose that this time you randomly draw two cards, one at a time, and **without replacement**.

- G_1= first card is green
- G_2= second card is green

- **a**Draw a tree diagram of the situation.
- **b**>$\mathrm{P}(G_1 \text{ AND } G_2) =$
- **c**$\mathrm{P}(\text{at least one green}) =$
- **d**$\mathrm{P}(G_2|G_1) =$
- **e**Are G_2 and G_1 independent events? Explain why or why not.

Solution:

- **b** $(\frac{5}{8})(\frac{4}{7})$
- **c** $(\frac{5}{8})(\frac{3}{7}) + (\frac{3}{8})(\frac{5}{7}) + (\frac{5}{8})(\frac{4}{7})$
- **d** $\frac{4}{7}$
- **e**No

Exercise:

Problem: Roll two fair dice. Each die has 6 faces.

- **a**List the sample space.
- **b**Let A be the event that either a 3 or 4 is rolled first, followed by an even number. Find $\mathrm{P(A)}$.
- **c**Let B be the event that the sum of the two rolls is at most 7. Find $\mathrm{P(B)}$.
- **d**In words, explain what "$\mathrm{P(A|B)}$" represents. Find $\mathrm{P(A|B)}$.
- **e**Are A and B mutually exclusive events? Explain your answer in 1 - 3 complete sentences, including numerical justification.
- **f**Are A and B independent events? Explain your answer in 1 - 3 complete sentences, including numerical justification.

Exercise:

Problem:

A special deck of cards has 10 cards. Four are green, three are blue, and three are red. When a card is picked, the color of it is recorded. An experiment consists of first picking a card and then tossing a coin.

- **a**List the sample space.
- **b**Let A be the event that a blue card is picked first, followed by landing a head on the coin toss. Find $\mathrm{P(A)}$.

- **c**Let B be the event that a red or green is picked, followed by landing a head on the coin toss. Are the events A and B mutually exclusive? Explain your answer in 1 - 3 complete sentences, including numerical justification.
- **d**Let C be the event that a red or blue is picked, followed by landing a head on the coin toss. Are the events A and C mutually exclusive? Explain your answer in 1 - 3 complete sentences, including numerical justification.

Solution:

- **a** $\{\mathrm{GH,GT,BH,BT,RH,RT}\}$
- **b** $\frac{3}{20}$
- **c**Yes
- **d**No

Exercise:

Problem: An experiment consists of first rolling a die and then tossing a coin:

- **a**List the sample space.
- **b**Let A be the event that either a 3 or 4 is rolled first, followed by landing a head on the coin toss. Find $\mathrm{P(A)}$.
- **c**Let B be the event that a number less than 2 is rolled, followed by landing a head on the coin toss. Are the events A and B mutually exclusive? Explain your answer in 1 - 3 complete sentences, including numerical justification.

Exercise:

Problem:

An experiment consists of tossing a nickel, a dime and a quarter. Of interest is the side the coin lands on.

- **a**List the sample space.
- **b**Let A be the event that there are at least two tails. Find $\mathrm{P(A)}$.
- **c**Let B be the event that the first and second tosses land on heads. Are the events A and B mutually exclusive? Explain your answer in 1 - 3 complete sentences, including justification.

Solution:

- **a** $\{\mathrm{(HHH),(HHT),(HTH),(HTT),(THH),(THT),(TTH),(TTT)}\}$
- **b** $\frac{4}{8}$
- **c**Yes

Exercise:

Problem: Consider the following scenario:

- Let $\mathrm{P(C)} = 0.4$
- Let $\mathrm{P(D)} = 0.5$
- Let $\mathrm{P(C|D)} = 0.6$

- **a**Find $\mathrm{P(C\ AND\ D)}$.
- **b**Are C and D mutually exclusive? Why or why not?
- **c**Are C and D independent events? Why or why not?
- **d**Find $\mathrm{P(C\ AND\ D)}$.
- **e**Find $\mathrm{P(D|C)}$.

Exercise:

Problem: E and F mutually exclusive events. $P(E) = 0.4$; $P(F) = 0.5$. Find $P(E \mid F)$.

Solution:

0

Exercise:

Problem: J and K are independent events. $\mathrm{P(J \mid K)} = 0.3$. Find $P(J)$.

Exercise:

Problem: U and V are mutually exclusive events. $P(U) = 0.26$; $P(V) = 0.37$. Find:

- **a**$\mathrm{P(U\ AND\ V)}$ =
- **b** $\mathrm{P(U \mid V)}$ =
- **c**$\mathrm{P(U\ OR\ V)}$ =

Solution:

- **a**0
- **b** 0
- **c**0.63

Exercise:

Problem:

Q and R are independent events. $\mathrm{P(Q)} = 0.4$; $\mathrm{P(Q\ AND\ R)} = 0.1$. Find $\mathrm{P(R)}$.

Exercise:

Problem: Y and Z are independent events.

- **a** Rewrite the basic Addition Rule $\mathrm{P(Y\ OR\ Z) = P(Y) + P(Z) - P(Y\ AND\ Z)}$ using the information that Y and Z are independent events.
- **b** Use the rewritten rule to find $\mathrm{P(Z)}$ if $\mathrm{P(Y\ OR\ Z) = 0.71}$ and $\mathrm{P(Y) = 0.42}$.

Solution:

- **b**0.5

Exercise:

Problem: G and H are mutually exclusive events. $P(G) = 0.5$; $P(H) = 0.3$

- **a**Explain why the following statement MUST be false: $P(H \mid G) = 0.4$.
- **b**Find: $\mathrm{P(H\ OR\ G)}$.
- **c**Are G and H independent or dependent events? Explain in a complete sentence.

Exercise:

Problem:

The following are real data from Santa Clara County, CA. As of March 31, 2000, there was a total of 3059 documented cases of AIDS in the county. They were grouped into the following categories (*Source: Santa Clara County Public H.D.*):

	Homosexual/Bisexual	**IV Drug User***	**Heterosexual Contact**	**Other**	**Totals**
Female	0	70	136	49	____
Male	2146	463	60	135	____
Totals	____	____	____	____	____

* includes homosexual/bisexual IV drug users

Suppose one of the persons with AIDS in Santa Clara County is randomly selected. Compute the following:

- **a** P(person is female) =
- **b**P(person has a risk factor Heterosexual Contact) =
- **c**P(person is female OR has a risk factor of IV Drug User) =
- **d**P(person is female AND has a risk factor of Homosexual/Bisexual) =
- **e**P(person is male AND has a risk factor of IV Drug User) =
- **f**P(female GIVEN person got the disease from heterosexual contact) =

Solution:

The completed contingency table is as follows:

	Homosexual/Bisexual	**IV Drug User***	**Heterosexual Contact**	**Other**	**Totals**
Female	0	70	136	49	**255**
Male	2146	463	60	135	**2804**
Totals	**2146**	**533**	**196**	**174**	**3059**

* includes homosexual/bisexual IV drug users

- **a** $\frac{255}{3059}$
- **b** $\frac{196}{3059}$
- **c** $\frac{718}{3059}$
- **d**0
- **e** $\frac{463}{3059}$
- **f** $\frac{136}{196}$

Exercise:

Problem:

A previous year, the weights of the members of the **San Francisco 49ers** and the **Dallas Cowboys** were published in the *San Jose Mercury News*. The factual data are compiled into the following table.

Shirt#	≤ 210	211-250	251-290	290≤
1-33	21	5	0	0
34-66	6	18	7	4
66-99	6	12	22	5

For the following, suppose that you randomly select one player from the 49ers or Cowboys.

- **a**Find the probability that his shirt number is from 1 to 33.
- **b**Find the probability that he weighs at most 210 pounds.
- **c**Find the probability that his shirt number is from 1 to 33 AND he weighs at most 210 pounds.
- **d**Find the probability that his shirt number is from 1 to 33 OR he weighs at most 210 pounds.
- **e**Find the probability that his shirt number is from 1 to 33 GIVEN that he weighs at most 210 pounds.
- **f**If having a shirt number from 1 to 33 and weighing at most 210 pounds were independent events, then what should be true about $\mathrm{P}(\text{Shirt}\#\ 1\text{-}33 \mid \leq 210 \text{ pounds})$?

Exercise:

Problem:

Approximately 249,000,000 people live in the United States. Of these people, 31,800,000 speak a language other than English at home. Of those who speak another language at home, over 50 percent speak Spanish. (*Source: U.S. Bureau of the Census, 1990 Census*)

Let: E = speak English at home; E’ = speak another language at home; S = speak Spanish at home

Finish each probability statement by matching the correct answer.

Probability Statements	Answers
a. P(E') =	i. 0.8723
b. P(E) =	ii. > 0.50
c. P(S) =	iii. 0.1277

Probability Statements	Answers
d. P(S|E') =	iv. > 0.0639

Solution:

- **a**iii
- **b**i
- **c**iv
- **d**ii

The next two questions refer to the following: The percent of licensed U.S. drivers (from a recent year) that are female is 48.60. Of the females, 5.03% are age 19 and under; 81.36% are age 20 - 64; 13.61% are age 65 or over. Of the licensed U.S. male drivers, 5.04% are age 19 and under; 81.43% are age 20 - 64; 13.53% are age 65 or over. (Source: Federal Highway Administration, U.S. Dept. of Transportation)

Try these multiple choice questions.

The next three questions refer to the following table of data obtained from [*www.baseball-almanac.com*](www.baseball-almanac.com) showing hit information for 4 well known baseball players. Suppose that one hit from the table is randomly selected.

NAME	Single	Double	Triple	Home Run	TOTAL HITS
Babe Ruth	1517	506	136	714	2873
Jackie Robinson	1054	273	54	137	1518
Ty Cobb	3603	174	295	114	4189
Hank Aaron	2294	624	98	755	3771
TOTAL	8471	1577	583	1720	12351

Exercise:

Problem: Find $\text{P}(\text{hit was made by Babe Ruth})$.

- **A** $\frac{1518}{2873}$
- **B** $\frac{2873}{12351}$
- **C** $\frac{583}{12351}$
- **D** $\frac{4189}{12351}$

Solution:

B

Exercise:

Problem: Find $\mathrm{P}(\text{hit was made by Ty Cobb} \mid \text{The hit was a Home Run})$

- **A** $\frac{4189}{12351}$
- **B** $\frac{1141}{1720}$
- **C** $\frac{1720}{4189}$
- **D** $\frac{114}{12351}$

Solution:

B

Exercise:

Problem:

Are the hit being made by Hank Aaron and the hit being a double independent events?

- **A** Yes, because $\mathrm{P}(\text{hit by Hank Aaron} \mid \text{hit is a double}) = \mathrm{P}(\text{hit by Hank Aaron})$
- **B** No, because $\mathrm{P}(\text{hit by Hank Aaron} \mid \text{hit is a double}) \neq \mathrm{P}(\text{hit is a double})$
- **C** No, because $\mathrm{P}(\text{hit is by Hank Aaron} \mid \text{hit is a double}) \neq \mathrm{P}(\text{hit by Hank Aaron})$
- **D** Yes, because $\mathrm{P}(\text{hit is by Hank Aaron} \mid \text{hit is a double}) = \mathrm{P}(\text{hit is a double})$

Solution:

C

Review

This module provides a number of homework/review exercises related to Probability.

The first six exercises refer to the following study: In a survey of 100 stocks on NASDAQ, the average percent increase for the past year was 9% for NASDAQ stocks. Answer the following:

Exercise:

Problem: The “average increase” for all NASDAQ stocks is the:

- **A**Population
- **B**Statistic
- **C**Parameter
- **D**Sample
- **E**Variable

Solution:

- **C** Parameter

Exercise:

Problem: All of the NASDAQ stocks are the:

- **A**Population
- **B**Statistic
- **C**Parameter
- **D**Sample
- **E**Variable

Solution:

- **A** Population

Exercise:

Problem: 9% is the:

- **A**Population
- **B**Statistic
- **C**Parameter
- **D**Sample
- **E**Variable

Solution:

- **B** Statistic

Exercise:

Problem: The 100 NASDAQ stocks in the survey are the:

- **A**Population
- **B**Statistic
- **C**Parameter
- **D**Sample
- **E**Variable

Solution:

- **D** Sample

Exercise:

Problem: The percent increase for one stock in the survey is the:

- **A**Population
- **B**Statistic

- **C**Parameter
- **D**Sample
- **E**Variable

Solution:

- **E** Variable

Exercise:

Problem:

Would the data collected be qualitative, quantitative – discrete, or quantitative – continuous?

Solution:

quantitative - continuous

The next two questions refer to the following study: Thirty people spent two weeks around Mardi Gras in New Orleans. Their two-week weight gain is below. (Note: a loss is shown by a negative weight gain.)

Weight Gain	Frequency
-2	3
-1	5
0	2

Weight Gain	**Frequency**
1	4
4	13
6	2
11	1

Exercise:

Problem: Calculate the following values:

- **a**The average weight gain for the two weeks
- **b**The standard deviation
- **c**The first, second, and third quartiles

Solution:

- **a** 2.27
- **b** 3.04
- **c** -1, 4, 4

Exercise:

Problem: Construct a histogram and a boxplot of the data.

Introduction
This module serves as the introduction to Discrete Random Variables in the Elementary Statistics textbook/collection.

Student Learning Objectives

By the end of this chapter, the student should be able to:

- Recognize and understand discrete probability distribution functions, in general.
- Calculate and interpret expected values.
- Recognize the binomial probability distribution and apply it appropriately.

Introduction

A student takes a 10 question true-false quiz. Because the student had such a busy schedule, he or she could not study and randomly guesses at each answer. What is the probability of the student passing the test with at least a 70%?

Small companies might be interested in the number of long distance phone calls their employees make during the peak time of the day. Suppose the average is 20 calls. What is the probability that the employees make more than 20 long distance phone calls during the peak time?

These two examples illustrate two different types of probability problems involving discrete random variables. Recall that discrete data are data that you can count. A **random variable** describes the outcomes of a statistical experiment both in words. The values of a random variable can vary with each repetition of an experiment.

In this chapter, you will study probability problems involving discrete random distributions. You will also study long-term averages associated with them.

Random Variable Notation

Upper case letters like X or Y denote a random variable. Lower case letters like x or y denote the value of a random variable. If X **is a random variable, then** X **is defined in words.**

For example, let X = the number of heads you get when you toss three fair coins. The sample space for the toss of three fair coins is $\text{TTT THH HTH HHT HTT THT TTH HHH}$. Then, x = 0, 1, 2, 3. X is in words and x is a number. Notice that for this example, the x values are countable outcomes. Because you can count the possible values that X can take on and the outcomes are random (the x values 0, 1, 2, 3), X is a discrete random variable.

Optional Collaborative Classroom Activity

Toss a coin 10 times and record the number of heads. After all members of the class have completed the experiment (tossed a coin 10 times and counted the number of heads), fill in the chart using a heading like the one below. Let X = the number of heads in 10 tosses of the coin.

X	**Frequency of X**	**Relative Frequency of X**

- Which value(s) of X occurred most frequently?
- If you tossed the coin 1,000 times, what values would X take on? Which value(s) of X do you think would occur most frequently?

- What does the relative frequency column sum to?

Glossary

Random Variable (RV)
: see **Variable**

Variable (Random Variable)
: A characteristic of interest in a population being studied. Common notation for variables are upper case Latin letters X, Y, Z,...; common notation for a specific value from the domain (set of all possible values of a variable) are lower case Latin letters x, y, z,.... For example, if X is the number of children in a family, then x represents a specific integer 0, 1, 2, 3, ... Variables in statistics differ from variables in intermediate algebra in two following ways.

  - The domain of the random variable (RV) is not necessarily a numerical set; the domain may be expressed in words; for example, if X = hair color then the domain is {black, blond, gray, green, orange}.
  - We can tell what specific value x of the Random Variable X takes only after performing the experiment.

Probability Distribution Function (PDF) for a Discrete Random Variable
This module introduces the Probability Distribution Function (PDF) and its characteristics.

A discrete **probability distribution function** has two characteristics:

- Each probability is between 0 and 1, inclusive.
- The sum of the probabilities is 1.

Example:
A child psychologist is interested in the number of times a newborn baby's crying wakes its mother after midnight. For a random sample of 50 mothers, the following information was obtained. Let X = the number of times a newborn wakes its mother after midnight. For this example, x = 0, 1, 2, 3, 4, 5.
$\mathrm{P}(\mathrm{x})$ = probability that X takes on a value x.

x	$\mathrm{P}(\mathrm{x})$
0	$\mathrm{P}(\mathrm{x}{=}0) = \frac{2}{50}$
1	$\mathrm{P}(\mathrm{x}{=}1) = \frac{11}{50}$
2	$\mathrm{P}(\mathrm{x}{=}2) = \frac{23}{50}$
3	$\mathrm{P}(\mathrm{x}{=}3) = \frac{9}{50}$
4	$\mathrm{P}(\mathrm{x}{=}4) = \frac{4}{50}$
5	

	$P(x=5) = \frac{1}{50}$

X takes on the values 0, 1, 2, 3, 4, 5. This is a discrete PDF because

1. Each $P(x)$ is between 0 and 1, inclusive.
2. The sum of the probabilities is 1, that is,

Equation:

$$\frac{2}{50} + \frac{11}{50} + \frac{23}{50} + \frac{9}{50} + \frac{4}{50} + \frac{1}{50} = 1$$

Example:
Suppose Nancy has classes **3 days** a week. She attends classes 3 days a week **80%** of the time, **2 days 15%** of the time, **1 day 4%** of the time, and **no days 1%** of the time. Suppose one week is randomly selected.

Exercise:

Problem:

Let X = the number of days Nancy ______________________ .

Solution:

Let X = the number of days Nancy **attends class per week**.

Exercise:

Problem: X takes on what values?

Solution:

0, 1, 2, and 3

Exercise:

Problem:

Suppose one week is randomly chosen. Construct a probability distribution table (called a PDF table) like the one in the previous example. The table should have two columns labeled x and $\mathrm{P(x)}$. What does the $\mathrm{P(x)}$ column sum to?

Solution:

x	$\mathrm{P(x)}$
0	0.01
1	0.04
2	0.15
3	0.80

Glossary

Probability Distribution Function (PDF)
: A mathematical description of a discrete random variable (RV), given either in the form of an equation (formula) , or in the form of a table listing all the possible outcomes of an experiment and the probability associated with each outcome.

Example:
A biased coin with probability 0.7 for a head (in one toss of the coin) is tossed 5 times. We are interested in the number of heads (the RV X = the number of heads). X is Binomial, so $X \sim B(5, 0.7)$ and $P(X = x) = \binom{5}{x} .7^x .3^{5-x}$ or in the form of the table:

x	$P(X = x)$
0	0.0024
1	0.0284
2	0.1323
3	0.3087
4	0.3602
5	0.1681

Mean or Expected Value and Standard Deviation
This module explores the Law of Large Numbers, the phenomenon where an experiment performed many times will yield cumulative results closer and closer to the theoretical mean over time.

The **expected value** is often referred to as the **"long-term"average or mean** . This means that over the long term of doing an experiment over and over, you would **expect** this average.

The **mean** of a random variable X is μ. If we do an experiment many times (for instance, flip a fair coin, as Karl Pearson did, 24,000 times and let X = the number of heads) and record the value of X each time, the average is likely to get closer and closer to μ as we keep repeating the experiment. This is known as the **Law of Large Numbers**.

Note: To find the expected value or long term average, μ, simply multiply each value of the random variable by its probability and add the products.

A Step-by-Step Example
A men's soccer team plays soccer 0, 1, or 2 days a week. The probability that they play 0 days is 0.2, the probability that they play 1 day is 0.5, and the probability that they play 2 days is 0.3. Find the long-term average, μ, or expected value of the days per week the men's soccer team plays soccer.

To do the problem, first let the random variable X = the number of days the men's soccer team plays soccer per week. X takes on the values 0, 1, 2. Construct a PDF table, adding a column $\mathrm{xP(x)}$. In this column, you will multiply each x value by its probability.

x	$\mathrm{P(x)}$	$x\mathrm{P(x)}$
0	0.2	(0)(0.2) = 0
1	0.5	(1)(0.5) = 0.5
2	0.3	(2)(0.3) = 0.6

Expected Value TableThis table is called an expected value table. The table helps you calculate the expected value or long-term average.

Add the last column to find the long term average or expected value: $(0)(0.2)+(1)(0.5)+(2)(0.3)= 0 + 0.5 + 0.6 = 1.1$.

The expected value is 1.1. The men's soccer team would, on the average, expect to play soccer 1.1 days per week. The number 1.1 is the long term average or expected value if the men's soccer team plays soccer week after week after week. We say $\mu=1.1$

Example:
Find the expected value for the example about the number of times a newborn baby's crying wakes its mother after midnight. The expected value is the expected number of times a newborn wakes its mother after midnight.

x	$\mathrm{P(X)}$	$x\mathrm{P(X)}$
0	$\mathrm{P(x{=}0)} = \frac{2}{50}$	$(0)\left(\frac{2}{50}\right) = 0$

x	P(X)	xP(X)
1	$P(x=1) = \frac{11}{50}$	$(1)\left(\frac{11}{50}\right) = \frac{11}{50}$
2	$P(x=2) = \frac{23}{50}$	$(2)\left(\frac{23}{50}\right) = \frac{46}{50}$
3	$P(x=3) = \frac{9}{50}$	$(3)\left(\frac{9}{50}\right) = \frac{27}{50}$
4	$P(x=4) = \frac{4}{50}$	$(4)\left(\frac{4}{50}\right) = \frac{16}{50}$
5	$P(x=5) = \frac{1}{50}$	$(5)\left(\frac{1}{50}\right) = \frac{5}{50}$

You expect a newborn to wake its mother after midnight 2.1 times, on the average.

Add the last column to find the expected value. μ = Expected Value = $\frac{105}{50} = 2.1$

Exercise:

Problem:

Go back and calculate the expected value for the number of days Nancy attends classes a week. Construct the third column to do so.

Solution:

2.74 days a week.

Example:

Suppose you play a game of chance in which five numbers are chosen from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. A computer randomly selects five numbers from 0 to 9 with replacement. You pay $2 to play and could profit $100,000 if you match all 5 numbers in order (you get your $2 back plus

$100,000). Over the long term, what is your **expected** profit of playing the game?
To do this problem, set up an expected value table for the amount of money you can profit.
Let X = the amount of money you profit. The values of x are not 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Since you are interested in your profit (or loss), the values of x are 100,000 dollars and -2 dollars.
To win, you must get all 5 numbers correct, in order. The probability of choosing one correct number is $\frac{1}{10}$ because there are 10 numbers. You may choose a number more than once. The probability of choosing all 5 numbers correctly and in order is:

Equation:

$$\frac{1}{10} * \frac{1}{10} * \frac{1}{10} * \frac{1}{10} * \frac{1}{10} *= 1*10^{-5} = 0.00001$$

Therefore, the probability of winning is 0.00001 and the probability of losing is

Equation:

$$1 - 0.00001 = 0.99999$$

The expected value table is as follows.

	x	P(x)	xP(x)
Loss	-2	0.99999	(-2)(0.99999)=-1.99998
Profit	100,000	0.00001	(100000)(0.00001)=1

Add the last column. -1.99998 + 1 = -0.99998

Since -0.99998 is about -1, you would, on the average, expect to lose approximately one dollar for each game you play. However, each time you play, you either lose $2 or profit $100,000. The $1 is the average or expected LOSS per game after playing this game over and over.

Example:

Suppose you play a game with a biased coin. You play each game by tossing the coin once. $\mathrm{P}(\text{heads}) = \frac{2}{3}$ and $\mathrm{P}(\text{tails}) = \frac{1}{3}$. If you toss a head, you pay $6. If you toss a tail, you win $10. If you play this game many times, will you come out ahead?

Exercise:

Problem: Define a random variable X.

Solution:

X = amount of profit

Exercise:

Problem: Complete the following expected value table.

	x	____	____
WIN	10	$\frac{1}{3}$	____
LOSE	____	____	$\frac{-12}{3}$

Solution:

	x	$\mathrm{P(x)}$	$x\mathrm{P(x)}$
WIN	10	$\frac{1}{3}$	$\frac{10}{3}$
LOSE	**-6**	$\frac{2}{3}$	$\frac{-12}{3}$

Exercise:

Problem: What is the expected value, μ? Do you come out ahead?

Solution:

Add the last column of the table. The expected value $\mu = \frac{-2}{3}$. You lose, on average, about 67 cents each time you play the game so you do not come out ahead.

Like data, probability distributions have standard deviations. To calculate the standard deviation (σ) of a probability distribution, find each deviation from its expected value, square it, multiply it by its probability, add the products, and take the square root . To understand how to do the calculation, look at the table for the number of days per week a men's soccer team plays soccer. To find the standard deviation, add the entries in the column labeled $(x-\mu)^2 \cdot P\left(x\right)$ and take the square root.

x	P(x)	xP(x)	(x -μ)^{2}P(x)
0	0.2	(0)(0.2) = 0	$(0-1.1)^2\left(.2\right)=0.242$
1	0.5	(1)(0.5) = 0.5	$(1-1.1)^2\left(.5\right)=0.005$
2	0.3	(2)(0.3) = 0.6	$(2-1.1)^2\left(.3\right)=0.243$

Add the last column in the table. $0.242+0.005+0.243=0.490$. The standard deviation is the square root of 0.49. $\sigma=\sqrt{0.49}=0.7$

Generally for probability distributions, we use a calculator or a computer to calculate μ and σ to reduce roundoff error. For some probability distributions, there are short-cut formulas that calculate μ and σ.

Glossary

Expected Value

Expected arithmetic average when an experiment is repeated many times. (Also called the mean). Notations: $E(x)$, μ. For a discrete random variable (RV) with probability distribution function $P(x)$,the definition can also be written in the form $E(x)=\mu=\sum\text{xP}(x)$.

Mean

A number that measures the central tendency. A common name for mean is 'average.' The term 'mean' is a shortened form of 'arithmetic mean.' By definition, the mean for a sample (denoted by x) is $x=\frac{\text{Sum of all values in the sample}}{\text{Number of values in the sample}}$, and the mean for a population (denoted by μ) is $\mu=\frac{\text{Sum of all values in the population}}{\text{Number of values in the population}}$.

Common Discrete Probability Distribution Functions

This module serves as a lead-in for several types of common discrete probability distribution functions, including binomial, geometric, hypergeometric, and Poisson.

Some of the more common discrete probability functions are binomial, geometric, hypergeometric, and Poisson. Most elementary courses do not cover the geometric, hypergeometric, and Poisson. Your instructor will let you know if he or she wishes to cover these distributions.

A probability distribution function is a pattern. You try to fit a probability problem into a **pattern** or distribution in order to perform the necessary calculations. These distributions are tools to make solving probability problems easier. Each distribution has its own special characteristics. Learning the characteristics enables you to distinguish among the different distributions.

Binomial
This module describes the characteristics of a binomial experiment and the binomial probability distribution function.

The characteristics of a binomial experiment are:

1. There are a fixed number of trials. Think of trials as repetitions of an experiment. The letter n denotes the number of trials.
2. There are only 2 possible outcomes, called "success" and, "failure" for each trial. The letter p denotes the probability of a success on one trial and q denotes the probability of a failure on one trial. $p+q=1$.
3. The n trials are independent and are repeated using identical conditions. Because the n trials are independent, the outcome of one trial does not help in predicting the outcome of another trial. Another way of saying this is that for each individual trial, the probability, p, of a success and probability, q, of a failure remain the same. For example, randomly guessing at a true - false statistics question has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose Joe always guesses correctly on any statistics true - false question with probability $p=0.6$. Then, $q=0.4$ .This means that for every true - false statistics question Joe answers, his probability of success ($p=0.6$) and his probability of failure ($q=0.4$) remain the same.

The outcomes of a binomial experiment fit a **binomial probability distribution**. The random variable $X=$ the number of successes obtained in the n independent trials.

The mean, μ, and variance, σ^2, for the binomial probability distribution is $\mu=\mathrm{np}$ and $\sigma^2=\mathrm{npq}$. The standard deviation, σ, is then $\sigma=\sqrt{\mathrm{npq}}$.

Any experiment that has characteristics 2 and 3 and where n = 1 is called a **Bernoulli Trial** (named after Jacob Bernoulli who, in the late 1600s, studied them extensively). A binomial experiment takes place when the number of successes is counted in one or more Bernoulli Trials.

Example:
At ABC College, the withdrawal rate from an elementary physics course is 30% for any given term. This implies that, for any given term, 70% of the students stay in the class for the entire term. A "success" could be defined as an individual who withdrew. The random variable is X = the number of students who withdraw from the randomly selected elementary physics class.

Example:
Suppose you play a game that you can only either win or lose. The probability that you win any game is 55% and the probability that you lose is 45%. Each game you play is independent. If you play the game 20 times, what is the probability that you win 15 of the 20 games? Here, if you define X = the number of wins, then X takes on the values 0, 1, 2, 3, ..., 20. The probability of a success is $p = 0.55$. The probability of a failure is $q = 0.45$. The number of trials is $n = 20$. The probability question can be stated mathematically as $P(x = 15)$.

Example:
A fair coin is flipped 15 times. Each flip is independent. What is the probability of getting more than 10 heads? Let X = the number of heads in 15 flips of the fair coin. X takes on the values 0, 1, 2, 3, ..., 15. Since the coin is fair, p = 0.5 and q = 0.5. The number of trials is n = 15. The probability question can be stated mathematically as $P(x > 10)$.

Example:
Approximately 70% of statistics students do their homework in time for it to be collected and graded. Each student does homework independently. In a statistics class of 50 students, what is the probability that at least 40 will do their homework on time? Students are selected randomly.
Exercise:

Problem:

This is a binomial problem because there is only a success or a ___________, there are a definite number of trials, and the probability of a success is 0.70 for each trial.

Solution:

failure

Exercise:

Problem:

If we are interested in the number of students who do their homework, then how do we define X?

Solution:

X = the number of statistics students who do their homework on time

Exercise:

Problem: What values does x take on?

Solution:

0, 1, 2, …, 50

Exercise:

Problem: What is a "failure", in words?

Solution:

Failure is a student who does not do his or her homework on time.

The probability of a success is p = 0.70. The number of trial is n = 50.

Exercise:

Problem: If $p + q = 1$, then what is q?

Solution:

q = 0.30

Exercise:

Problem:

The words "at least" translate as what kind of inequality for the probability question $P(x\_\_\_\_40)$.

Solution:

greater than or equal to (≥)

The probability question is $P(x \geq 40)$.

Notation for the Binomial: B = Binomial Probability Distribution Function

X ~ $B(n, p)$

Read this as "X is a random variable with a binomial distribution." The parameters are n and p. n = number of trials p = probability of a success on each trial

Example:

It has been stated that about 41% of adult workers have a high school diploma but do not pursue any further education. If 20 adult workers are randomly selected, find the probability that at most 12 of them have a high school diploma but do not pursue any further education. How many adult

workers do you expect to have a high school diploma but do not pursue any further education?

Let X = the number of workers who have a high school diploma but do not pursue any further education.

X takes on the values 0, 1, 2, ..., 20 where n = 20 and p = 0.41. q = 1 - 0.41 = 0.59. X ~ $B(20, 0.41)$

Find $P(x \leq 12)$. $P(x \leq 12) = 0.9738$. (calculator or computer)

Using the TI-83+ or the TI-84 calculators, the calculations are as follows. Go into 2nd DISTR. The syntax for the instructions are

To calculate (x = value): binompdf(n, p, number) If "number" is left out, the result is the binomial probability table.

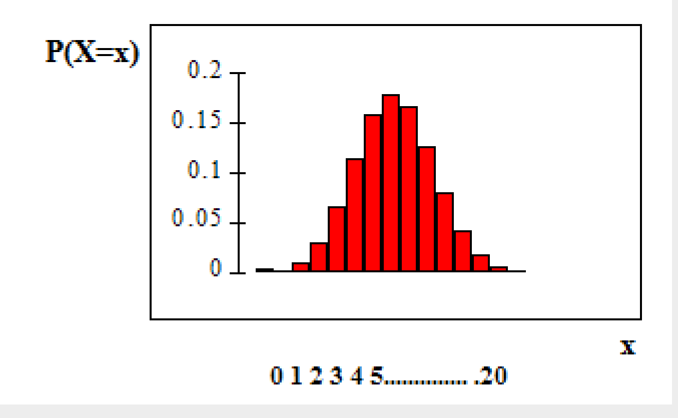
To calculate $P(x \leq \text{value})$: binomcdf(n, p, number) If "number" is left out, the result is the cumulative binomial probability table.

For this problem: After you are in 2nd DISTR, arrow down to binomcdf. Press ENTER. Enter 20,.41,12). The result is $P(x \leq 12) = 0.9738$.

Note: If you want to find $P(x = 12)$, use the pdf (binompdf). If you want to find P(x>12), use 1 - binomcdf(20,.41,12).

The probability at most 12 workers have a high school diploma but do not pursue any further education is 0.9738

The graph of x ~ $B(20, 0.41)$ is:



The y-axis contains the probability of x, where X = the number of workers who have only a high school diploma.
The number of adult workers that you expect to have a high school diploma but not pursue any further education is the mean, $\mu = \text{np} = (20)(0.41) = 8.2$.
The formula for the variance is $\sigma^2 = \text{npq}$. The standard deviation is $\sigma = \sqrt{\text{npq}}$. $\sigma = \sqrt{(20)(0.41)(0.59)} = 2.20$.

Example:
The following example illustrates a problem that is **not** binomial. It violates the condition of independence. ABC College has a student advisory committee made up of 10 staff members and 6 students. The committee wishes to choose a chairperson and a recorder. What is the probability that the chairperson and recorder are both students? All names of the committee are put into a box and two names are drawn **without replacement**. The first name drawn determines the chairperson and the second name the recorder. There are two trials. However, the trials are not independent because the outcome of the first trial affects the outcome of the second trial. The probability of a student on the first draw is $\frac{6}{16}$. The probability of a student on the second draw is $\frac{5}{15}$, when the first draw produces a student. The probability is $\frac{6}{15}$ when the first draw produces a staff member. The probability of drawing a student's name changes for each of the trials and, therefore, violates the condition of independence.

Glossary

Bernoulli Trials
An experiment with the following characteristics:

- There are only 2 possible outcomes called “success” and “failure” for each trial.
- The probability p of a success is the same for any trial (so the probability $q = 1 - p$ of a failure is the same for any trial).

Binomial Distribution

A discrete random variable (RV) which arises from Bernoulli trials. There are a fixed number, n, of independent trials. “Independent” means that the result of any trial (for example, trial 1) does not affect the results of the following trials, and all trials are conducted under the same conditions. Under these circumstances the binomial RV X is defined as the number of successes in n trials. The notation is: $X \sim B(n,p)$. The mean is $\mu = \mathrm{np}$ and the standard deviation is $\sigma = \sqrt{\mathrm{npq}}$. The probability of exactly x successes in n trials is $P(X = x) = \binom{n}{x} p^x q^{n-x}$.

Summary of the Discrete Probability Functions
This module provides a review of the binomial, geometric, hypergeometric, and Poisson probability distribution functions and their properties.

Formula

Binomial

$X \sim B(n, p)$

X = the number of successes in n independent trials

n = the number of independent trials

X takes on the values $x =$ 0,1, 2, 3, ...,n

p = the probability of a success for any trial

q = the probability of a failure for any trial

$p + q = 1 \qquad q = 1 - p$

The mean is $\mu = \text{np}$. The standard deviation is $\sigma = \sqrt{\text{npq}}$.

Practice 1: Discrete Distribution
This module provides students an opportunity to practice applying concepts related to discrete distributions. This practice exercise asks students to calculate several values based on the data provided.

Student Learning Outcomes

- The student will analyze the properties of a discrete distribution.

Given:

A ballet instructor is interested in knowing what percent of each year's class will continue on to the next, so that she can plan what classes to offer. Over the years, she has established the following probability distribution.

- Let X = the number of years a student will study ballet with the teacher.
- Let $\mathrm{P}(\mathrm{x})$ = the probability that a student will study ballet x years.

Organize the Data

Complete the table below using the data provided.

x	P(x)	x*P(x)
1	0.10	
2	0.05	
3	0.10	

---

x	P(x)	x*P(x)
4		
5	0.30	
6	0.20	
7	0.10	

Exercise:

Problem: In words, define the Random Variable X.

Exercise:

Problem: $\mathrm{P}(\mathrm{x} = 4) =$

Exercise:

Problem: $P(x < 4) =$

Exercise:

Problem:

On average, how many years would you expect a child to study ballet with this teacher?

Discussion Question

Exercise:

Problem: What does the column "$\mathrm{P}(\mathrm{x})$" sum to and why?

Exercise:

Problem: What does the column "x*P(x)" sum to and why?

Practice 2: Binomial Distribution
This module provides a practice of Binomial Distribution as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcomes

- The student will construct the Binomial Distribution.

Given

The Higher Education Research Institute at UCLA collected data from 203,967 incoming first-time, full-time freshmen from 270 four-year colleges and universities in the U.S. 71.3% of those students replied that, yes, they believe that same-sex couples should have the right to legal marital status. (*Source: http://heri.ucla.edu/PDFs/pubs/TFS/Norms/Monographs/TheAmericanFreshman2011.pdf)*.)

Suppose that you randomly pick 8 first-time, full-time freshmen from the survey. You are interested in the number that believes that same sex-couples should have the right to legal marital status

Interpret the Data

Exercise:

Problem: In words, define the random Variable X.

Solution:

X= the number that reply "yes"

Exercise:

Problem: X~____________

Solution:

$B(8,0.713)$

Exercise:

Problem: What values does the random variable X take on?

Solution:

0,1,2,3,4,5,6,7,8

Exercise:

Problem: Construct the probability distribution function (PDF).

x	P(x)

Exercise:

Problem:

On average (u), how many would you expect to answer yes?

Solution:

5.7

Exercise:

Problem: What is the standard deviation (σ) ?

Solution:

1.28

Exercise:

Problem:

What is the probability that at most 5 of the freshmen reply “yes”?

Solution:

0.4151

Exercise:

Problem:

What is the probability that at least 2 of the freshmen reply “yes”?

Solution:

0.9990

Exercise:

Problem:

Construct a histogram or plot a line graph. Label the horizontal and vertical axes with words. Include numerical scaling.

Exercises
This module provides a number of homework exercises related to Discrete Random Variables.
Exercise:

Problem: 1. Complete the PDF and answer the questions.

x	$P(X=x)$	$x \cdot P(X=x)$
0	0.3	
1	0.2	
2		
3	0.4	

- **a**Find the probability that $X=2$.
- **b** Find the expected value.

Solution:

- **a**0.1
- **b**1.6

Exercise:

Problem:

Suppose that you are offered the following "deal." You roll a die. If you roll a 6, you win $10. If you roll a 4 or 5, you win $5. If you roll a 1, 2, or 3, you pay $6.

- **a**What are you ultimately interested in here (the value of the roll or the money you win)?
- **b**In words, define the Random Variable X.
- **c**List the values that X may take on.
- **d**Construct a PDF.
- **e**Over the long run of playing this game, what are your expected average winnings per game?
- **f**Based on numerical values, should you take the deal? Explain your decision in complete sentences.

Exercise:

Problem:

A venture capitalist, willing to invest $1,000,000, has three investments to choose from. The first investment, a software company, has a 10% chance of returning $5,000,000 profit, a 30% chance of returning $1,000,000 profit, and a 60% chance of losing the million dollars. The second company, a hardware company, has a 20% chance of returning $3,000,000 profit, a 40% chance of returning $1,000,000 profit, and a 40% chance of losing the million dollars. The third company, a biotech firm, has a 10% chance of returning $6,000,000 profit, a 70% of no profit or loss, and a 20% chance of losing the million dollars.

- **a**Construct a PDF for each investment.
- **b**Find the expected value for each investment.
- **c**Which is the safest investment? Why do you think so?
- **d**Which is the riskiest investment? Why do you think so?
- **e**Which investment has the highest expected return, on average?

Solution:

- **b**\$200,000;\$600,000;\$400,000
- **c**third investment
- **d**first investment
- **e**second investment

Exercise:

Problem:

A theater group holds a fund-raiser. It sells 100 raffle tickets for \$5 apiece. Suppose you purchase 4 tickets. The prize is 2 passes to a Broadway show, worth a total of \$150.

- **a**What are you interested in here?
- **b**In words, define the Random Variable X.
- **c**List the values that X may take on.
- **d**Construct a PDF.
- **e**If this fund-raiser is repeated often and you always purchase 4 tickets, what would be your expected average winnings per game?

Exercise:

Problem:

Suppose that 20,000 married adults in the United States were randomly surveyed as to the number of children they have. The results are compiled and are used as theoretical probabilities. Let X = the number of children

x	$P(X = x)$	$x \cdot P(X = x)$

0	0.10	
1	0.20	
2	0.30	
3		
4	0.10	
5	0.05	
6 (or more)	0.05	

- **a**Find the probability that a married adult has 3 children.
- **b**In words, what does the expected value in this example represent?
- **c** Find the expected value.
- **d** Is it more likely that a married adult will have 2 – 3 children or 4 – 6 children? How do you know?

Solution:

- **a**0.2
- **c**2.35
- **d**2-3 children

Exercise:

Problem:

Suppose that the PDF for the number of years it takes to earn a Bachelor of Science (B.S.) degree is given below.

x	$P(X = x)$
3	0.05
4	0.40
5	0.30
6	0.15
7	0.10

- **a**In words, define the Random Variable X.
- **b** What does it mean that the values 0, 1, and 2 are not included for X on the PDF?
- **c**On average, how many years do you expect it to take for an individual to earn a B.S.?

For each problem:

- **a**In words, define the Random Variable X.
- **b**List the values hat X may take on.
- **c**Give the distribution of X. X~

Then, answer the questions specific to each individual problem.
Exercise:

Problem:

Six different colored dice are rolled. Of interest is the number of dice that show a “1.”

- **d**On average, how many dice would you expect to show a “1”?
- **e**Find the probability that all six dice show a “1.”

- **f** Is it more likely that 3 or that 4 dice will show a “1”? Use numbers to justify your answer numerically.

Solution:

- **a** X = the number of dice that show a 1
- **b** 0,1,2,3,4,5,6
- **c** $X \sim B\left(6, \frac{1}{6}\right)$
- **d** 1
- **e** 0.00002
- **f** 3 dice

Exercise:

Problem:

According to a 2003 publication by Waits and Lewis *(source: http://nces.ed.gov/pubs2003/2003017.pdf)*, by the end of 2002, 92% of U.S. public two-year colleges offered distance learning courses. Suppose you randomly pick 13 U.S. public two-year colleges. We are interested in the number that offer distance learning courses.

- **d** On average, how many schools would you expect to offer such courses?
- **e** Find the probability that at most 6 offer such courses.
- **f** Is it more likely that 0 or that 13 will offer such courses? Use numbers to justify your answer numerically and answer in a complete sentence.

Exercise:

Problem:

A school newspaper reporter decides to randomly survey 12 students to see if they will attend Tet festivities this year. Based on past years, she knows that 18% of students attend Tet festivities. We are interested in the number of students who will attend the festivities.

- **d**How many of the 12 students do we expect to attend the festivities?
- **e**Find the probability that at most 4 students will attend.
- **f**Find the probability that more than 2 students will attend.

Solution:

- **a** X = the number of students that will attend Tet.
- **b**0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
- **c** $X \sim B(12, 0.18)$
- **d**2.16
- **e**0.9511
- **f**0.3702

Exercise:

Problem:

Suppose that about 85% of graduating students attend their graduation. A group of 22 graduating students is randomly chosen.

- **d**How many are expected to attend their graduation?
- **e**Find the probability that 17 or 18 attend.
- **f**Based on numerical values, would you be surprised if all 22 attended graduation? Justify your answer numerically.

Exercise:

Problem:

At The Fencing Center, 60% of the fencers use the foil as their main weapon. We randomly survey 25 fencers at The Fencing Center. We are interested in the numbers that do **not** use the foil as their main weapon.

- **d**How many are expected to **not** use the foil as their main weapon?

- **e**Find the probability that six do **not** use the foil as their main weapon.
- **f**Based on numerical values, would you be surprised if all 25 did **not** use foil as their main weapon? Justify your answer numerically.

Solution:

- **a** X = the number of fencers that do **not** use foil as their main weapon
- **b**0, 1, 2, 3,... 25
- **c** X~$\mathrm{B}(25{,}0.40)$
- **d**10
- **e**0.0442
- **f**Yes

Exercise:

Problem:

Approximately 8% of students at a local high school participate in after-school sports all four years of high school. A group of 60 seniors is randomly chosen. Of interest is the number that participated in after-school sports all four years of high school.

- **d**How many seniors are expected to have participated in after-school sports all four years of high school?
- **e**Based on numerical values, would you be surprised if none of the seniors participated in after-school sports all four years of high school? Justify your answer numerically.
- **f**Based upon numerical values, is it more likely that 4 or that 5 of the seniors participated in after-school sports all four years of high school? Justify your answer numerically.

Try these multiple choice problems.

For the next three problems: The probability that the San Jose Sharks will win any given game is 0.3694 based on their 13 year win history of 382 wins out of 1034 games played (as of a certain date). Their 2005 schedule for November contains 12 games. Let X= number of games won in November 2005

Exercise:

Problem:

The expected number of wins for the month of November 2005 is:

- **A** 1.67
- **B** 12
- **C** $\frac{382}{1043}$
- **D** 4.43

Solution:

D: 4.43

Exercise:

Problem:

What is the probability that the San Jose Sharks win 6 games in November?

- **A**0.1476
- **B**0.2336
- **C**0.7664
- **D**0.8903

Solution:

A: 0.1476

Exercise:

Problem:

Find the probability that the San Jose Sharks win at least 5 games in November.

- **A**0.3694
- **B**0.5266
- **C**0.4734
- **D**0.2305

Solution:

C: 0.4734

Review

This module provides a number of homework/review exercises summarizing topics related to Discrete Random Variables.

Exercise:

Problem:

A sociologist wants to know the opinions of employed adult women about government funding for day care. She obtains a list of 520 members of a local business and professional women's club and mails a questionnaire to 100 of these women selected at random. 68 questionnaires are returned. What is the population in this study?

- **A**All employed adult women
- **B**All the members of a local business and professional women's club
- **C**The 100 women who received the questionnaire
- **D**All employed women with children

Solution:

A

The next two questions refer to the following: An article from The San Jose Mercury News was concerned with the racial mix of the 1500 students at Prospect High School in Saratoga, CA. The table summarizes the results. (Male and female values are approximate.)

			Ethnic Group		

			Ethnic Group		
Gender	White	Asian	Hispanic	Black	American Indian
Male	400	168	115	35	16
Female	440	132	140	40	14

Exercise:

Problem: Find the probability that a student is Asian or Male.

Solution:

0.5773

Exercise:

Problem:

Find the probability that a student is Black given that the student is Female.

Solution:

0.0522

Exercise:

Problem:

A sample of pounds lost, in a certain month, by individual members of a weight reducing clinic produced the following statistics:

- Mean = 5 lbs.
- Median = 4.5 lbs.
- Mode = 4 lbs.

- Standard deviation = 3.8 lbs.
- First quartile = 2 lbs.
- Third quartile = 8.5 lbs.

The correct statement is:

- **A**One fourth of the members lost exactly 2 pounds.
- **B**The middle fifty percent of the members lost from 2 to 8.5 lbs.
- **C**Most people lost 3.5 to 4.5 lbs.
- **D**All of the choices above are correct.

Solution:

B

Exercise:

Problem:

What does it mean when a data set has a standard deviation equal to zero?

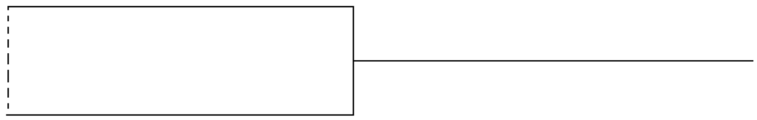
- **A**All values of the data appear with the same frequency.
- **B**The mean of the data is also zero.
- **C**All of the data have the same value.
- **D**There are no data to begin with.

Solution:

C

Exercise:

Problem: The statement that best describes the illustration below is:



- **A**The mean is equal to the median.
- **B**There is no first quartile.
- **C**The lowest data value is the median.
- **D**The median equals $\frac{(Q1+Q3)}{2}$

Solution:

C

Exercise:

Problem:

A “friend” offers you the following “deal.” For a $10 fee, you may pick an envelope from a box containing 100 seemingly identical envelopes. However, each envelope contains a coupon for a free gift.

- 10 of the coupons are for a free gift worth $6.
- 80 of the coupons are for a free gift worth $8.
- 6 of the coupons are for a free gift worth $12.
- 4 of the coupons are for a free gift worth $40.

Based upon the financial gain or loss over the long run, should you play the game?

- **A**Yes, I expect to come out ahead in money.
- **B**No, I expect to come out behind in money.
- **C**It doesn’t matter. I expect to break even.

Solution:

B

The next four questions refer to the following: Recently, a nurse commented that when a patient calls the medical advice line claiming to have **the flu**, the chance that he/she truly has **the flu** (and not just a nasty cold) is only about 4%. Of the next 25 patients calling in claiming to have **the flu**, we are interested in how many actually have **the flu**.

Exercise:

Problem: Define the Random Variable and list its possible values.

Solution:

X = the number of patients calling in claiming to have **the flu**, who actually have **the flu**. X = 0, 1, 2, ...25

Exercise:

Problem: State the distribution of X .

Solution:

$B(25,0.04)$

Exercise:

Problem:

Find the probability that at least 4 of the 25 patients actually have **the flu**.

Solution:

0.0165

Exercise:

Problem:

On average, for every 25 patients calling in, how many do you expect to have **the flu**?

Solution:

1

The next two questions refer to the following: Different types of writing can sometimes be distinguished by the number of letters in the words used. A student interested in this fact wants to study the number of letters of words used by Tom Clancy in his novels. She opens a Clancy novel at random and records the number of letters of the first 250 words on the page.

Exercise:

Problem: What kind of data was collected?

- **A**qualitative
- **B**quantitative - continuous
- **C**quantitative – discrete

Solution:

C

Exercise:

Problem: What is the population under study?

Solution:

All words used by Tom Clancy in his novels

Introduction
This module serves as an introduction to the Continuous Random Variables chapter in the Elementary Statistics textbook.

Student Learning Objectives

By the end of this chapter, the student should be able to:

- Recognize and understand continuous probability density functions in general.
- Recognize the uniform probability distribution and apply it appropriately.

Introduction

Continuous random variables have many applications. Baseball batting averages, IQ scores, the length of time a long distance telephone call lasts, the amount of money a person carries, the length of time a computer chip lasts, and SAT scores are just a few. The field of reliability depends on a variety of continuous random variables.

This chapter gives an introduction to continuous random variables and the many continuous distributions. We will be studying these continuous distributions for several chapters.

The characteristics of continuous random variables are:

- The outcomes are measured, not counted.
- Geometrically, the probability of an outcome is equal to an area under a mathematical curve called the density curve, $f(x)$.
- Each individual value has zero probability of occurring. Instead we find the probability that the value is between two endpoints.

We will start with a simplest continuous distributions, the **Uniform**.

Note: The values of discrete and continuous random variables can be ambiguous. For example, if X is equal to the number of miles (to the nearest mile) you drive to work, then X is a discrete random variable. You count the miles. If X is the distance you drive to work, then you measure values of X and X is a continuous random variable. How the random variable is defined is very important.

Glossary

Uniform Distribution
: A continuous random variable (RV) that has equally likely outcomes over the domain, $a < x < b$. Often referred as the **Rectangular distribution** because the graph of the pdf has the form of a rectangle. Notation: X~$U(a,b)$. The mean is $\mu = \frac{a+b}{2}$ and the standard deviation is $\sigma = \sqrt{\frac{(b-a)^2}{12}}$ The probability density function is $f(X) = \frac{1}{b-a}$ for $a < X < b$ or $a \leq X \leq b$. The cumulative distribution is $P(X \leq x) = \frac{x-a}{b-a}$.

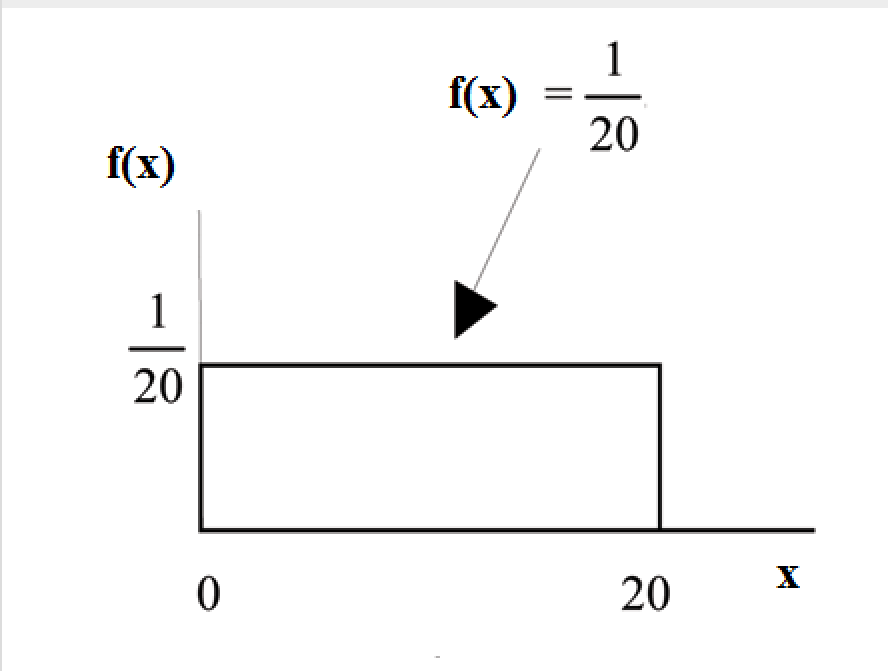
Exponential Distribution
: A continuous random variable (RV) that appears when we are interested in the intervals of time between some random events, for example, the length of time between emergency arrivals at a hospital. Notation: X~$\mathrm{Exp}(m)$. The mean is $\mu = \frac{1}{m}$ and the standard deviation is $\sigma = \frac{1}{m}$. The probability density function is $f(x) = \mathrm{me}^{-\mathrm{mx}}$, $x \geq 0$ and the cumulative distribution function is $P(X \leq x) = 1 - e^{-\mathrm{mx}}$.

Continuous Probability Functions
This module introduces the continuous probability function and explores the relationship between the probability of X and the area under the curve of f(X).

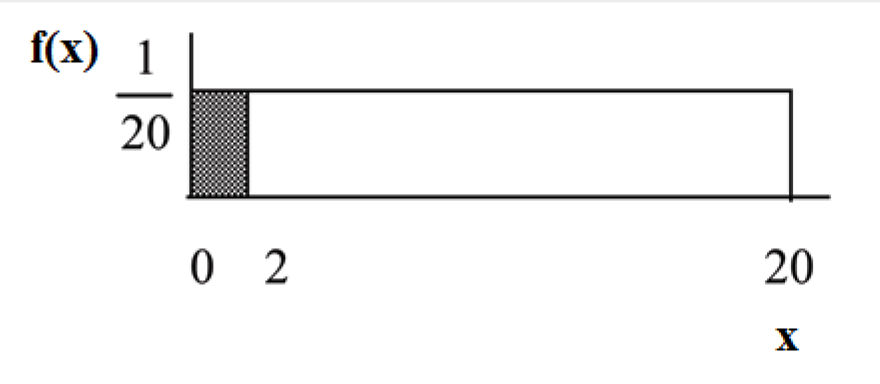
We begin by defining a continuous probability density function. We use the function notation $f(x)$. Intermediate algebra may have been your first formal introduction to functions. In the study of probability, the functions we study are special. We define the function $f(x)$ so that the area between it and the x-axis is equal to a probability. Since the maximum probability is one, the maximum area is also one.

For continuous probability distributions, PROBABILITY = AREA.

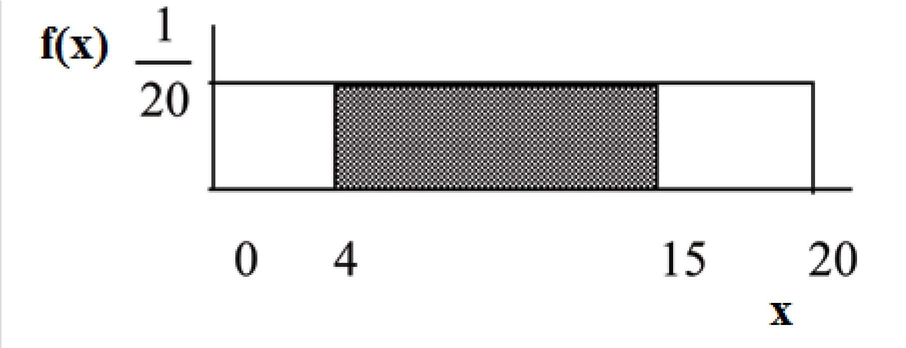
Example:
Consider the function $f(x) = \frac{1}{20}$ for $0 \leq x \leq 20$. x = a real number. The graph of $f(x) = \frac{1}{20}$ is a horizontal line. However, since $0 \leq x \leq 20$, $f(x)$ is restricted to the portion between $x = 0$ and $x = 20$, inclusive .



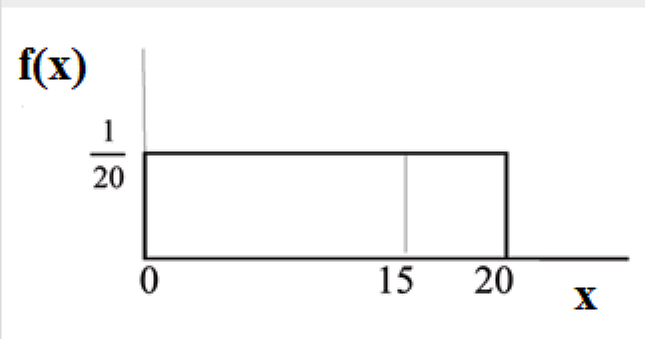
$f(x) = \frac{1}{20}$ **for** $0 \leq x \leq 20$.
The graph of $f(x) = \frac{1}{20}$ is a horizontal line segment when $0 \leq x \leq 20$.
The area between $f(x) = \frac{1}{20}$ where $0 \leq x \leq 20$ and the x-axis is the area of a rectangle with base = 20 and height $= \frac{1}{20}$.
$\text{AREA} = 20 \cdot \frac{1}{20} = 1$
This particular function, where we have restricted x so that the area between the function and the x-axis is 1, is an example of a continuous probability density function. It is used as a tool to calculate probabilities.
Suppose we want to find the area between $f(x) = \frac{1}{20}$ **and the x-axis where** $0 < x < 2$ **.**



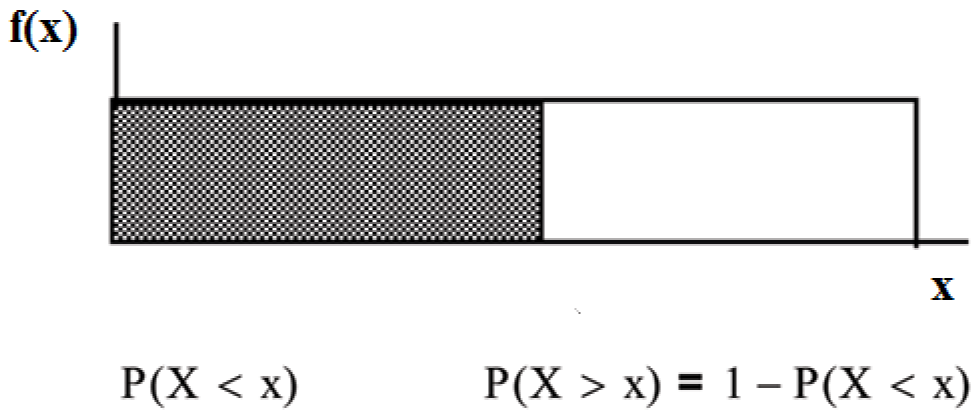
$\text{AREA} = (2 - 0) \cdot \frac{1}{20} = 0.1$
$(2 - 0) = 2 = \text{base of a rectangle}$
$\frac{1}{20}$ = the height.
The area corresponds to a probability. The probability that x is between 0 and 2 is 0.1, which can be written mathematically as
$\text{P}(0<\text{x}<2) = \text{P}(\text{x}<2) = 0.1$.
Suppose we want to find the area between $f(x) = \frac{1}{20}$ **and the x-axis where** $4 < x < 15$ **.**



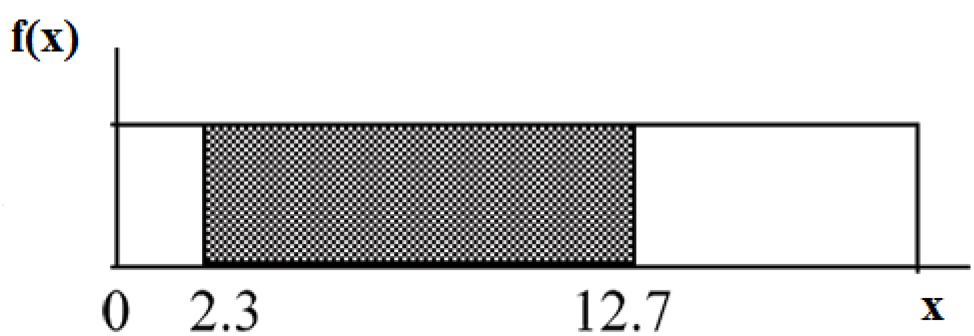
AREA $= (15-4) \cdot \frac{1}{20} = 0.55$
$(15-4) = 11 =$ the base of a rectangle
$\frac{1}{20}$ = the height.
The area corresponds to the probability $P(4 < x < 15) = 0.55$.
Suppose we want to find $P(x{=}15)$**.** On an x-y graph, $x{=}15$ is a vertical line. A vertical line has no width (or 0 width). Therefore, $P(x{=}15) =$ (base)(height) $= (0)\left(\frac{1}{20}\right) = 0$.



$P(X \leq x)$ (can be written as $P(X < x)$ for continuous distributions) is called the cumulative distribution function or CDF. Notice the "less than or equal to" symbol. We can use the CDF to calculate $P(X > x)$. The CDF gives "area to the left" and $P(X > x)$ gives "area to the right." We calculate $P(X > x)$ for continuous distributions as follows: $P(X > x) = 1 - P(X < x)$.



Label the graph with f(x) and x. Scale the x and y axes with the maximum x and y values. $f(x) = \frac{1}{20}$, $0 \leq x \leq 20$.



$P(2.3 < x < 12.7) = (\text{base})(\text{height}) = (12.7 - 2.3)\left(\frac{1}{20}\right) = 0.52$

The Uniform Distribution
Continuous Random Variable: Uniform Distribution is part of the collection col10555 written by Barbara Illowsky and Susan Dean. It describes the properties of the Uniform Distribution with contributions from Roberta Bloom.

Example:
The previous problem is an example of the **uniform probability distribution**.
Illustrate the uniform distribution. The data that follows are 55 smiling times, in seconds, of an eight-week old baby.

10.4	19.6	18.8	13.9	17.8	16.8	21.6	17.9	12.5	11.1	4.9
12.8	14.8	22.8	20.0	15.9	16.3	13.4	17.1	14.5	19.0	22.8
1.3	0.7	8.9	11.9	10.9	7.3	5.9	3.7	17.9	19.2	9.8
5.8	6.9	2.6	5.8	21.7	11.8	3.4	2.1	4.5	6.3	10.7
8.9	9.4	9.4	7.6	10.0	3.3	6.7	7.8	11.6	13.8	18.6

sample mean = 11.49 and sample standard deviation = 6.23
We will assume that the smiling times, in seconds, follow a uniform distribution between 0 and 23 seconds, inclusive. This means that any smiling time from 0 to and including 23 seconds is **equally likely**. The histogram that could be constructed from the sample is an empirical distribution that closely matches the theoretical uniform distribution.
Let X = length, in seconds, of an eight-week old baby's smile.
The notation for the uniform distribution is
X ~ $U(\mathrm{a},b)$ where a = the lowest value of x and b = the highest value of x.
The probability density function is $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$.
For this example, x ~ $U(0{,}23)$ and $f(x) = \frac{1}{23-0}$ for 0$\leq$ x $\leq$23.
Formulas for the theoretical mean and standard deviation are
$\mu = \frac{a+b}{2}$ and $\sigma = \sqrt{\frac{(b-a)^2}{12}}$
For this problem, the theoretical mean and standard deviation are
$\mu = \frac{0+23}{2} = 11.50$ seconds and $\sigma = \sqrt{\frac{(23-0)^2}{12}} = 6.64$ seconds
Notice that the theoretical mean and standard deviation are close to the sample mean and standard deviation.

Example:
Exercise:

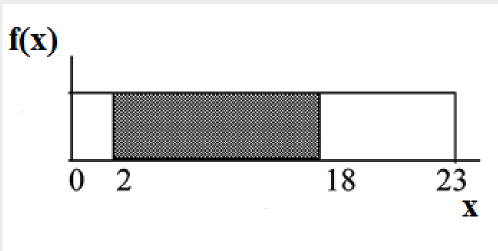
Problem:

What is the probability that a randomly chosen eight-week old baby smiles between 2 and 18 seconds?

Solution:

Find $P(2 < x < 18)$.

$P(2 < x < 18) = (\text{base})(\text{height}) = \left(18 - 2\right) \cdot \frac{1}{23} = \frac{16}{23}$.



Exercise:

Problem: Find the 90th percentile for an eight week old baby's smiling time.

Solution:

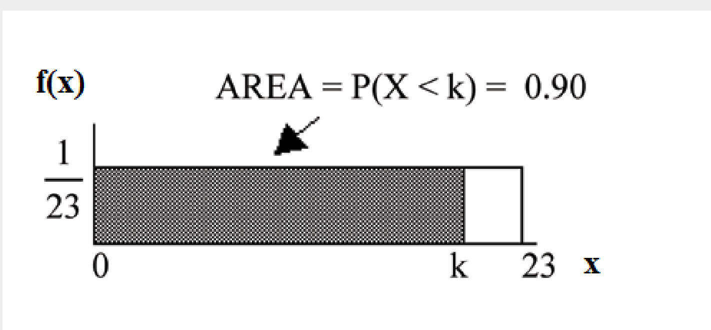
Ninety percent of the smiling times fall below the 90th percentile, k, so $P(x < k) = 0.90$

$P(x < k) = 0.90$

$(\text{base})(\text{height}) = 0.90$

$\left(k - 0\right) \cdot \frac{1}{23} = 0.90$

$k = 23 \cdot 0.90 = 20.7$



Exercise:

Problem:

Find the probability that a random eight week old baby smiles more than 12 seconds **KNOWING** that the baby smiles **MORE THAN 8 SECONDS**.

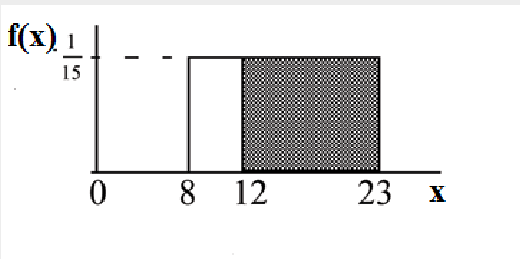
Solution:

Find $P(x > 12|x > 8)$ There are two ways to do the problem. **For the first way**, use the fact that this is a **conditional** and changes the sample space. The graph illustrates the new sample space. You already know the baby smiled more than 8 seconds.

Write a new $f(x)$: $f(x) = \frac{1}{23-8} = \frac{1}{15}$

for $8 < x < 23$

$P(x > 12|x > 8) = \left(23 - 12\right) \cdot \frac{1}{15} = \frac{11}{15}$



For the second way, use the conditional formula from **Probability Topics** with the original distribution $X \sim U(0, 23)$:

$P\left(A\middle|B\right) = \frac{P(A \text{ AND } B)}{P(B)}$ For this problem, A is $(x > 12)$ and B is $(x > 8)$.

So, $P(x > 12|x > 8) = \frac{(x>12 \text{ AND } x>8)}{P(x>8)} = \frac{P(x>12)}{P(x>8)} = \frac{\frac{11}{23}}{\frac{15}{23}} = 0.733$

Example:
Uniform: The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between 0 and 15 minutes, inclusive.
Exercise:

Problem: What is the probability that a person waits fewer than 12.5 minutes?

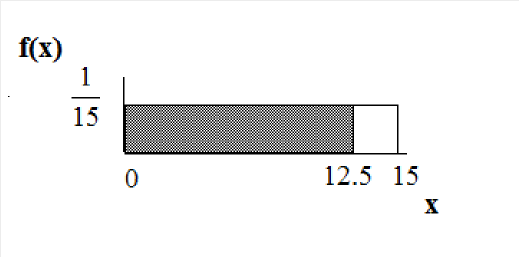
Solution:

Let X = the number of minutes a person must wait for a bus. a = 0 and b = 15. x~$U(0, 15)$. Write the probability density function. $f(x) = \frac{1}{15-0} = \frac{1}{15}$ for 0≤ x ≤15.

Find $P(x < 12.5)$. Draw a graph.

$P(x < k) = (\text{base})(\text{height}) = \left(12.5 - 0\right) \cdot \frac{1}{15} = 0.8333$

The probability a person waits less than 12.5 minutes is 0.8333.



Exercise:

Problem: On the average, how long must a person wait?

Find the mean, μ, and the standard deviation, σ.

Solution:

$\mu = \frac{a+b}{2} = \frac{15+0}{2} = 7.5$. On the average, a person must wait 7.5 minutes.

$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(15-0)^2}{12}} = 4.3$. The Standard deviation is 4.3 minutes.

Exercise:

Problem: Ninety percent of the time, the time a person must wait falls below what value?

Note: This asks for the 90th percentile.

Solution:

Find the 90th percentile. Draw a graph. Let k = the 90th percentile.

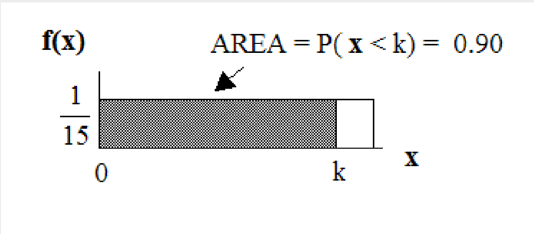
$P(x < k) = (\text{base})(\text{height}) = \left(k - 0\right) \cdot \left(\frac{1}{15}\right)$

$0.90 = k \cdot \frac{1}{15}$

$\mathrm{k} = (0.90)(15) = 13.5$

k is sometimes called a critical value.

The 90th percentile is 13.5 minutes. Ninety percent of the time, a person must wait at most 13.5 minutes.



Example:
Uniform: Suppose the time it takes a nine-year old to eat a donut is between 0.5 and 4 minutes, inclusive. Let X = the time, in minutes, it takes a nine-year old child to eat a donut. Then X ~ $U(0.5, 4)$.

Exercise:

Problem:

The probability that a randomly selected nine-year old child eats a donut in at least two minutes is _______.

Solution:

0.5714

Exercise:

Problem:

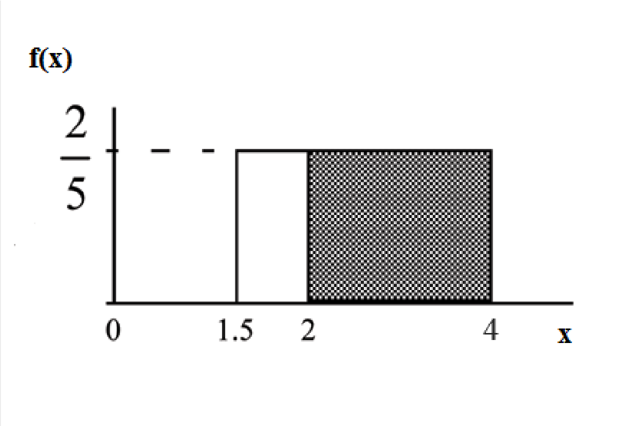
Find the probability that a different nine-year old child eats a donut in more than 2 minutes given that the child has already been eating the donut for more than 1.5 minutes.

The second probability question has a **conditional** (refer to "Probability Topics"). You are asked to find the probability that a nine-year old child eats a donut in more than 2 minutes given that the child has already been eating the donut for more than 1.5 minutes. Solve the problem two different ways (see the first example). You must reduce the sample space. **First way**: Since you already know the child has already been eating the donut for more than 1.5 minutes, you are no longer starting at $\mathrm{a} = 0.5$ minutes. Your starting point is 1.5 minutes.

Write a new f(x):

$f(x) = \frac{1}{4-1.5} = \frac{2}{5}$ for 1.5$\le x \le$4.

Find $P(x > 2 | x > 1.5)$. Draw a graph.



$P(x > 2|x > 1.5) = (\text{base})(\text{new height}) = (4 - 2)(2/5) = ?$

Solution:

$\frac{4}{5}$

The probability that a nine-year old child eats a donut in more than 2 minutes given that the child has already been eating the donut for more than 1.5 minutes is $\frac{4}{5}$.

Second way: Draw the original graph for $x \sim U(0.5, 4)$. Use the conditional formula

$P(x > 2|x > 1.5) = \frac{P(x>2 \text{ AND } x>1.5)}{P(x>1.5)} = \frac{P(x>2)}{P(x>1.5)} = \frac{\frac{2}{3.5}}{\frac{2.5}{3.5}} = 0.8 = \frac{4}{5}$

Note: See "Summary of the Uniform and Exponential Probability Distributions" for a full summary.

Example:

Uniform: Ace Heating and Air Conditioning Service finds that the amount of time a repairman needs to fix a furnace is uniformly distributed between 1.5 and 4 hours. Let x = the time needed to fix a furnace. Then $x \sim U(1.5, 4)$.

1. Find the problem that a randomly selected furnace repair requires more than 2 hours.
2. Find the probability that a randomly selected furnace repair requires less than 3 hours.
3. Find the 30th percentile of furnace repair times.
4. The longest 25% of repair furnace repairs take at least how long? (In other words: Find the minimum time for the longest 25% of repair times.) What percentile does this represent?
5. Find the mean and standard deviation

Exercise:

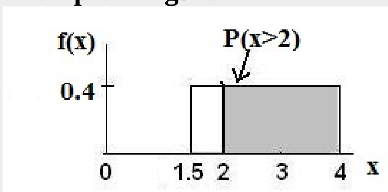
Problem: Find the probability that a randomly selected furnace repair requires longer than 2 hours.

Solution:

To find $f(x)$: $f(x) = \frac{1}{4-1.5} = \frac{1}{2.5}$ so $f(x)$ =0.4

P(x>2) = (base)(height) = (4 − 2)(0.4) = 0.8

Example 4 Figure 1



Uniform Distribution between 1.5 and 4 with shaded area between 2 and 4 representing the probability that the repair time x is greater than 2

Exercise:

Problem:

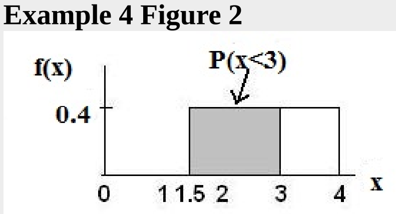
Find the probability that a randomly selected furnace repair requires less than 3 hours. Describe how the graph differs from the graph in the first part of this example.

Solution:

$P(x<3)$ = (base)(height) = (3 − 1.5)(0.4) = 0.6

The graph of the rectangle showing the entire distribution would remain the same. However the graph should be shaded between x=1.5 and x=3. Note that the shaded area starts at x=1.5 rather than at x=0; since X~U(1.5,4), x can not be less than 1.5.

Example 4 Figure 2



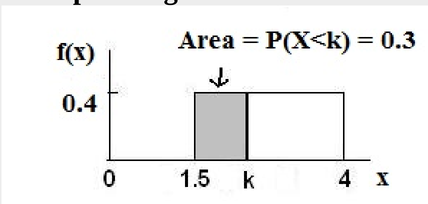
Uniform Distribution between 1.5 and 4 with shaded area between 1.5 and 3 representing the probability that the repair time x is less than 3

Exercise:

Problem: Find the 30th percentile of furnace repair times.

Solution:

Example 4 Figure 3



Uniform Distribution between 1.5 and 4 with an area of 0.30 shaded to the left, representing the shortest 30% of repair times.

$$P(x < k) = 0.30$$

$$P(x < k) = (\text{base})(\text{height}) = (k - 1.5) \cdot (0.4)$$

- **0.3 = (k − 1.5) (0.4)** ; Solve to find k:
- 0.75 = k − 1.5 , obtained by dividing both sides by 0.4
- **k = 2.25** , obtained by adding 1.5 to both sides

The 30th percentile of repair times is 2.25 hours. 30% of repair times are 2.5 hours or less.

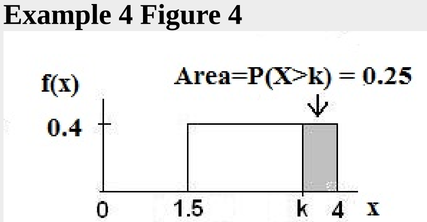
Exercise:

Problem:

The **longest 25%** of furnace repair times take **at least** how long? (Find the minimum time for the longest 25% of repairs.)

Solution:

Example 4 Figure 4



Uniform Distribution between 1.5 and 4 with an area of 0.25 shaded to the right representing the longest 25% of repair times.

$$P(x > k) = 0.25$$

$$P(x > k) = (\text{base})(\text{height}) = (4 - k) \cdot (0.4)$$

- **0.25 = (4 − k)(0.4)** ; Solve for k:

- 0.625 = 4 − k , obtained by dividing both sides by 0.4
- −3.375 = −k , obtained by subtracting 4 from both sides
- **k=3.375**

The longest 25% of furnace repairs take at least 3.375 hours (3.375 hours or longer).

Note: Since 25% of repair times are 3.375 hours or longer, that means that 75% of repair times are 3.375 hours or less. 3.375 hours is the **75th percentile** of furnace repair times.

Exercise:

Problem: Find the mean and standard deviation

Solution:

$\mu = \frac{a+b}{2}$ and $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

$\mu = \frac{1.5+4}{2} = 2.75$ hours and $\sigma = \sqrt{\frac{(4-1.5)^2}{12}} = 0.7217$ hours

Note: See "Summary of the Uniform and Exponential Probability Distributions" for a full summary.

**Example 5 contributed by Roberta Bloom

Glossary

Conditional Probability
: The likelihood that an event will occur given that another event has already occurred.

Uniform Distribution
: A continuous random variable (RV) that has equally likely outcomes over the domain, $a < x < b$. Often referred as the **Rectangular distribution** because the graph of the pdf has the form of a rectangle. Notation: $X \text{~} U(a,b)$. The mean is $\mu = \frac{a+b}{2}$ and the standard deviation is $\sigma = \sqrt{\frac{(b-a)^2}{12}}$ The probability density function is $f(x) = \frac{1}{b-a}$ for $a < x < b$ or $a \le x \le b$. The cumulative distribution is $P(X \le x) = \frac{x-a}{b-a}$.

Practice: Uniform Distribution
In this module the student will explore the properties of data with a uniform distribution.

Student Learning Outcomes

- The student will analyze data following a uniform distribution.

Given

The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.

Describe the Data

Exercise:

Problem: What is being measured here?

Solution:

The age of cars in the staff parking lot

Exercise:

Problem: In words, define the Random Variable X.

Solution:

X = The age (in years) of cars in the staff parking lot

Exercise:

Problem: Are the data discrete or continuous?

Solution:

Continuous

Exercise:

Problem: The interval of values for x is:

Solution:

0.5 - 9.5

Exercise:

Problem: The distribution for X is:

Solution:

$X \sim U(0.5, 9.5)$

Probability Distribution

Exercise:

Problem: Write the probability density function.

Solution:

$f(x) = \frac{1}{9}$

Exercise:

Problem: Graph the probability distribution.

- **a**Sketch the graph of the probability distribution.

- **b**Identify the following values:
    - **i**Lowest value for x:
    - **ii**Highest value for x:
    - **iii**Height of the rectangle:
    - **iv**Label for x-axis (words):
    - **v**Label for y-axis (words):

Solution:

- **b.i**0.5
- **b.ii**9.5
- **b.iii** $\frac{1}{9}$
- **b.iv**Age of Cars
- **b.v** $f(x)$

Random Probability

Exercise:

Problem:

Find the probability that a randomly chosen car in the lot was less than 4 years old.

- **a**Sketch the graph. Shade the area of interest.

- **b**Find the probability. $P(x < 4)$ =

Solution:

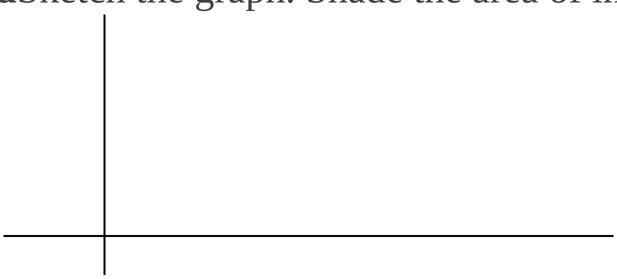
- **b.** $\frac{3.5}{9}$

Exercise:

Problem:

Out of just the cars less than 7.5 years old, find the probability that a randomly chosen car in the lot was less than 4 years old.

- **a**Sketch the graph. Shade the area of interest.



- **b**Find the probability. $P(x < 4 \mid x < 7.5)$ =

Solution:

- **b** $\frac{3.5}{7}$

Exercise:
Discussion Question

Problem:

What has changed in the previous two problems that made the solutions different?

Quartiles

Exercise:

Problem: Find the average age of the cars in the lot.

Solution:

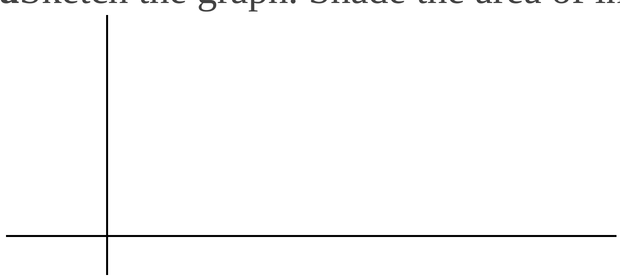
μ = 5

Exercise:

Problem:

Find the third quartile of ages of cars in the lot. This means you will have to find the value such that $\frac{3}{4}$, or 75%, of the cars are at most (less than or equal to) that age.

- **a**Sketch the graph. Shade the area of interest.



- **b**Find the value k such that $P(x < k) = 0.75$.
- **c**The third quartile is:

Solution:

- **b**k = 7.25

Summary of The Uniform Distribution
This module provides a summary of formulas and definitions related to Continuous Random Variables.

Formula

Uniform

X = a real number between a and b (in some instances, X can take on the values a and b). a = smallest X ; b = largest X

X ~ U(a,b)

The mean is $\mu = \frac{a+b}{2}$

The standard deviation is $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

Probability density function: $f(X) = \frac{1}{b-a}$ for $a \le X \le b$

Area to the Left of x: $P(X < x) = \text{(base)(height)}$

Area to the Right of x: $P(X > x) = \text{(base)(height)}$

Area Between c and d:
$P(c < X < d) = \text{(base)(height)} = (d - c)\text{(height)}.$

Exercises
This module provides a number of homework exercises related to Continuous Random Variables.

For each probability and percentile problem, DRAW THE PICTURE!
Exercise:

Problem:

Consider the following experiment. You are one of 100 people enlisted to take part in a study to determine the percent of nurses in America with an R.N. (registered nurse) degree. You ask nurses if they have an R.N. degree. The nurses answer “yes” or “no.” You then calculate the percentage of nurses with an R.N. degree. You give that percentage to your supervisor.

- **a**What part of the experiment will yield discrete data?
- **b** What part of the experiment will yield continuous data?

Exercise:

Problem:

When age is rounded to the nearest year, do the data stay continuous, or do they become discrete? Why?

Exercise:

Problem:

Births are approximately uniformly distributed between the 52 weeks of the year. They can be said to follow a Uniform Distribution from 1 – 53 (spread of 52 weeks).

- **a** X~
- **b** Graph the probability distribution.
- **c** $f(x)$=
- **d** μ=
- **e** σ=

- **f** Find the probability that a person is born at the exact moment week 19 starts. That is, find $P(X = 19)$.
- **g** $P(2 < X < 31) =$
- **h** Find the probability that a person is born after week 40.
- **i** $P(12 < X \mid X < 28)$=
- **j** Find the 70th percentile.
- **k** Find the minimum for the upper quarter.

Solution:

- **a** X~$U(1{,}53)$
- **c** $f(x) = \frac{1}{52}$ where $1 \leq x \leq 53$
- **d** 27
- **e**15.01
- **f** 0
- **g** $\frac{29}{52}$
- **h** $\frac{13}{52}$
- **i** $\frac{16}{27}$
- **j** 37.4
- **k** 40

Exercise:

Problem:

A random number generator picks a number from 1 to 9 in a uniform manner.

- **a** X~
- **b** Graph the probability distribution.
- **c** $f(x) =$
- **d** $\mu =$
- **e** $\sigma =$
- **f** $P(3.5 < X < 7.25) =$
- **g** $P(X > 5.67) =$
- **h** $P(X > 5 \mid X > 3) =$

- **i** Find the 90th percentile.

Exercise:

Problem:

The speed of cars passing through the intersection of Blossom Hill Road and the Almaden Expressway varies from 10 to 35 mph and is uniformly distributed. None of the cars travel over 35 mph through the intersection.

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** What is the probability that the speed of a car is at most 30 mph?
- **h** What is the probability that the speed of a car is between 16 and 22 mph.
- **i** $P(20 < X < 53) =$ State this in a probability question (similar to **g** and **h**), draw the picture, and find the probability.
- **j** Find the 90th percentile. This means that 90% of the time, the speed is less than _____ mph while passing through the intersection per minute.
- **k** Find the 75th percentile. In a complete sentence, state what this means. (See **j**.)
- **l** Find the probability that the speed is more than 24 mph given (or knowing that) it is at least 15 mph.

Solution:

- **b** X~$U(10{,}35)$
- **d** $f(x) = \frac{1}{25}$ where $10 \leq X \leq 35$
- **e** $\frac{45}{2}$

- **f** 7.22
- **g** $\frac{4}{5}$
- **h** $\frac{6}{25}$
- **i** $\frac{3}{5}$
- **j** 32.5
- **k** 28.75
- **l** $\frac{11}{20}$

Exercise:

Problem:

According to a study by Dr. John McDougall of his live-in weight loss program at St. Helena Hospital, the people who follow his program lose between 6 and 15 pounds a month until they approach trim body weight. Let's suppose that the weight loss is uniformly distributed. We are interested in the weight loss of a randomly selected individual following the program for one month. (Source: **The McDougall Program for Maximum Weight Loss** by John A. McDougall, M.D.)

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** Find the probability that the individual lost more than 10 pounds in a month.
- **h** Suppose it is known that the individual lost more than 10 pounds in a month. Find the probability that he lost less than 12 pounds in the month.
- **i** $P(7 < X < 13 \mid X > 9) =$ State this in a probability question (similar to g and h), draw the picture, and find the probability.

Exercise:

Problem:

A subway train on the Red Line arrives every 8 minutes during rush hour. We are interested in the length of time a commuter must wait for a train to arrive. The time follows a uniform distribution.

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** Find the probability that the commuter waits less than one minute.
- **h** Find the probability that the commuter waits between three and four minutes.
- **i** 60% of commuters wait more than how long for the train? State this in a probability question (similar to **g** and **h**), draw the picture, and find the probability.

Solution:

- **b** X~$U(0,8)$
- **d** $f(x) = \frac{1}{8}$ where $0 \le X \le 8$
- **e** 4
- **f** 2.31
- **g** $\frac{1}{8}$
- **h** $\frac{1}{8}$
- **i** 3.2

Exercise:

Problem:

The age of a first grader on September 1 at Garden Elementary School is uniformly distributed from 5.8 to 6.8 years. We randomly select one first grader from the class.

- **a** $X =$
- **b** X~
- **c** Graph the probability distribution.
- **d** $f(x) =$
- **e** $\mu =$
- **f** $\sigma =$
- **g** Find the probability that she is over 6.5 years.
- **h** Find the probability that she is between 4 and 6 years.
- **i** Find the 70th percentile for the age of first graders on September 1 at Garden Elementary School.

Try these multiple choice problems

The next three questions refer to the following information. The Sky Train from the terminal to the rental car and long term parking center is supposed to arrive every 8 minutes. The waiting times for the train are known to follow a uniform distribution.
Exercise:

Problem: What is the average waiting time (in minutes)?

- **A** 0.0000
- **B** 2.0000
- **C** 3.0000
- **D** 4.0000

Solution:

D

Exercise:

Problem: Find the 30th percentile for the waiting times (in minutes).

- **A** 2.0000
- **B** 2.4000
- **C** 2.750
- **D** 3.000

Solution:

B

Exercise:

Problem:

The probability of waiting more than 7 minutes given a person has waited more than 4 minutes is?

- **A** 0.1250
- **B** 0.2500
- **C** 0.5000
- **D** 0.7500

Solution:

B

Review

This module provides a number of homework/review problems related to Continuous Random Variables.

[link] – [link] refer to the following study: A recent study of mothers of junior high school children in Santa Clara County reported that 76% of the mothers are employed in paid positions. Of those mothers who are employed, 64% work full-time (over 35 hours per week), and 36% work part-time. However, out of all of the mothers in the population, 49% work full-time. The population under study is made up of mothers of junior high school children in Santa Clara County.

Let E =employed, Let F =full-time employment

Exercise:

Problem:

- **a** Find the percent of all mothers in the population that NOT employed.
- **b** Find the percent of mothers in the population that are employed part-time.

Solution:

- **a** 24%
- **b** 27%

Exercise:

Problem:

The type of employment is considered to be what type of data?

Solution:

Qualitative

Exercise:

Problem: In symbols, what does the 36% represent?

Solution:

$P(\mathrm{PT} \mid E)$

Exercise:

Problem:

Find the probability that a randomly selected person from the population will be employed OR work full-time.

Solution:

0.7336

Exercise:

Problem:

Based upon the above information, are being employed AND working part-time:

- **a** mutually exclusive events? Why or why not?
- **b** independent events? Why or why not?

Solution:

- **a** No,
- **b** No,

[link] - [link] refer to the following: We randomly pick 10 mothers from the above population. We are interested in the number of the mothers that are employed. Let X =number of mothers that are employed.

Exercise:

Problem: State the distribution for X.

Solution:

$B(10,0.76)$

Exercise:

Problem: Find the probability that at least 6 are employed.

Solution:

0.9330

Exercise:

Problem:

A person invests $1000 in stock of a company that hopes to go public in 1 year.

- The probability that the person will lose all his money after 1 year (i.e. his stock will be worthless) is 35%.
- The probability that the person's stock will still have a value of $1000 after 1 year (i.e. no profit and no loss) is 60%.
- The probability that the person's stock will increase in value by $10,000 after 1 year (i.e. will be worth $11,000) is 5%.

Find the expected PROFIT after 1 year.

Solution:

$150

Exercise:

Problem:

Rachel's piano cost $3000. The average cost for a piano is $4000 with a standard deviation of $2500. Becca's guitar cost $550. The average cost for a guitar is $500 with a standard deviation of $200. Matt's drums cost $600. The average cost for drums is $700 with a standard deviation of $100. Whose cost was lowest when compared to his or her own instrument? Justify your answer.

Solution:

Matt

Exercise:

Problem:

For the following data, which of the measures of central tendency would be the LEAST useful: mean, median, mode? Explain why. Which would be the MOST useful? Explain why.

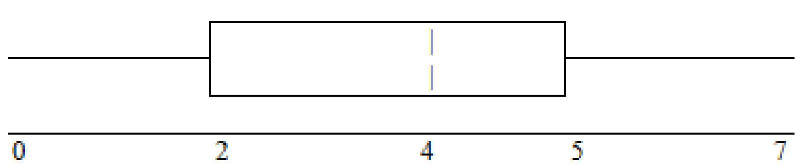
4,6,6,12,18,18,18,200

Solution:

Mean

Exercise:

Problem:



For each statement below, explain why each is either true or false.

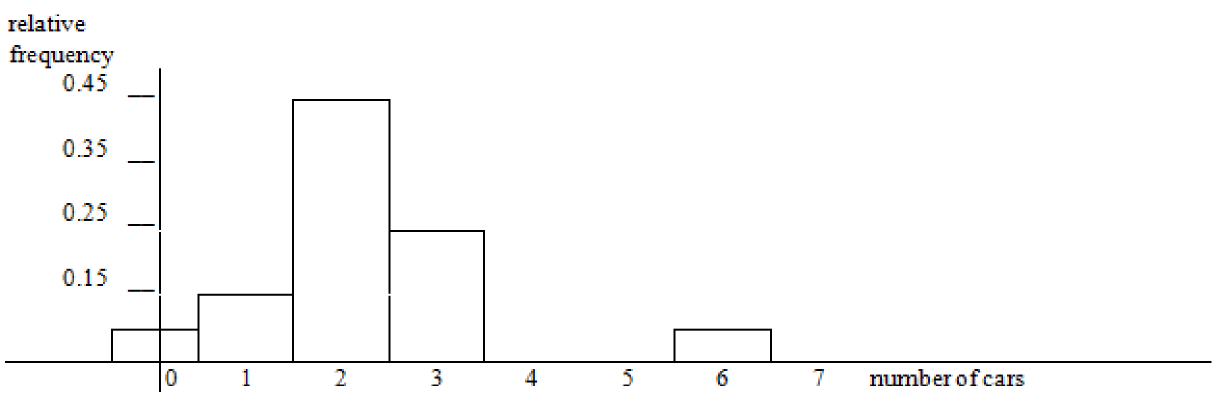
- **a** 25% of the data are at most 5.

- **b** There is the same amount of data from 4 – 5 as there is from 5 – 7.
- **c** There are no data values of 3.
- **d** 50% of the data are 4.

Solution:

- **a** False
- **b** True
- **c** False
- **d** False

[link] – [link] **refer to the following:** 64 faculty members were asked the number of cars they owned (including spouse and children's cars). The results are given in the following graph:



Exercise:

Problem: Find the approximate number of responses that were "3."

Solution:

16

Exercise:

Problem:

Find the first, second and third quartiles. Use them to construct a box plot of the data.

Solution:

2,2,3

[link] – [link] refer to the following study done of the Girls soccer team "Snow Leopards":

Hair Style		Hair Color	
	blond	brown	black
ponytail	3	2	5
plain	2	2	1

Suppose that one girl from the Snow Leopards is randomly selected.

Exercise:

Problem:

Find the probability that the girl has black hair GIVEN that she wears a ponytail.

Solution:

$\frac{5}{10} = 0.5$

Exercise:

Problem:

Find the probability that the girl wears her hair plain OR has brown hair.

Solution:

$\frac{7}{15}$

Exercise:

Problem:

Find the probability that the girl has blond hair AND that she wears her hair plain.

Solution:

$\frac{2}{15}$

The Normal Distribution

Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Recognize the normal probability distribution and apply it appropriately.
- Recognize the standard normal probability distribution and apply it appropriately.
- Compare normal probabilities by converting to the standard normal distribution.

Introduction

The normal, a continuous distribution, is the most important of all the distributions. It is widely used and even more widely abused. Its graph is bell-shaped. You see the bell curve in almost all disciplines. Some of these include psychology, business, economics, the sciences, nursing, and, of course, mathematics. Some of your instructors may use the normal distribution to help determine your grade. Most IQ scores are normally distributed. Often real estate prices fit a normal distribution. The normal distribution is extremely important but it cannot be applied to everything in the real world.

In this chapter, you will study the normal distribution, the standard normal, and applications associated with them.

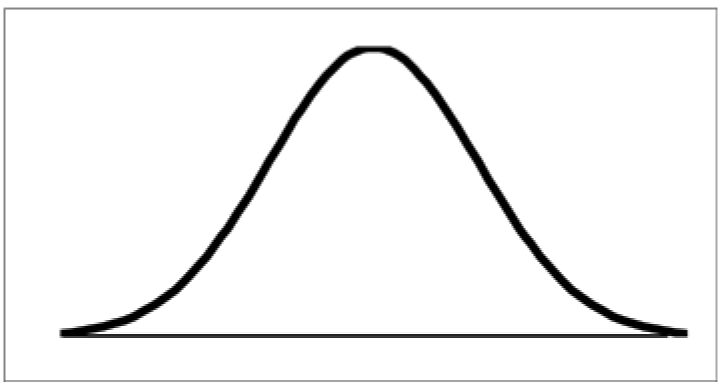
Optional Collaborative Classroom Activity

Your instructor will record the heights of both men and women in your class, separately. Draw histograms of your data. Then draw a smooth curve through each histogram. Is each curve somewhat bell-shaped? Do you think that if you had recorded 200 data values for men and 200 for women that the curves would look bell-shaped? Calculate the mean for each data set. Write the means on the x-axis of the appropriate graph below the peak.

Shade the approximate area that represents the probability that one randomly chosen male is taller than 72 inches. Shade the approximate area that represents the probability that one randomly chosen female is shorter than 60 inches. If the total area under each curve is one, does either probability appear to be more than 0.5?

The normal distribution has two parameters (two numerical descriptive measures), the mean (μ) and the standard deviation (σ). If X is a quantity to be measured that has a normal distribution with mean (μ) and the standard deviation (σ), we designate this by writing

NORMAL: X~N(μ, σ)



The probability density function is a rather complicated function. **Do not memorize it**. It is not necessary.

$$f\left(x\right) = \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2}$$

The cumulative distribution function is $P(X < x)$. It is calculated either by a calculator or a computer or it is looked up in a table. Technology has made the tables basically obsolete. For that reason, as well as the fact that there are various table formats, we are not including table instructions in this chapter. See the NOTE in this chapter in **Calculation of Probabilities**.

The curve is symmetrical about a vertical line drawn through the mean, μ. In theory, the mean is the same as the median since the graph is symmetric about μ. As the notation indicates, the normal distribution depends only on the mean and the standard deviation. Since the area under the curve must

equal one, a change in the standard deviation, σ, causes a change in the shape of the curve; the curve becomes fatter or skinnier depending on σ. A change in μ causes the graph to shift to the left or right. This means there are an infinite number of normal probability distributions. One of special interest is called the **standard normal distribution**.

Glossary

Normal Distribution

A continuous random variable (RV) with pdf $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean of the distribution and σ is the standard deviation. Notation: X ~ $N(\mu, \sigma)$. If $\mu = 0$ and $\sigma = 1$, the RV is called **the standard normal distribution**.

The Standard Normal Distribution

The **standard normal distribution** is a normal distribution of **standardized values called z-scores. A z-score is measured in units of the standard deviation.** For example, if the mean of a normal distribution is 5 and the standard deviation is 2, the value 11 is 3 standard deviations above (or to the right of) the mean. The calculation is:

Equation:

$$x = \mu + (z)\sigma = 5 + (3)(2) = 11$$

The z-score is 3.

The mean for the standard normal distribution is 0 and the standard deviation is 1. The transformation

$z = \frac{x-\mu}{\sigma}$ produces the distribution $Z \sim N(0,1)$. The value x comes from a normal distribution with mean μ and standard deviation σ.

Glossary

Standard Normal Distribution

A continuous random variable (RV) $X \sim N(0,1)$.. When X follows the standard normal distribution, it is often noted as $Z \sim N(0,1)$.

z-score

The linear transformation of the form $z = \frac{x-\mu}{\sigma}$. If this transformation is applied to any normal distribution $X \sim N(\mu,\sigma)$, the result is the standard normal distribution $Z \sim N(0,1)$. If this transformation is applied to any specific value x of the RV with mean μ and standard deviation σ , the result is called the z-score of x. Z-scores allow us to compare data that are normally distributed but scaled differently.

Z-scores

If X is a normally distributed random variable and X~N(μ, σ), then the z-score is:
Equation:

$$z = \frac{x - \mu}{\sigma}$$

The z-score tells you how many standard deviations that the value x is above (to the right of) or below (to the left of) the mean, μ. Values of x that are larger than the mean have positive z-scores and values of x that are smaller than the mean have negative z-scores. If x equals the mean, then x has a z-score of 0.

Example:
Suppose X ~ N(5, 6). This says that X is a normally distributed random variable with mean μ = 5 and standard deviation σ = 6. Suppose x = 17. Then:
Equation:

$$z = \frac{x - \mu}{\sigma} = \frac{17 - 5}{6} = 2$$

This means that x = 17 is **2 standard deviations** (2σ) above or to the right of the mean μ = 5. The standard deviation is σ = 6.
Notice that:
Equation:

$$5 + 2 \cdot 6 = 17 \qquad \text{(The pattern is } \mu + z\sigma = x.\text{)}$$

Now suppose x=1. Then:
Equation:

$$z = \frac{x - \mu}{\sigma} = \frac{1 - 5}{6} = -0.67 \qquad \text{(rounded to two decimal places)}$$

This means that $\mathrm{x} = 1$ **is 0.67 standard deviations** $(\text{- } 0.67\sigma)$ **below or to the left of the mean** $\mu = 5$**. Notice that:**
$5 + (-0.67)(6)$ is approximately equal to 1 (This has the pattern $\mu + (-0.67)\sigma = 1$)
Summarizing, when z is positive, x is above or to the right of μ and when z is negative, x is to the left of or below μ.

Example:
Some doctors believe that a person can lose 5 pounds, on the average, in a month by reducing his/her fat intake and by exercising consistently. Suppose weight loss has a normal distribution. Let X = the amount of weight lost (in pounds) by a person in a month. Use a standard deviation of 2 pounds. X~N$(5, 2)$. Fill in the blanks.

Exercise:

Problem:

Suppose a person **lost** 10 pounds in a month. The z-score when $\mathrm{x} = 10$ pounds is $\mathrm{z} = 2.5$ (verify). This z-score tells you that $\mathrm{x} = 10$ is _________ standard deviations to the _________ (right or left) of the mean ______ (What is the mean?).

Solution:

This z-score tells you that $\mathrm{x} = 10$ is **2.5** standard deviations to the **right** of the mean **5**.

Exercise:

Problem:

Suppose a person **gained** 3 pounds (a negative weight loss). Then z = ___________. This z-score tells you that $\mathrm{x} =$ -3 is _________ standard deviations to the ___________ (right or left) of the mean.

Solution:

z = **-4**. This z-score tells you that x = -3 is **4** standard deviations to the **left** of the mean.

Suppose the random variables X and Y have the following normal distributions: X ~N$(5, 6)$ and Y ~ N$(2, 1)$. If x = 17, then $z = 2$. (This was previously shown.) If y = 4, what is z?

Equation:

$$z = \frac{y - \mu}{\sigma} = \frac{4 - 2}{1} = 2 \qquad \text{where } \mu\text{=2 and } \sigma\text{=1.}$$

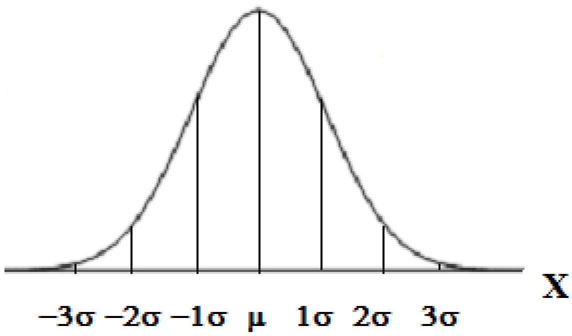
The z-score for y = 4 is z = 2. This means that 4 is z = 2 standard deviations to the right of the mean. Therefore, x = 17 and y = 4 are both 2 (of **their**) standard deviations to the right of **their** respective means.

The z-score allows us to compare data that are scaled differently. To understand the concept, suppose X ~N$(5, 6)$ represents weight gains for one group of people who are trying to gain weight in a 6 week period and Y ~N$(2, 1)$ measures the same weight gain for a second group of people. A negative weight gain would be a weight loss. Since x = 17 and y = 4 are each 2 standard deviations to the right of their means, they represent the same weight gain **relative to their means**.

The Empirical Rule

If X is a random variable and has a normal distribution with mean μ and standard deviation σ then the **Empirical Rule** says (See the figure below)

- About 68.27% of the x values lie between -1σ and +1σ of the mean μ (within 1 standard deviation of the mean).
- About 95.45% of the x values lie between -2σ and +2σ of the mean μ (within 2 standard deviations of the mean).
- About 99.73% of the x values lie between -3σ and +3σ of the mean μ (within 3 standard deviations of the mean). Notice that almost all the x values lie within 3 standard deviations of the mean.
- The z-scores for +1σ and –1σ are +1 and -1, respectively.
- The z-scores for +2σ and –2σ are +2 and -2, respectively.
- The z-scores for +3σ and –3σ are +3 and -3 respectively.



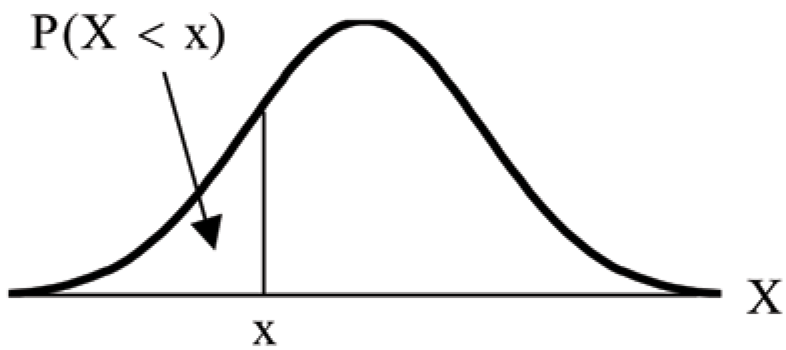
The Empirical Rule is also known as the 68-95-99.7 Rule.

Example:
Suppose X has a normal distribution with mean 50 and standard deviation 6.

- About 68.27% of the x values lie between -1σ = (-1)(6) = -6 and 1σ = (1)(6) = 6 of the mean 50. The values 50 - 6 = 44 and 50 + 6 = 56 are within 1 standard deviation of the mean 50. The z-scores are -1 and +1 for 44 and 56, respectively.
- About 95.45% of the x values lie between -2σ = (-2)(6) = -12 and 2σ = (2)(6) = 12 of the mean 50. The values 50 - 12 = 38 and 50 + 12 = 62 are within 2 standard deviations of the mean 50. The z-scores are -2 and 2 for 38 and 62, respectively.
- About 99.73% of the x values lie between -3σ = (-3)(6) = -18 and 3σ = (3)(6) = 18 of the mean 50. The values 50 - 18 = 32 and 50 + 18 = 68 are within 3 standard deviations of the mean 50. The z-scores are -3 and +3 for 32 and 68, respectively.

Normal Distribution: Areas to the Left and Right of x

The arrow in the graph below points to the area to the left of x. This area is represented by the probability $P(X < x)$**.** Normal tables, computers, and calculators provide or calculate the probability $P(X < x)$.



The area to the right is then $P(X > x) = 1 - P(X < x)$**.**

Remember, $P(X < x) =$ **Area to the left** of the vertical line through x.

$P(X > x) = 1 - P(X < x) =$. **Area to the right** of the vertical line through x

$P(X < x)$ is the same as $P(X \leq x)$ and $P(X > x)$ is the same as $P(X \geq x)$ for continuous distributions.

Calculations of Probabilities

Probabilities are calculated by using technology. There are instructions in the chapter for the TI-83+ and TI-84 calculators.

Note: In the Table of Contents for **Collaborative Statistics**, entry **15. Tables** has a link to a table of normal probabilities. Use the probability tables if so desired, instead of a calculator. The tables include instructions for how to use then.

Example:
If the area to the left is 0.0228, then the area to the right is $1 - 0.0228 = 0.9772$.

Example:
The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of 5.

Exercise:

Problem:

Find the probability that a randomly selected student scored more than 65 on the exam.

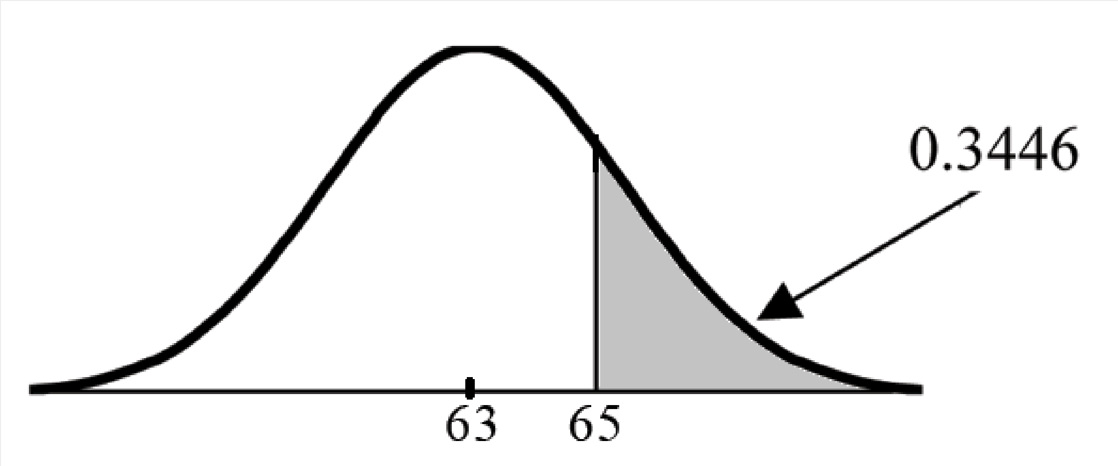
Solution:

Let $X =$ a score on the final exam. X~$N(63, 5)$, where $\mu = 63$ and $\sigma = 5$

Draw a graph.

Then, find $P(x > 65)$.

$P(x > 65) = 0.3446$ (calculator or computer)



The probability that one student scores more than 65 is 0.3446.

Using the TI-83+ or the TI-84 calculators, the calculation is as follows. Go into `2nd DISTR`.

After pressing `2nd DISTR`, press `2:normalcdf`.

The syntax for the instructions are shown below.

normalcdf(lower value, upper value, mean, standard deviation) For this problem: normalcdf(65,1E99,63,5) = 0.3446. You get 1E99 (= 10^{99}) by pressing `1`, the `EE` key (a 2nd key) and then `99`. Or, you can enter `10^99` instead. The number 10^{99} is way out in the right tail of the normal curve. We are calculating the area between 65 and 10^{99}. In some instances, the lower number of the area might be -1E99 (= -10^{99}). The number -10^{99} is way out in the left tail of the normal curve.

Note: The TI probability program calculates a z-score and then the probability from the z-score. Before technology, the z-score was looked up in a standard normal probability table (because the math involved is too cumbersome) to find the probability. In this example,

a standard normal table with area to the left of the z-score was used. You calculate the z-score and look up the area to the left. The probability is the area to the right.

$z = \frac{65-63}{5} = 0.4$. Area to the left is 0.6554.
$P(x > 65) = P(z > 0.4) = 1 - 0.6554 = 0.3446$

Exercise:

Problem:

Find the probability that a randomly selected student scored less than 85.

Solution:

Draw a graph.

Then find $P(x < 85)$. Shade the graph. $P(x < 85) = 1$ (calculator or computer)

The probability that one student scores less than 85 is approximately 1 (or 100%).

The TI-instructions and answer are as follows:

normalcdf(0,85,63,5) = 1 (rounds to 1)

Exercise:

Problem:

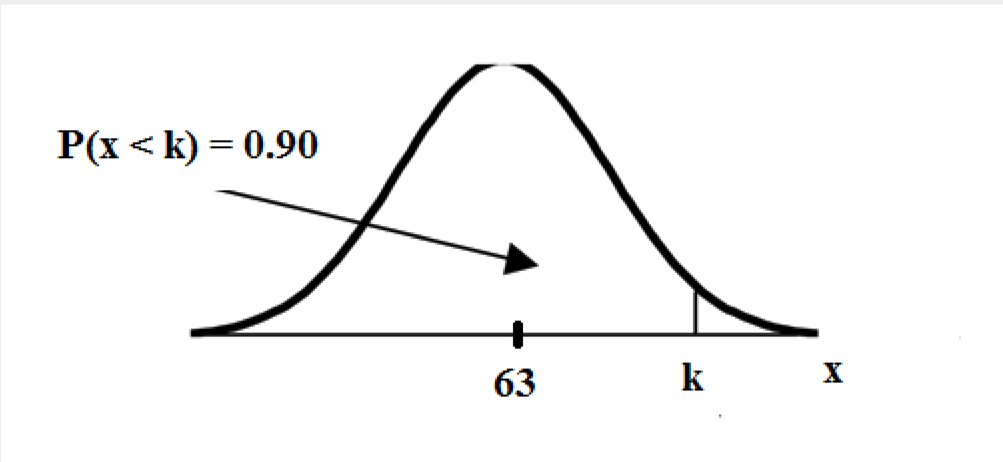
Find the 90th percentile (that is, find the score k that has 90 % of the scores below k and 10% of the scores above k).

Solution:

Find the 90th percentile. For each problem or part of a problem, draw a new graph. Draw the x-axis. Shade the area that corresponds to the 90th percentile.

Let k = the 90th percentile. k is located on the x-axis. $P(x < k)$ is the area to the left of k. The 90th percentile k separates the exam scores into those that are the same or lower than k and those that are the same or higher. Ninety percent of the test scores are the same or lower than k and 10% are the same or higher. k is often called a **critical value**.

$k = 69.4$ (calculator or computer)



The 90th percentile is 69.4. This means that 90% of the test scores fall at or below 69.4 and 10% fall at or above. For the TI-83+ or TI-84 calculators, use `invNorm` in `2nd DISTR`. invNorm(area to the left, mean, standard deviation) For this problem, invNorm(0.90,63,5) = 69.4

Exercise:

Problem:

Find the 70th percentile (that is, find the score k such that 70% of scores are below k and 30% of the scores are above k).

Solution:

Find the 70th percentile.

Draw a new graph and label it appropriately. $k = 65.6$

The 70th percentile is 65.6. This means that 70% of the test scores fall at or below 65.5 and 30% fall at or above.

invNorm(0.70,63,5) = 65.6

Example:
A computer is used for office work at home, research, communication, personal finances, education, entertainment, social networking and a myriad of other things. Suppose that the average number of hours a household personal computer is used for entertainment is 2 hours per day. Assume the times for entertainment are normally distributed and the standard deviation for the times is half an hour.

Exercise:

Problem:

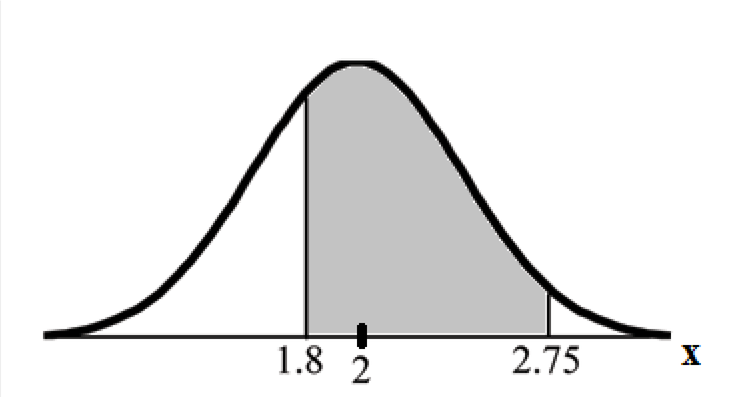
Find the probability that a household personal computer is used between 1.8 and 2.75 hours per day.

Solution:

Let X = the amount of time (in hours) a household personal computer is used for entertainment. x~$N(2, 0.5)$ where $\mu = 2$ and $\sigma = 0.5$.

Find $P(1.8 < x < 2.75)$.

The probability for which you are looking is the area **between** $x = 1.8$ and $x = 2.75$. $P(1.8 < x < 2.75) = 0.5886$



normalcdf(1.8,2.75,2,0.5) = 0.5886

The probability that a household personal computer is used between 1.8 and 2.75 hours per day for entertainment is 0.5886.

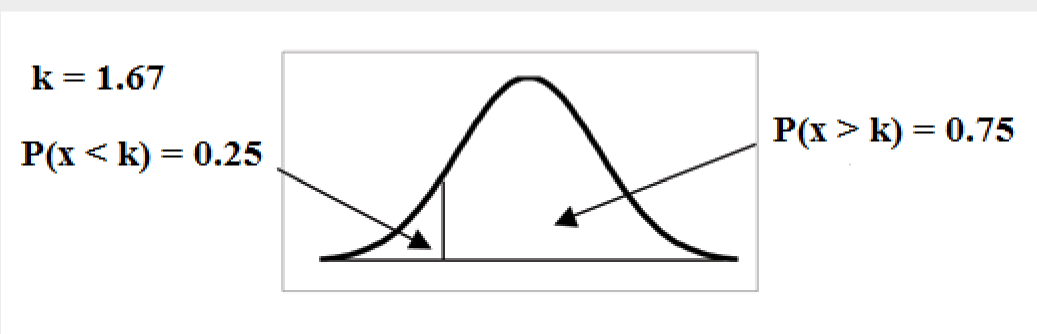
Exercise:

Problem:

Find the maximum number of hours per day that the bottom quartile of households use a personal computer for entertainment.

Solution:

To find the maximum number of hours per day that the bottom quartile of households uses a personal computer for entertainment, **find the 25th percentile,** k, where $P(x < k) = 0.25$.



invNorm(0.25,2,.5) = 1.66

The maximum number of hours per day that the bottom quartile of households uses a personal computer for entertainment is 1.66 hours.

Summary of Formulas

Formula

Normal Probability Distribution

$X \sim N \;\; \mu \;\; \sigma$

μ = the mean $\qquad$ σ = the standard deviation

Formula

Standard Normal Probability Distribution

$Z \sim N$

z = a standardized value (z-score)

mean = 0 $\qquad$ standard deviation = 1

Formula

Finding the kth Percentile

To find the **kth** percentile when the z-score is known: $k \quad \mu \quad z \; \sigma$

Formula

z-score

$z \quad \frac{x \quad \mu}{\sigma}$

Formula

Finding the area to the left

The area to the left: $P \;\; X \quad x$

Formula

Finding the area to the right

The area to the right: $P \;\; X \quad x \qquad\qquad P \;\; X \quad x$

Practice: The Normal Distribution

Student Learning Outcomes

- The student will analyze data following a normal distribution.

Given

The life of Sunshine CD players is normally distributed with a mean of 4.1 years and a standard deviation of 1.3 years. A CD player is guaranteed for 3 years. We are interested in the length of time a CD player lasts.

Normal Distribution

Exercise:

Problem: Define the Random Variable X in words. $X =$

Exercise:

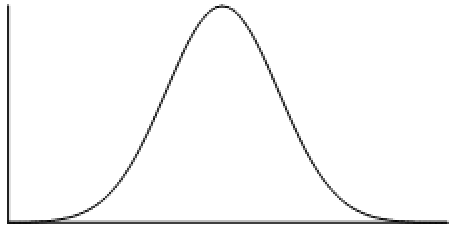
Problem: X~

Exercise:

Problem:

Find the probability that a CD player will break down during the guarantee period.

- **a** Sketch the situation. Label and scale the axes. Shade the region corresponding to the probability.



- **b** $P(0 < x <$ ________$) =$ ________ (Use zero (0) for the minimum value of x.)

Solution:

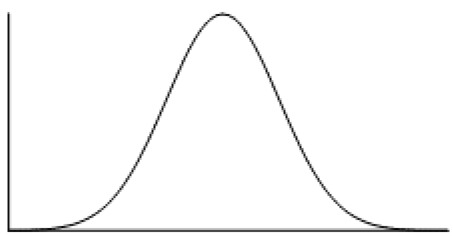
- **b** 3,0.1979

Exercise:

Problem:

Find the probability that a CD player will last between 2.8 and 6 years.

- **a** Sketch the situation. Label and scale the axes. Shade the region corresponding to the probability.



- **b** $P($______ $< x <$ ______$) =$ ________

Solution:

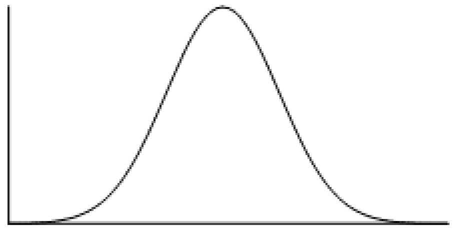
- **b** 2.8,6,0.7694

Exercise:

Problem:

Find the 70th percentile of the distribution for the time a CD player lasts.

- **a** Sketch the situation. Label and scale the axes. Shade the region corresponding to the lower 70%.



- **b** $P(x < k) =$ __________. Therefore, $k =$ ___________.

Solution:

- **b** 0.70 4.78years

Exercises

Exercise:

Problem:

According to a study done by De Anza students, the height for Asian adult males is normally distributed with an average of 66 inches and a standard deviation of 2.5 inches. Suppose one Asian adult male is randomly chosen. Let $X=$height of the individual.

- **a** X~_______(_______,_______)
- **b** Find the probability that the person is between 65 and 69 inches. Include a sketch of the graph and write a probability statement.
- **c** Would you expect to meet many Asian adult males over 72 inches? Explain why or why not, and justify your answer numerically.
- **d** The middle 40% of heights fall between what two values? Sketch the graph and write the probability statement.

Solution:

- **a** $N(66,2.5)$
- **b**0.5404
- **c**No
- **d**Between 64.7 and 67.3 inches

Exercise:

Problem:

IQ is normally distributed with a mean of 100 and a standard deviation of 15. Suppose one individual is randomly chosen. Let $X=$IQ of an individual.

- **a** X~_______(_______,_______)
- **b** Find the probability that the person has an IQ greater than 120. Include a sketch of the graph and write a probability statement.

- **c** Mensa is an organization whose members have the top 2% of all IQs. Find the minimum IQ needed to qualify for the Mensa organization. Sketch the graph and write the probability statement.
- **d** The middle 50% of IQs fall between what two values? Sketch the graph and write the probability statement.

Exercise:

Problem:

The percent of fat calories that a person in America consumes each day is normally distributed with a mean of about 36 and a standard deviation of 10. Suppose that one individual is randomly chosen. Let X =percent of fat calories.

- **a** X~_______(_______,_______)
- **b** Find the probability that the percent of fat calories a person consumes is more than 40. Graph the situation. Shade in the area to be determined.
- **c** Find the maximum number for the lower quarter of percent of fat calories. Sketch the graph and write the probability statement.

Solution:

- **a** $N(36,10)$
- **b** 0.3446
- **c** 29.3

Exercise:

Problem:

Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet.

- **a** If $X =$ distance in feet for a fly ball, then X~ _______(_______,_______)
- **b** If one fly ball is randomly chosen from this distribution, what is the probability that this ball traveled fewer than 220 feet? Sketch the graph. Scale the horizontal axis X. Shade the region corresponding to the probability. Find the probability.
- **c** Find the 80th percentile of the distribution of fly balls. Sketch the graph and write the probability statement.

Exercise:

Problem:

In China, 4-year-olds average 3 hours a day unsupervised. Most of the unsupervised children live in rural areas, considered safe. Suppose that the standard deviation is 1.5 hours and the amount of time spent alone is normally distributed. We randomly survey one Chinese 4-year-old living in a rural area. We are interested in the amount of time the child spends alone per day. (Source: **San Jose Mercury News**)

- **a** In words, define the random variable X. $X =$
- **b** X~
- **c** Find the probability that the child spends less than 1 hour per day unsupervised. Sketch the graph and write the probability statement.
- **d** What percent of the children spend over 10 hours per day unsupervised?
- **e** 70% of the children spend at least how long per day unsupervised?

Solution:

- **a** the time (in hours) a 4-year-old in China spends unsupervised per day
- **b** $N(3{,}1.5)$
- **c** 0.0912
- **d** 0

- **e** 2.21 hours

Exercise:

Problem:

In the 1992 presidential election, Alaska's 40 election districts averaged 1956.8 votes per district for President Clinton. The standard deviation was 572.3. (There are only 40 election districts in Alaska.) The distribution of the votes per district for President Clinton was bell-shaped. Let $X =$ number of votes for President Clinton for an election district. (Source: **The World Almanac and Book of Facts**)

- **a** State the approximate distribution of X. X~
- **b** Is 1956.8 a population mean or a sample mean? How do you know?
- **c** Find the probability that a randomly selected district had fewer than 1600 votes for President Clinton. Sketch the graph and write the probability statement.
- **d** Find the probability that a randomly selected district had between 1800 and 2000 votes for President Clinton.
- **e** Find the third quartile for votes for President Clinton.

Exercise:

Problem:

Suppose that the duration of a particular type of criminal trial is known to be normally distributed with a mean of 21 days and a standard deviation of 7 days.

- **a** In words, define the random variable X. $X =$
- **b** X~
- **c** If one of the trials is randomly chosen, find the probability that it lasted at least 24 days. Sketch the graph and write the probability statement.
- **d** 60% of all of these types of trials are completed within how many days?

Solution:

- **a**The duration of a criminal trial
- **b** $N(21,7)$
- **c** 0.3341
- **d** 22.77

Exercise:

Problem:

Terri Vogel, an amateur motorcycle racer, averages 129.71 seconds per 2.5 mile lap (in a 7 lap race) with a standard deviation of 2.28 seconds . The distribution of her race times is normally distributed. We are interested in one of her randomly selected laps. (Source: log book of Terri Vogel)

- **a** In words, define the random variable X. $X =$
- **b** X~
- **c** Find the percent of her laps that are completed in less than 130 seconds.
- **d** The fastest 3% of her laps are under ________ .
- **e** The middle 80% of her laps are from ________ seconds to ________ seconds.

Exercise:

Problem:

Thuy Dau, Ngoc Bui, Sam Su, and Lan Voung conducted a survey as to how long customers at Lucky claimed to wait in the checkout line until their turn. Let X =time in line. Below are the ordered real data (in minutes):

0.50	4.25	5	6	7.25
1.75	4.25	5.25	6	7.25
2	4.25	5.25	6.25	7.25
2.25	4.25	5.5	6.25	7.75
2.25	4.5	5.5	6.5	8
2.5	4.75	5.5	6.5	8.25
2.75	4.75	5.75	6.5	9.5
3.25	4.75	5.75	6.75	9.5
3.75	5	6	6.75	9.75
3.75	5	6	6.75	10.75

- **a** Calculate the sample mean and the sample standard deviation.
- **b** Construct a histogram. Start the $x-$axis at -0.375 and make bar widths of 2 minutes.
- **c** Draw a smooth curve through the midpoints of the tops of the bars.
- **d** In words, describe the shape of your histogram and smooth curve.
- **e** Let the sample mean approximate μ and the sample standard deviation approximate σ. The distribution of X can then be approximated by X~
- **f** Use the distribution in (e) to calculate the probability that a person will wait fewer than 6.1 minutes.
- **g** Determine the cumulative relative frequency for waiting less than 6.1 minutes.
- **h** Why aren’t the answers to (f) and (g) exactly the same?
- **i** Why are the answers to (f) and (g) as close as they are?

- **j** If only 10 customers were surveyed instead of 50, do you think the answers to (f) and (g) would have been closer together or farther apart? Explain your conclusion.

Solution:

- **a** The sample mean is 5.51 and the sample standard deviation is 2.15
- **e** $N(5.51, 2.15)$
- **f** 0.6081
- **g** 0.64

Exercise:

Problem:

Suppose that Ricardo and Anita attend different colleges. Ricardo's GPA is the same as the average GPA at his school. Anita's GPA is 0.70 standard deviations above her school average. In complete sentences, explain why each of the following statements may be false.

- **a** Ricardo's actual GPA is lower than Anita's actual GPA.
- **b** Ricardo is not passing since his z-score is zero.
- **c** Anita is in the 70th percentile of students at her college.

Exercise:

Problem:

Below is a sample of the maximum capacity (maximum number of spectators) of sports stadiums. The table does not include horse racing or motor racing stadiums. (Source: **http://en.wikipedia.org/wiki/List_of_stadiums_by_capacity**)

40,000	40,000	45,050	45,500	46,249	48,134
49,133	50,071	50,096	50,466	50,832	51,100
51,500	51,900	52,000	52,132	52,200	52,530
52,692	53,864	54,000	55,000	55,000	55,000
55,000	55,000	55,000	55,082	57,000	58,008
59,680	60,000	60,000	60,492	60,580	62,380
62,872	64,035	65,000	65,050	65,647	66,000
66,161	67,428	68,349	68,976	69,372	70,107
70,585	71,594	72,000	72,922	73,379	74,500
75,025	76,212	78,000	80,000	80,000	82,300

- **a** Calculate the sample mean and the sample standard deviation for the maximum capacity of sports stadiums (the data).
- **b** Construct a histogram of the data.
- **c** Draw a smooth curve through the midpoints of the tops of the bars of the histogram.
- **d** In words, describe the shape of your histogram and smooth curve.
- **e** Let the sample mean approximate μ and the sample standard deviation approximate σ. The distribution of X can then be approximated by X~
- **f** Use the distribution in (e) to calculate the probability that the maximum capacity of sports stadiums is less than 67,000 spectators.
- **g** Determine the cumulative relative frequency that the maximum capacity of sports stadiums is less than 67,000 spectators. Hint: Order the data and count the sports stadiums that have a

maximum capacity less than 67,000. Divide by the total number of sports stadiums in the sample.
- **h** Why aren’t the answers to (f) and (g) exactly the same?

Solution:

- **a** The sample mean is 60,136.4 and the sample standard deviation is 10,468.1.
- **e** $N(60136.4,10468.1)$
- **f** 0.7440
- **g** 0.7167

Try These Multiple Choice Questions

The questions below refer to the following: The patient recovery time from a particular surgical procedure is normally distributed with a mean of 5.3 days and a standard deviation of 2.1 days.

Exercise:

Problem: What is the median recovery time?

- **A** 2.7
- **B** 5.3
- **C** 7.4
- **D** 2.1

Solution:

B

Exercise:

Problem:

What is the z-score for a patient who takes 10 days to recover?

- **A** 1.5
- **B** 0.2
- **C** 2.2
- **D** 7.3

Solution:

C

Exercise:

Problem:

What is the probability of spending more than 2 days in recovery?

- **A** 0.0580
- **B** 0.8447
- **C** 0.0553
- **D** 0.9420

Solution:

D

Exercise:

Problem: The 90th percentile for recovery times is?

- **A** 8.89
- **B** 7.07
- **C** 7.99
- **D** 4.32

Solution:

C

The questions below refer to the following: The length of time to find a parking space at 9 A.M. follows a normal distribution with a mean of 5 minutes and a standard deviation of 2 minutes.

Exercise:

Problem:

Based upon the above information and numerically justified, would you be surprised if it took less than 1 minute to find a parking space?

- **A** Yes
- **B** No
- **C** Unable to determine

Solution:

A

Exercise:

Problem:

Find the probability that it takes at least 8 minutes to find a parking space.

- **A** 0.0001
- **B** 0.9270
- **C** 0.1862
- **D** 0.0668

Solution:

D

Exercise:

Problem:

Seventy percent of the time, it takes more than how many minutes to find a parking space?

- **A** 1.24
- **B** 2.41
- **C** 3.95
- **D** 6.05

Solution:

C

Exercise:

Problem:

If the mean is significantly greater than the standard deviation, which of the following statements is true?

- **I** The data cannot follow the uniform distribution.
- **II** The data cannot follow the exponential distribution..
- **III** The data cannot follow the normal distribution.

- **A** I only
- **B** II only
- **C** III only
- **D** I, II, and III

Solution:

B

Review

The next two questions refer to: $X \sim U(3, 13)$
Exercise:

Problem: Explain which of the following are false and which are true.

- **a** $f(x) = \frac{1}{10}, 3 \le x \le 13$
- **b** There is no mode.
- **c** The median is less than the mean.
- **d** $P(X > 10) = P(X \le 6)$

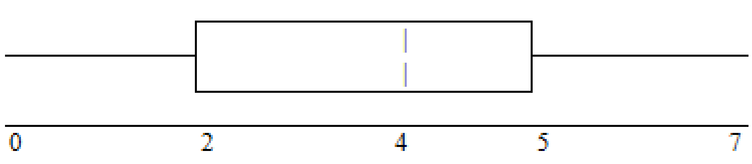
Solution:

- **a** True
- **b** True
- **c** False – the median and the mean are the same for this symmetric distribution
- **d** True

Exercise:

Problem: Calculate:

- **a** Mean
- **b** Median
- **c** 65th percentile.



Solution:

- **a** 8

- **b** 8
- **c** $P(X < k) = 0.65 = (k - 3) * \left(\frac{1}{10}\right)$. $k = 9.5$

Exercise:

Problem: Which of the following is true for the above box plot?

- **a** 25% of the data are at most 5.
- **b** There is about the same amount of data from 4 – 5 as there is from 5 – 7.
- **c** There are no data values of 3.
- **d** 50% of the data are 4.

Solution:

- **a** False – $\frac{3}{4}$ of the data are at most 5
- **b** True – each quartile has 25% of the data
- **c** False – that is unknown
- **d** False – 50% of the data are 4 or less

Exercise:

Problem:

If $P(G \mid H) = P(G)$, then which of the following is correct?

- **A** G and H are mutually exclusive events.
- **B** $P(G) = P(H)$
- **C** Knowing that H has occurred will affect the chance that G will happen.
- **D** G and H are independent events.

Solution:

D

Exercise:

Problem:

If $P(J) = 0.3$, $P(K) = 0.6$, and J and K are independent events, then explain which are correct and which are incorrect.

- **A** $P(J \text{ and } K) = 0$
- **B** $P(J \text{ or } K) = 0.9$
- **C** $P(J \text{ or } K) = 0.72$
- **D** $P(J) \neq P(J \mid K)$

Solution:

- **A** False - J and K are independent so they are not mutually exclusive which would imply dependency (meaning P(J and K) is not 0).
- **B** False - see answer C.
- **C** True - P(J or K) = P(J) + P(K) - P(J and K) = P(J) + P(K) - P(J)P(K) = 0.3 + 0.6 - (0.3)(0.6) = 0.72. Note that P(J and K) = P(J)P(K) because J and K are independent.
- **D** False - J and K are independent so P(J) = P(J|K).

The Central Limit Theorem
This module provides a brief introduction to the Central Limit Theorem.

Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Recognize the Central Limit Theorem problems.
- Classify continuous word problems by their distributions.
- Apply and interpret the Central Limit Theorem for Means.
- Apply and interpret the Central Limit Theorem for Sums.

Introduction

Why are we so concerned with means? Two reasons are that they give us a middle ground for comparison and they are easy to calculate. In this chapter, you will study means and the Central Limit Theorem.

The Central Limit Theorem (CLT for short) is one of the most powerful and useful ideas in all of statistics. Both alternatives are concerned with drawing finite samples of size n from a population with a known mean, μ, and a known standard deviation, σ. The first alternative says that if we collect samples of size n and n is "large enough," calculate each sample's mean, and create a histogram of those means, then the resulting histogram will tend to have an approximate normal bell shape. The second alternative says that if we again collect samples of size n that are "large enough," calculate the sum of each sample and create a histogram, then the resulting histogram will again tend to have a normal bell-shape.

In either case, it does not matter what the distribution of the original population is, or whether you even need to know it. The important fact is that the sample means and the sums tend to follow the normal distribution. And, the rest you will learn in this chapter.

The size of the sample, n, that is required in order to be to be 'large enough' depends on the original population from which the samples are drawn. If the original population is far from normal then more observations are

needed for the sample means or the sample sums to be normal. **Sampling is done with replacement.**

Optional Collaborative Classroom Activity

Do the following example in class: Suppose 8 of you roll 1 fair die 10 times, 7 of you roll 2 fair dice 10 times, 9 of you roll 5 fair dice 10 times, and 11 of you roll 10 fair dice 10 times.

Each time a person rolls more than one die, he/she calculates the sample **mean** of the faces showing. For example, one person might roll 5 fair dice and get a 2, 2, 3, 4, 6 on one roll.

The mean is $\frac{2+2+3+4+6}{5} = 3.4.$ The 3.4 is one mean when 5 fair dice are rolled. This same person would roll the 5 dice 9 more times and calculate 9 more means for a total of 10 means.

Your instructor will pass out the dice to several people as described above. Roll your dice 10 times. For each roll, record the faces and find the mean. Round to the nearest 0.5.

Your instructor (and possibly you) will produce one graph (it might be a histogram) for 1 die, one graph for 2 dice, one graph for 5 dice, and one graph for 10 dice. Since the "mean" when you roll one die, is just the face on the die, what distribution do these **means** appear to be representing?

Draw the graph for the means using 2 dice. Do the sample means show any kind of pattern?

Draw the graph for the means using 5 dice. Do you see any pattern emerging?

Finally, draw the graph for the means using 10 dice. Do you see any pattern to the graph? What can you conclude as you increase the number of dice?

As the number of dice rolled increases from 1 to 2 to 5 to 10, the following is happening:

1. The mean of the sample means remains approximately the same.
2. The spread of the sample means (the standard deviation of the sample means) gets smaller.
3. The graph appears steeper and thinner.

You have just demonstrated the Central Limit Theorem (CLT).

The Central Limit Theorem tells you that as you increase the number of dice, **the sample means tend toward a normal distribution (the sampling distribution).**

Glossary

Average

A number that describes the central tendency of the data. There are a number of specialized averages, including the arithmetic mean, weighted mean, median, mode, and geometric mean.

Central Limit Theorem

Given a random variable (RV) with known mean μ and known standard deviation σ. We are sampling with size n and we are interested in two new RVs - the sample mean, $\bar{X}$, and the sample sum, ΣX. If the size n of the sample is sufficiently large, then $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\Sigma X \sim N\left(\mathrm{n}\mu, \sqrt{n}\sigma\right)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

The Central Limit Theorem for Sample Means (Averages)

Suppose X is a random variable with a distribution that may be known or unknown (it can be any distribution). Using a subscript that matches the random variable, suppose:

- **a**μ_X = the mean of X
- **b**σ_X = the standard deviation of X

If you draw random samples of size n, then as n increases, the random variable X which consists of sample means, tends to be **normally distributed** and

$X \sim N\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right)$

The Central Limit Theorem for Sample Means says that if you keep drawing larger and larger samples (like rolling 1, 2, 5, and, finally, 10 dice) and **calculating their means** the sample means form their own **normal distribution** (the sampling distribution). The normal distribution has the same mean as the original distribution and a variance that equals the original variance divided by n, the sample size. n is the number of values that are averaged together not the number of times the experiment is done.

To put it more formally, if you draw random samples of size n,the distribution of the random variable X, which consists of sample means, is called the **sampling distribution of the mean**. The sampling distribution of the mean approaches a normal distribution as n, the sample size, increases.

The random variable X has a different z-score associated with it than the random variable X. x is the value of X in one sample.
Equation:

$$z = \frac{x - \mu_X}{\left(\frac{\sigma_X}{\sqrt{n}}\right)}$$

μ_X is both the average of X and of X.

$\sigma_X = \frac{\sigma_X}{\sqrt{n}}$ = standard deviation of X and is called the **standard error of the mean.**

Example:
An unknown distribution has a mean of 90 and a standard deviation of 15. Samples of size n = 25 are drawn randomly from the population.
Exercise:

Problem:

Find the probability that the **sample mean** is between 85 and 92.

Solution:

Let X = one value from the original unknown population. The probability question asks you to find a probability for the **sample mean**.

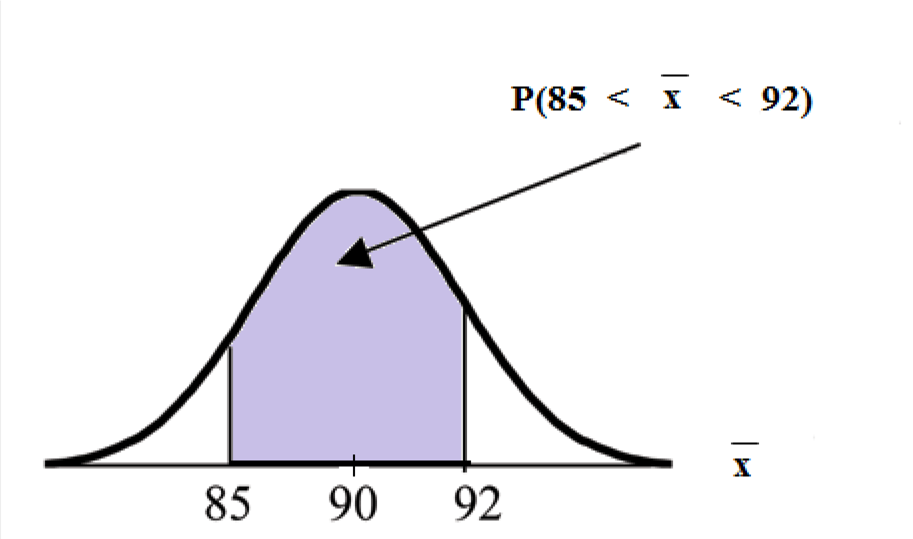
Let X = the mean of a sample of size 25. Since $\mu_X = 90$, $\sigma_X = 15$, and $n = 25$;

then X ~ $N\left(90, \frac{15}{\sqrt{25}}\right)$

Find $P(85 < x < 92)$ Draw a graph.

$P(85 < x < 92) = 0.6997$

The probability that the sample mean is between 85 and 92 is 0.6997.



TI-83 or 84: `normalcdf`(lower value, upper value, mean, standard error of the mean)

The parameter list is abbreviated (lower value, upper value, μ, $\frac{\sigma}{\sqrt{n}}$)

`normalcdf`$(85{,}92{,}90{,}\frac{15}{\sqrt{25}}) = 0.6997$

Exercise:

Problem:

Find the value that is 2 standard deviations above the expected value (it is 90) of the sample mean.

Solution:

To find the value that is 2 standard deviations above the expected value 90, use the formula

value = $\mu_X + (\#\text{ofSTDEVs})\left(\frac{\sigma_X}{\sqrt{n}}\right)$

value = $90 + 2 \cdot \frac{15}{\sqrt{25}} = 96$

So, the value that is 2 standard deviations above the expected value is 96.

Example:
The length of time, in hours, it takes an "over 40" group of people to play one soccer match is normally distributed with a **mean of 2 hours** and a **standard deviation of 0.5 hours**. A **sample of size** n **= 50** is drawn randomly from the population.

Exercise:

Problem:

Find the probability that the **sample mean** is between 1.8 hours and 2.3 hours.

Solution:

Let X = the time, in hours, it takes to play one soccer match.

The probability question asks you to find a probability for the **sample mean time, in hours**, it takes to play one soccer match.

Let X = the **mean** time, in hours, it takes to play one soccer match.

If $\mu_X =$ __________, $\sigma_X =$ ___________, and $n =$ ____________, then X~N(_______, _______) by the Central Limit Theorem for Means.

$\mu_X = \mathbf{2}$, $\sigma_X = \mathbf{0.5}$, $n = \mathbf{50}$, and X~$N\left(2, \frac{0.5}{\sqrt{50}}\right)$

Find $P(1.8 < x < 2.3)$. Draw a graph.

$P(1.8 < x < 2.3) = 0.9977$

`normalcdf`$(1.8,2.3,2,\frac{.5}{\sqrt{50}}) = 0.9977$

The probability that the mean time is between 1.8 hours and 2.3 hours is _______.

Glossary

Average

A number that describes the central tendency of the data. There are a number of specialized averages, including the arithmetic mean, weighted mean, median, mode, and geometric mean.

Central Limit Theorem

Given a random variable (RV) with known mean μ and known standard deviation σ. We are sampling with size n and we are interested in two new RVs - the sample mean, $\bar{X}$, and the sample sum, ΣX. If the size n of the sample is sufficiently large, then $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\Sigma X \sim N\left(\mathrm{n}\mu, \sqrt{n}\sigma\right)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

Normal Distribution

A continuous random variable (RV) with pdf $\mathrm{f(x)} = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean of the distribution and σ is the standard deviation. Notation: $X \sim N(\mu, \sigma)$. If $\mu = 0$ and $\sigma = 1$, the RV is called **the standard normal distribution**.

Standard Error of the Mean

The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$.

The Central Limit Theorem for Sums

Suppose X is a random variable with a distribution that may be **known or unknown** (it can be any distribution) and suppose:

- **a** $\mu_X =$ the mean of X
- **b** $\sigma_X =$ the standard deviation of X

If you draw random samples of size n, then as n increases, the random variable ΣX which consists of sums tends to be **normally distributed** and

$$\Sigma X \sim N\left(n \cdot \mu_X, \sqrt{n} \cdot \sigma_X\right)$$

The Central Limit Theorem for Sums says that if you keep drawing larger and larger samples and taking their sums, the sums form their own normal distribution (the sampling distribution) which approaches a normal distribution as the sample size increases. **The normal distribution has a mean equal to the original mean multiplied by the sample size and a standard deviation equal to the original standard deviation multiplied by the square root of the sample size.**

The random variable ΣX has the following z-score associated with it:

- **a**Σx is one sum.
- **b**$z = \frac{\Sigma x - n \cdot \mu_X}{\sqrt{n} \cdot \sigma_X}$

- **a**$n \cdot \mu_X =$ the mean of ΣX
- **b** $\sqrt{n} \cdot \sigma_X =$ standard deviation of ΣX

Example:
An unknown distribution has a mean of 90 and a standard deviation of 15. A sample of size 80 is drawn randomly from the population.
Exercise:

Problem:

- **a**Find the probability that the sum of the 80 values (or the total of the 80 values) is more than 7500.
- **b**Find the sum that is 1.5 standard deviations above the mean of the sums.

Solution:

Let X = one value from the original unknown population. The probability question asks you to find a probability for **the sum (or total of) 80 values.**

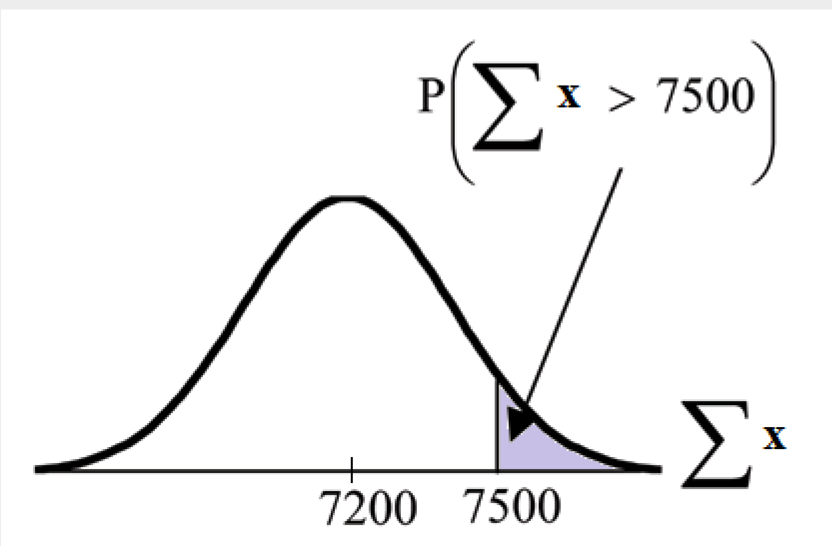
ΣX = the sum or total of 80 values. Since $\mu_X = 90$, $\sigma_X = 15$, and $n = 80$, then

$$\Sigma X \sim N\left(80 \cdot 90, \sqrt{80} \cdot 15\right)$$

- mean of the sums = $n \cdot \mu_X = (80)(90) = 7200$
- standard deviation of the sums = $\sqrt{n} \cdot \sigma_X = \sqrt{80} \cdot 15$
- sum of 80 values = $\Sigma\text{x} = 7500$

- **a**Find $P(\Sigma\text{x} > 7500)$

$P(\Sigma\text{x} > 7500) = 0.0127$



`normalcdf`(lower value, upper value, mean of sums, `stdev` of sums)

The parameter list is abbreviated (lower, upper, $n \cdot \mu_X, \sqrt{n} \cdot \sigma_X$)

`normalcdf`(7500,1E99, $80 \cdot 90, \sqrt{80} \cdot 15\Big) = 0.0127$

Reminder: $1\text{E}99 = 10^{99}$. Press the `EE` key for E.

- **b**Find Σx where z = 1.5:

Σx = $n \cdot \mu_X$ + $z \cdot \sqrt{n} \cdot \sigma_X$ = (80)(90) + (1.5)($\sqrt{80}$) (15) = 7401.2

Glossary

Central Limit Theorem
: Given a random variable (RV) with known mean μ and known standard deviation σ. We are sampling with size n and we are interested in two new RVs - the sample mean, $\bar{X}$, and the sample sum, ΣX. If the size n of the sample is sufficiently large, then $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\Sigma X \sim N\left(\mathrm{n\mu}, \sqrt{n}\sigma\right)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

Normal Distribution

A continuous random variable (RV) with pdf
$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean of the distribution and σ is the standard deviation. Notation: $X \sim N(\mu, \sigma)$. If $\mu = 0$ and $\sigma = 1$, the RV is called **the standard normal distribution**.

Using the Central Limit Theorem
Central Limit Theorem: Using the Central Limit Theorem is part of the collection col10555 written by Barbara Illowsky and Susan Dean. It covers how and when to use the Central Limit Theorem and has contributions from Roberta Bloom.

It is important for you to understand when to use the **CLT**. If you are being asked to find the probability of the mean, use the CLT for the mean. If you are being asked to find the probability of a sum or total, use the CLT for sums. This also applies to percentiles for means and sums.

Note: If you are being asked to find the probability of an **individual** value, do **not** use the CLT. **Use the distribution of its random variable.**

Examples of the Central Limit Theorem

Law of Large Numbers

The **Law of Large Numbers** says that if you take samples of larger and larger size from any population, then the mean x of the sample tends to get closer and closer to μ. From the Central Limit Theorem, we know that as n gets larger and larger, the sample means follow a normal distribution. The larger n gets, the smaller the standard deviation gets. (Remember that the standard deviation for X is $\frac{\sigma}{\sqrt{n}}$.) This means that the sample mean x must be close to the population mean μ. We can say that μ is the value that the sample means approach as n gets larger. The Central Limit Theorem illustrates the Law of Large Numbers.

Central Limit Theorem for the Mean and Sum Examples

Example:

A study involving stress is done on a college campus among the students. **The stress scores follow a uniform distribution** with the lowest stress score equal to 1 and the highest equal to 5. Using a sample of 75 students, find:

1. The probability that the **mean stress score** for the 75 students is less than 2.
2. The 90th percentile for the **mean stress score** for the 75 students.
3. The probability that the **total of the 75 stress scores** is less than 200.
4. The 90th percentile for the **total stress score** for the 75 students.

Let X = one stress score.
Problems 1. and 2. ask you to find a probability or a percentile for a **mean**. Problems 3 and 4 ask you to find a probability or a percentile for a **total or sum**. The sample size, n, is equal to 75.
Since the individual stress scores follow a uniform distribution, X ~ $U(1, 5)$ where $a = 1$ and $b = 5$ (See Continuous Random Variables for the uniform).

$$\mu_X = \frac{a+b}{2} = \frac{1+5}{2} = 3$$

$$\sigma_X = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(5-1)^2}{12}} = 1.15$$

For problems 1. and 2., let X = the mean stress score for the 75 students. Then,

X ~ $N\left(3, \frac{1.15}{\sqrt{75}}\right)$ where $\mathrm{n} = 75.$

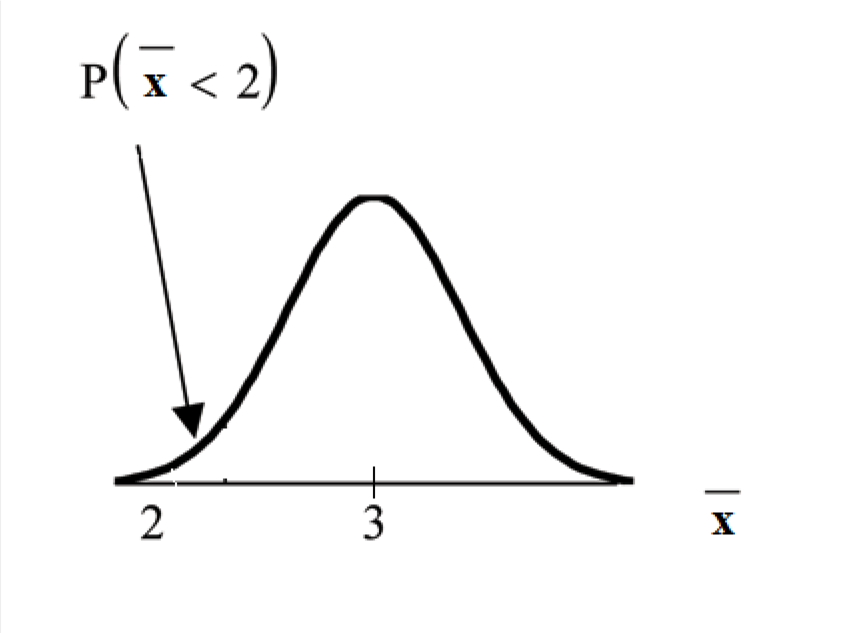
Exercise:

Problem: Find $P(x < 2)$. Draw the graph.

Solution:

$P(x < 2) = 0$

The probability that the mean stress score is less than 2 is about 0.



$\texttt{normalcdf}\left(1, 2, 3, \frac{1.15}{\sqrt{75}}\right) = 0$

Note: The smallest stress score is 1. Therefore, the smallest mean for 75 stress scores is 1.

Exercise:

Problem:

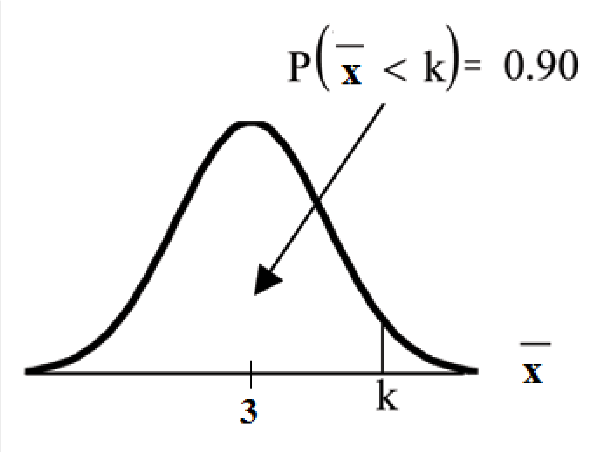
Find the 90th percentile for the mean of 75 stress scores. Draw a graph.

Solution:

Let k = the 90th precentile.

Find k where $P(x < k) = 0.90$.

$k = 3.2$



The 90th percentile for the mean of 75 scores is about 3.2. This tells us that 90% of all the means of 75 stress scores are at most 3.2 and 10% are at least 3.2.

`invNorm` $\left(.90, 3, \frac{1.15}{\sqrt{75}}\right) = 3.2$

For problems c and d, let ΣX = the sum of the 75 stress scores. Then, ΣX ~ $N\left[\left(75\right) \cdot \left(3\right), \sqrt{75} \cdot 1.15\right]$

Exercise:

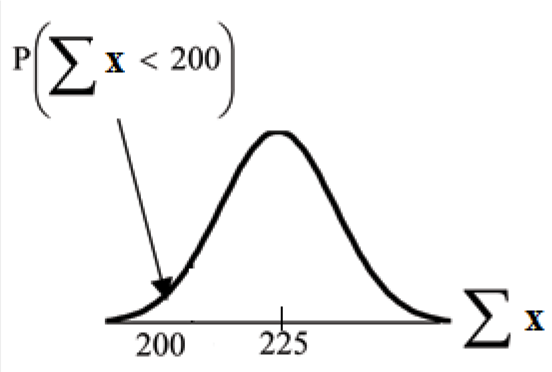
Problem: Find $P(\Sigma \text{x} < 200)$. Draw the graph.

Solution:

The mean of the sum of 75 stress scores is $75 \cdot 3 = 225$

The standard deviation of the sum of 75 stress scores is $\sqrt{75} \cdot 1.15 = 9.96$

$P(\Sigma \text{x} < 200) = 0$



The probability that the total of 75 scores is less than 200 is about 0.

`normalcdf` $\left(75, 200, 75 \cdot 3, \sqrt{75} \cdot 1.15\right) = 0.$

Note: The smallest total of 75 stress scores is 75 since the smallest single score is 1.

Exercise:

Problem:

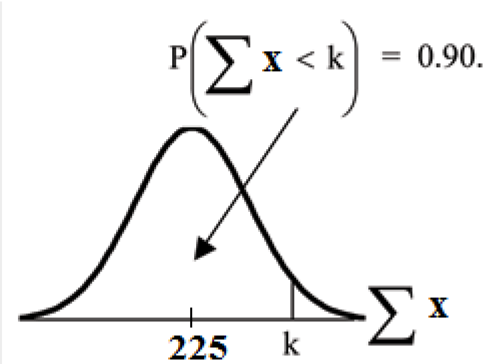
Find the 90th percentile for the total of 75 stress scores. Draw a graph.

Solution:

Let k = the 90th percentile.

Find k where $P(\Sigma\text{x} < k) = 0.90$.

$k = 237.8$



The 90th percentile for the sum of 75 scores is about 237.8. This tells us that 90% of all the sums of 75 scores are no more than 237.8 and 10% are no less than 237.8.

`invNorm` $\left(.90, 75 \cdot 3, \sqrt{75} \cdot 1.15\right) = 237.8$

Example:

Suppose that a market research analyst for a cell phone company conducts a study of their customers who exceed the time allowance included on their basic cell phone contract; the analyst finds that for those people who exceed the time included in their basic contract, the **excess time used** follows an **exponential distribution** with a mean of 22 minutes.

Consider a random sample of 80 customers who exceed the time allowance included in their basic cell phone contract.

Let X = the excess time used by one INDIVIDUAL cell phone customer who exceeds his contracted time allowance.

$X \sim \text{Exp}\left(\frac{1}{22}\right)$ From Chapter 5, we know that $\mu = 22$ and $\sigma = 22$.

Let X = the mean excess time used by a sample of $n = 80$ customers who exceed their contracted time allowance.

$X \sim N\left(22, \frac{22}{\sqrt{80}}\right)$ by the CLT for Sample Means

Exercise:

Problem:

Using the CLT to find Probability:

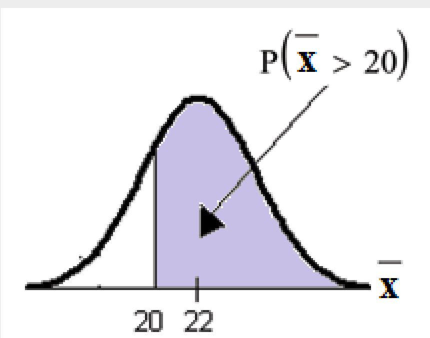
- **a**Find the probability that the mean excess time used by the 80 customers in the sample is longer than 20 minutes. This is asking us to find $P(x > 20)$ Draw the graph.
- **b** Suppose that one customer who exceeds the time limit for his cell phone contract is randomly selected. Find the probability that this individual customer's excess time is longer than 20 minutes. This is asking us to find $P(x > 20)$
- **c** Explain why the probabilities in (a) and (b) are different.

Solution:

Part a.
Find: $P(x > 20)$

$P(x > 20) = 0.7919$ using `normalcdf` $\left(20, 1E99, 22, \frac{22}{\sqrt{80}}\right)$

The probability is 0.7919 that the mean excess time used is more than 20 minutes, for a sample of 80 customers who exceed their contracted time allowance.



Note:$1E99 = 10^{99}$and-$1E99 = -10^{99}$. Press the

`EE`

key for E. Or just use 10^99 instead of 1E99.

Part b.
Find P(x>20) . Remember to use the exponential distribution for an **individual: X~Exp(1/22)**.

P(X>20) = e^(–(1/22)*20) or e^(–.04545*20) = 0.4029

Part c. Explain why the probabilities in (a) and (b) are different.

- $P(x > 20) = 0.4029$ but $P(x > 20) = 0.7919$
- The probabilities are not equal because we use different distributions to calculate the probability for individuals and for means.
- When asked to find the probability of an individual value, use the stated distribution of its random variable; do not use the CLT. Use the CLT with the normal distribution when you are being asked to find the probability for an mean.

Exercise:

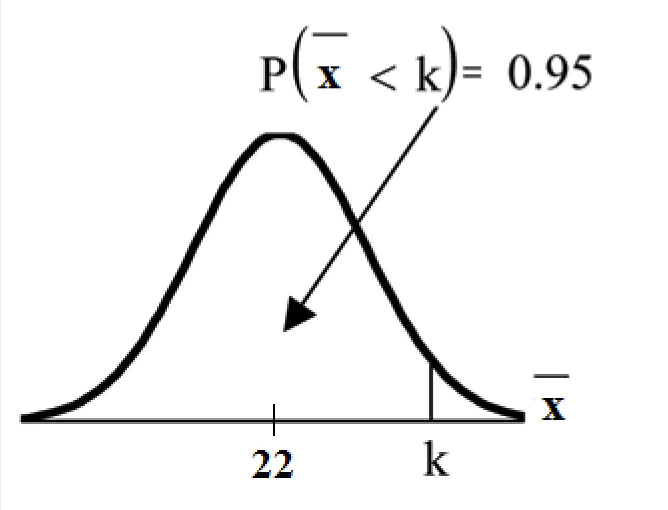
Problem:

Using the CLT to find Percentiles:
Find the 95th percentile for the **sample mean excess time** for samples of 80 customers who exceed their basic contract time allowances. Draw a graph.

Solution:

Let k = the 95th percentile. Find k where $P(x < k) = 0.95$

$k = 26.0$ using `invNorm`$\left(.95, 22, \frac{22}{\sqrt{80}}\right) = 26.0$



The 95th percentile for the **sample mean excess time used** is about 26.0 minutes for random samples of 80 customers who exceed their contractual allowed time.

95% of such samples would have means under 26 minutes; only 5% of such samples would have means above 26 minutes.

Note:(HISTORICAL): Normal Approximation to the Binomial

Historically, being able to compute binomial probabilities was one of the most important applications of the Central Limit Theorem. Binomial probabilities were displayed in a table in a book with a small value for n (say, 20). To calculate the probabilities with large values of n, you had to use the binomial formula which could be very complicated. Using the **Normal Approximation to the Binomial** simplified the process. To compute the Normal Approximation to the Binomial, take a simple random sample from a population. You must meet the conditions for a **binomial distribution**:

- there are a certain number n of independent trials
- the outcomes of any trial are success or failure

- each trial has the same probability of a success p

Recall that if X is the binomial random variable, then X~$B(n, p)$. The shape of the binomial distribution needs to be similar to the shape of the normal distribution. To ensure this, the quantities np and nq must both be greater than five ($np > 5$ and $nq > 5$; the approximation is better if they are both greater than or equal to 10). Then the binomial can be approximated by the normal distribution with mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$. Remember that $q = 1 - p$. In order to get the best approximation, add 0.5 to x or subtract 0.5 from x $($use $x + 0.5$ or $x - 0.5$ $)$. The number 0.5 is called the **continuity correction factor**.

Example:
Suppose in a local Kindergarten through 12th grade (K - 12) school district, 53 percent of the population favor a charter school for grades K - 5. A simple random sample of 300 is surveyed.

1. Find the probability that **at least 150** favor a charter school.
2. Find the probability that **at most 160** favor a charter school.
3. Find the probability that **more than 155** favor a charter school.
4. Find the probability that **less than 147** favor a charter school.
5. Find the probability that **exactly 175** favor a charter school.

Let $X =$ the number that favor a charter school for grades K - 5. X~ $B(n, p)$ where n =300 and $p = 0.53$. Since $np > 5$ and $nq > 5$, use the normal approximation to the binomial. The formulas for the mean and standard deviation are $\mu = np$ and $\sigma = \sqrt{npq}$. The mean is 159 and the standard deviation is 8.6447. The random variable for the normal distribution is Y. Y~$N(159, 8.6447)$. See **The Normal Distribution** for help with calculator instructions.
For Problem 1., you **include 150** so $P(x \geq 150)$ has normal approximation $P(Y \geq 149.5)$=0.8641.
`normalcdf` $(149.5, 10\hat{}99, 159, 8.6447) = 0.8641$.
For Problem 2., you **include 160** so $P(x \leq 160)$ has normal approximation $P(Y \leq 160.5)$=0.5689.

`normalcdf` $(0, 160.5, 159, 8.6447) = 0.5689$
For Problem 3., you **exclude 155** so $P(x > 155)$ has normal approximation $P(y > 155.5)$=0.6572.
`normalcdf` $(155.5, 10\hat{}99, 159, 8.6447) = 0.6572$
For Problem 4., you **exclude 147** so $P(x < 147)$ has normal approximation $P(Y < 146.5)$=0.0741.
`normalcdf` $(0, 146.5, 159, 8.6447) = 0.0741$
For Problem 5., $P(x$=175$)$ has normal approximation $P(174.5 < y < 175.5)$=0.0083.
`normalcdf` $(174.5, 175.5, 159, 8.6447) = 0.0083$
Because of calculators and computer software that easily let you calculate binomial probabilities for large values of n, it is not necessary to use the the Normal Approximation to the Binomial provided you have access to these technology tools. Most school labs have Microsoft Excel, an example of computer software that calculates binomial probabilities. Many students have access to the TI-83 or 84 series calculators and they easily calculate probabilities for the binomial. In an Internet browser, if you type in "binomial probability distribution calculation," you can find at least one online calculator for the binomial.
For **Example 3**, the probabilities are calculated using the binomial (n=300 and p=0.53) below. Compare the binomial and normal distribution answers. See **Discrete Random Variables** for help with calculator instructions for the binomial.
$P(x \geq 150)$: `1 - binomialcdf` $(300, 0.53, 149)$=0.8641
$P(x \leq 160)$: `binomialcdf` $(300, 0.53, 160)$=0.5684
$P(x > 155)$: `1 - binomialcdf` $(300, 0.53, 155)$=0.6576
$P(x < 147)$: `binomialcdf` $(300, 0.53, 146)$=0.0742
$P(x$=175$)$: (You use the binomial pdf.) `binomialpdf` $(175, 0.53, 146)$=0.0083

**Contributions made to Example 2 by Roberta Bloom

Glossary

Average

A number that describes the central tendency of the data. There are a number of specialized averages, including the arithmetic mean, weighted mean, median, mode, and geometric mean.

Central Limit Theorem

Given a random variable (RV) with known mean μ and known standard deviation σ. We are sampling with size n and we are interested in two new RVs - the sample mean, $\bar{X}$, and the sample sum, ΣX. If the size n of the sample is sufficiently large, then $\bar{X}$~ $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\Sigma X \sim N\left(\mathrm{n\mu}, \sqrt{n}\sigma\right)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

Exponential Distribution

A continuous random variable (RV) that appears when we are interested in the intervals of time between some random events, for example, the length of time between emergency arrivals at a hospital. Notation: X~$\mathrm{Exp}(m)$. The mean is $\mu = \frac{1}{m}$ and the standard deviation is $\sigma = \frac{1}{m}$. The probability density function is $f(x) = \mathrm{me}^{-\mathrm{mx}}, x \geq 0$ and the cumulative distribution function is $P(X \leq x) = 1 - e^{-\mathrm{mx}}$.

Mean

A number that measures the central tendency. A common name for mean is 'average.' The term 'mean' is a shortened form of 'arithmetic mean.' By definition, the mean for a sample (denoted by x) is $x = \frac{\text{Sum of all values in the sample}}{\text{Number of values in the sample}}$, and the mean for a population (denoted by μ) is $\mu = \frac{\text{Sum of all values in the population}}{\text{Number of values in the population}}$.

Uniform Distribution

A continuous random variable (RV) that has equally likely outcomes over the domain, $a < x < b$. Often referred as the **Rectangular distribution** because the graph of the pdf has the form of a rectangle. Notation: $X \sim U(a,b)$. The mean is $\mu = \frac{a+b}{2}$ and the standard deviation is $\sigma = \sqrt{\frac{(b-a)^2}{12}}$ The probability density function is $f(x) = \frac{1}{b-a}$ for $a < x < b$ or $a \le x \le b$. The cumulative distribution is $P(X \le x) = \frac{x-a}{b-a}$.

Summary of Formulas

Formula

Central Limit Theorem for Sample Means

$X \sim \mathrm{N}\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right)$ **The Mean** $\left(X\right)$**:** μ_X

Formula

Central Limit Theorem for Sample Means Z-Score and Standard Error of the Mean

$z = \frac{x-\mu_X}{\left(\frac{\sigma_X}{\sqrt{n}}\right)}$ **Standard Error of the Mean (Standard Deviation** $\left(X\right)$**):** $\frac{\sigma_X}{\sqrt{n}}$

Formula

Central Limit Theorem for Sums

$\Sigma\mathrm{X} \sim N\left[\left(n\right) \cdot \mu_X, \sqrt{n} \cdot \sigma_X\right]$ **Mean for Sums** $(\Sigma\mathrm{X})$**:** $n \cdot \mu_X$

Formula

Central Limit Theorem for Sums Z-Score and Standard Deviation for Sums

$z = \frac{\Sigma\mathrm{x}-n\cdot\mu_X}{\sqrt{n}\cdot\sigma_X}$ **Standard Deviation for Sums** $(\Sigma\mathrm{X})$**:** $\sqrt{n} \cdot \sigma_X$

Practice: The Central Limit Theorem

Student Learning Outcomes

- The student will calculate probabilities using the Central Limit Theorem.

Given

Yoonie is a personnel manager in a large corporation. Each month she must review 16 of the employees. From past experience, she has found that the reviews take her approximately 4 hours each to do with a population standard deviation of 1.2 hours. Let X be the random variable representing the time it takes her to complete one review. Assume X is normally distributed. Let $\overline{X}$ be the random variable representing the mean time to complete the 16 reviews. Let ΣX be the total time it takes Yoonie to complete all of the month's reviews. Assume that the 16 reviews represent a random set of reviews.

Distribution

Complete the distributions.

1. X ~
2. $\overline{X}$ ~
3. ΣX ~

Graphing Probability

For each problem below:

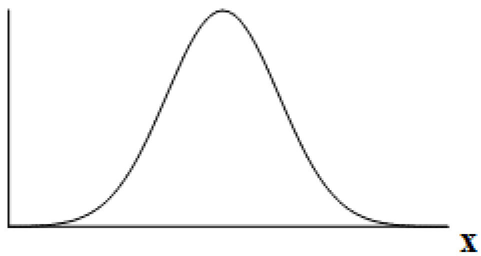
- **a** Sketch the graph. Label and scale the horizontal axis. Shade the region corresponding to the probability.
- **b** Calculate the value.

Exercise:

Problem:

Find the probability that **one** review will take Yoonie from 3.5 to 4.25 hours.

- **a**



- **b** $P($ ________ $< x <$ ________ $) =$ _______

Solution:

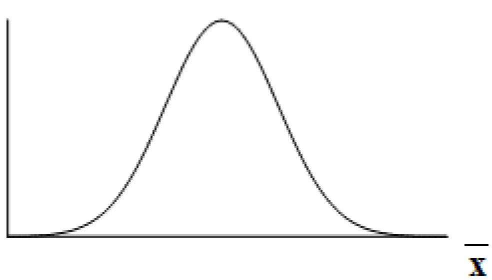
- **b** 3.5, 4.25, 0.2441

Exercise:

Problem:

Find the probability that the **mean** of a month's reviews will take Yoonie from 3.5 to 4.25 hrs.

- **a**



- **b** $P($________________ $)$ =_______

Solution:

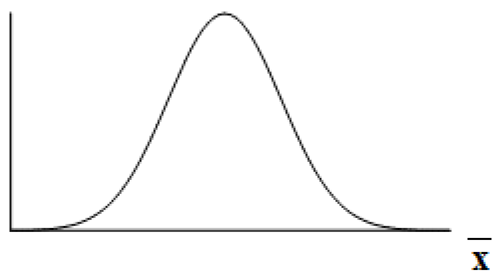
- **b** 0.7499

Exercise:

Problem:

Find the 95th percentile for the **mean** time to complete one month's reviews.

- **a**



- **b**The 95th Percentile=

Solution:

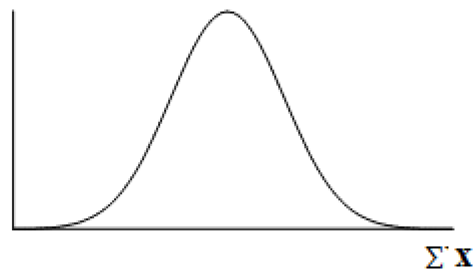
- **b** 4.49 hours

Exercise:

Problem:

Find the probability that the **sum** of the month's reviews takes Yoonie from 60 to 65 hours.

- **a**



- **b** The Probability=

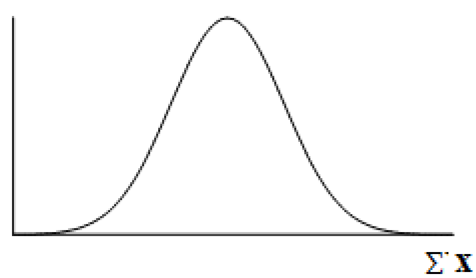
Solution:

- **b** 0.3802

Exercise:

Problem: Find the 95th percentile for the **sum** of the month’s reviews.

- **a**



- **b** The 95th percentile=

Solution:

- **b** 71.90

Discussion Question

Exercise:

Problem: What causes the probabilities in [link] and [link] to differ?

Exercises

The Central Limit Theorem: Homework is part of the collection col10555 written by Barbara Illowsky and Susan Dean.

Exercise:

Problem:

$X \sim N(60,9)$. Suppose that you form random samples of 25 from this distribution. Let X be the random variable of averages. Let ΣX be the random variable of sums. For **c - f**, sketch the graph, shade the region, label and scale the horizontal axis for X, and find the probability.

- **a** Sketch the distributions of X and X on the same graph.
- **b** X ~
- **c** $P(x < 60) =$
- **d** Find the 30th percentile for the mean.
- **e** $P(56 < x < 62) =$
- **f** $P(18 < x < 58) =$
- **g** Σx ~
- **h** Find the minimum value for the upper quartile for the sum.
- **i** $P(1400 < \Sigma x < 1550) =$

Solution:

- **b** Xbar~$N(60, \frac{9}{\sqrt{25}})$
- **c**0.5000
- **d**59.06
- **e**0.8536
- **f**0.1333
- **h**1530.35
- **i**0.8536

Exercise:

Problem:

Determine which of the following are true and which are false. Then, in complete sentences, justify your answers.

- **a** When the sample size is large, the mean of X is approximately equal to the mean of X.
- **b** When the sample size is large, X is approximately normally distributed.
- **c** When the sample size is large, the standard deviation of X is approximately the same as the standard deviation of X.

Exercise:

Problem:

The percent of fat calories that a person in America consumes each day is normally distributed with a mean of about 36 and a standard deviation of about 10. Suppose that 16 individuals are randomly chosen.

Let X =average percent of fat calories.

- **a** X ~______ (______ , ______)
- **b** For the group of 16, find the probability that the average percent of fat calories consumed is more than 5. Graph the situation and shade in the area to be determined.
- **c** Find the first quartile for the average percent of fat calories.

Solution:

- **a** $N(36, \frac{10}{\sqrt{16}})$
- **b** 1
- **c** 34.31

Exercise:

Problem:

Previously, De Anza statistics students estimated that the amount of change daytime statistics students carry is exponentially distributed with a mean of $0.88. Suppose that we randomly pick 25 daytime statistics students.

- **a** In words, $X =$
- **b** X~
- **c** In words, $X =$
- **d** X~ ______ (______ , ______)
- **e** Find the probability that an individual had between $0.80 and $1.00. Graph the situation and shade in the area to be determined.
- **f** Find the probability that the average of the 25 students was between $0.80 and $1.00. Graph the situation and shade in the area to be determined.
- **g** Explain the why there is a difference in (e) and (f).

Exercise:

Problem:

Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet. We randomly sample 49 fly balls.

- **a** If $X =$ average distance in feet for 49 fly balls, then X~ _______ (_______ , _______)
- **b** What is the probability that the 49 balls traveled an average of less than 240 feet? Sketch the graph. Scale the horizontal axis for X. Shade the region corresponding to the probability. Find the probability.
- **c** Find the 80th percentile of the distribution of the average of 49 fly balls.

Solution:

- **a** $N(250, \frac{50}{\sqrt{49}})$
- **b** 0.0808
- **c**256.01 feet

Exercise:

Problem:

Suppose that the weight of open boxes of cereal in a home with children is uniformly distributed from 2 to 6 pounds. We randomly survey 64 homes with children.

- **a** In words, $X =$
- **b** X~
- **c** $\mu_X =$
- **d** $\sigma_X =$
- **e** In words, $\Sigma X =$
- **f** ΣX~
- **g** Find the probability that the total weight of open boxes is less than 250 pounds.
- **h** Find the 35th percentile for the total weight of open boxes of cereal.

Exercise:

Problem:

Suppose that the duration of a particular type of criminal trial is known to have a mean of 21 days and a standard deviation of 7 days. We randomly sample 9 trials.

- **a** In words, $\Sigma X =$
- **b** ΣX~
- **c** Find the probability that the total length of the 9 trials is at least 225 days.
- **d** 90 percent of the total of 9 of these types of trials will last at least how long?

Solution:

- **a**The total length of time for 9 criminal trials
- **b** $N(189,21)$
- **c** 0.0432
- **d** 162.09

Exercise:

Problem:

According to the Internal Revenue Service, the average length of time for an individual to complete (record keep, learn, prepare, copy, assemble and send) IRS Form 1040 is 10.53 hours (without any attached schedules). The distribution is unknown. Let us assume that the standard deviation is 2 hours. Suppose we randomly sample 36 taxpayers.

- **a** In words, $X =$
- **b** In words, $X =$
- **c** X~
- **d** Would you be surprised if the 36 taxpayers finished their Form 1040s in an average of more than 12 hours? Explain why or why not in complete sentences.
- **e** Would you be surprised if one taxpayer finished his Form 1040 in more than 12 hours? In a complete sentence, explain why.

Exercise:

Problem:

Suppose that a category of world class runners are known to run a marathon (26 miles) in an average of 145 minutes with a standard deviation of 14 minutes. Consider 49 of the races.

Let $X =$ the average of the 49 races.

- **a** X~
- **b** Find the probability that the runner will average between 142 and 146 minutes in these 49 marathons.
- **c** Find the 80th percentile for the average of these 49 marathons.
- **d** Find the median of the average running times.

Solution:

- **a** $N(145, \frac{14}{\sqrt{49}})$
- **b** 0.6247
- **c** 146.68
- **d** 145 minutes

Exercise:

Problem:

The attention span of a two year-old is exponentially distributed with a mean of about 8 minutes. Suppose we randomly survey 60 two year-olds.

- **a** In words, $X =$
- **b** X~
- **c** In words, $X =$
- **d** X~
- **e** Before doing any calculations, which do you think will be higher? Explain why.
  - **i** the probability that an individual attention span is less than 10 minutes; or
  - **ii** the probability that the average attention span for the 60 children is less than 10 minutes? Why?
- **f** Calculate the probabilities in part (e).
- **g** Explain why the distribution for X is not exponential.

Exercise:

Problem:

Suppose that the length of research papers is uniformly distributed from 10 to 25 pages. We survey a class in which 55 research papers were turned in to a professor. The 55 research papers are considered a random collection of all papers. We are interested in the average length of the research papers.

- **a** In words, $X =$
- **b** X~
- **c** $\mu_X =$
- **d** $\sigma_X =$
- **e** In words, $X =$
- **f** X~
- **g** In words, $\Sigma X =$
- **h** ΣX~
- **i** Without doing any calculations, do you think that it's likely that the professor will need to read a total of more than 1050 pages? Why?
- **j** Calculate the probability that the professor will need to read a total of more than 1050 pages.
- **k** Why is it so unlikely that the average length of the papers will be less than 12 pages?

Solution:

- **b** $U(10{,}25)$
- **c**17.5
- **d** $\sqrt{\frac{225}{12}}$ = 4.3301
- **f**$N(17.5{,}0.5839)$
- **h**$N(962.5{,}32.11)$
- **j**0.0032

Exercise:

Problem:

The length of songs in a collector's CD collection is uniformly distributed from 2 to 3.5 minutes. Suppose we randomly pick 5 CDs from the collection. There is a total of 43 songs on the 5 CDs.

- **a** In words, $X =$
- **b** X~
- **c** In words, X=
- **d** X~
- **e** Find the first quartile for the average song length.
- **f** The IQR (interquartile range) for the average song length is from _______ to _______.

Exercise:

Problem:

Salaries for teachers in a particular elementary school district are normally distributed with a mean of $44,000 and a standard deviation of $6500. We randomly survey 10 teachers from that district.

- **a** In words, $X =$
- **b** In words, $X =$
- **c** X~
- **d** In words, $\Sigma X =$
- **e** ΣX~
- **f** Find the probability that the teachers earn a total of over $400,000.
- **g** Find the 90th percentile for an individual teacher's salary.
- **h** Find the 90th percentile for the average teachers' salary.
- **i** If we surveyed 70 teachers instead of 10, graphically, how would that change the distribution for X?
- **j** If each of the 70 teachers received a $3000 raise, graphically, how would that change the distribution for X?

Solution:

- **c** $N\left(44{,}000, \frac{6500}{\sqrt{10}}\right)$
- **e** $N(440{,}000, (\sqrt{10})(6500))$
- **f** 0.9742
- **g** \$52,330
- **h** \$46,634

Exercise:

Problem:

The distribution of income in some Third World countries is considered wedge shaped (many very poor people, very few middle income people, and few to many wealthy people). Suppose we pick a country with a wedge distribution. Let the average salary be \$2000 per year with a standard deviation of \$8000. We randomly survey 1000 residents of that country.

- **a** In words, $X =$
- **b** In words, $X =$
- **c** X ~
- **d** How is it possible for the standard deviation to be greater than the average?
- **e** Why is it more likely that the average of the 1000 residents will be from \$2000 to \$2100 than from \$2100 to \$2200?

Exercise:

Problem:

The average length of a maternity stay in a U.S. hospital is said to be 2.4 days with a standard deviation of 0.9 days. We randomly survey 80 women who recently bore children in a U.S. hospital.

- **a** In words, $X =$
- **b** In words, $X =$

- **c** X~
- **d** In words, $\Sigma X =$
- **e** ΣX~
- **f** Is it likely that an individual stayed more than 5 days in the hospital? Why or why not?
- **g** Is it likely that the average stay for the 80 women was more than 5 days? Why or why not?
- **h** Which is more likely:
  - **i**an individual stayed more than 5 days; or
  - **ii**the average stay of 80 women was more than 5 days?
- **i** If we were to sum up the women's stays, is it likely that, collectively they spent more than a year in the hospital? Why or why not?

Solution:

- **c** $N(2.4, \frac{0.9}{\sqrt{80}})$
- **e**$N(192{,}8.05)$
- **h**Individual

Exercise:

Problem:

In 1940 the average size of a U.S. farm was 174 acres. Let's say that the standard deviation was 55 acres. Suppose we randomly survey 38 farmers from 1940. (Source: U.S. Dept. of Agriculture)

- **a** In words, $X =$
- **b** In words, $X =$
- **c** X~
- **d** The IQR for X is from ________ acres to ________ acres.

Exercise:

Problem:

The stock closing prices of 35 U.S. semiconductor manufacturers are given below. (Source: **Wall Street Journal**)

8.625 30.25 27.625 46.75 32.875 18.25 5 0.125 2.9375 6.875 28.25 24.25 21 1.5 30.25 71 43.5 49.25 2.5625 31 16.5 9.5 18.5 18 9 10.5 16.625 1.25 18 12.875 7 12.875 2.875 60.25 29.25

- **a** In words, $X =$
- **b**
    - **i**$x =$
    - **ii** $s_x =$
    - **iii** $n =$
- **c** Construct a histogram of the distribution of the averages. Start at $x = -0.0005$. Make bar widths of 10.
- **d** In words, describe the distribution of stock prices.
- **e** Randomly average 5 stock prices together. (Use a random number generator.) Continue averaging 5 pieces together until you have 10 averages. List those 10 averages.
- **f** Use the 10 averages from (e) to calculate:
    - **i** $x =$
    - **ii** $s_x =$
- **g** Construct a histogram of the distribution of the averages. Start at $x = -0.0005$. Make bar widths of 10.
- **h** Does this histogram look like the graph in (c)?
- **i** In 1 - 2 complete sentences, explain why the graphs either look the same or look different?
- **j** Based upon the theory of the Central Limit Theorem, X~

Solution:

- **b**$20.71; $17.31; 35

- **d** Exponential distribution, X ~ $\text{Exp}(1/20.71)$
- **f** \$20.71; \$11.14
- **j** $N(20.71, \frac{17.31}{\sqrt{5}})$

Exercise:

Problem:

Use the Initial Public Offering data (see “Table of Contents) to do this problem.

- **a** In words, $X =$
- **b**
  - **i**$\mu_X =$
  - **ii** $\sigma_X =$
  - **iii** $n =$
- **c** Construct a histogram of the distribution. Start at $x = -0.50$. Make bar widths of \$5.
- **d** In words, describe the distribution of stock prices.
- **e** Randomly average 5 stock prices together. (Use a random number generator.) Continue averaging 5 pieces together until you have 15 averages. List those 15 averages.
- **f** Use the 15 averages from (e) to calculate the following:
  - **i**$x =$
  - **ii** $s_x =$
- **g** Construct a histogram of the distribution of the averages. Start at $x = -0.50$. Make bar widths of \$5.
- **h** Does this histogram look like the graph in (c)? Explain any differences.
- **i** In 1 - 2 complete sentences, explain why the graphs either look the same or look different?
- **j** Based upon the theory of the Central Limit Theorem, X ~

Try these multiple choice questions (Exercises19 - 23).

The next two questions refer to the following information: The time to wait for a particular rural bus is distributed uniformly from 0 to 75 minutes. 100 riders are randomly sampled to learn how long they waited.

Exercise:

Problem:

The 90th percentile sample average wait time (in minutes) for a sample of 100 riders is:

- **A** 315.0
- **B** 40.3
- **C** 38.5
- **D** 65.2

Solution:

B

Exercise:

Problem:

Would you be surprised, based upon numerical calculations, if the sample average wait time (in minutes) for 100 riders was less than 30 minutes?

- **A** Yes
- **B** No
- **C** There is not enough information.

Solution:

A

Exercise:

Problem:

Which of the following is NOT TRUE about the distribution for averages?

- **A** The mean, median and mode are equal
- **B** The area under the curve is one
- **C** The curve never touches the x-axis
- **D** The curve is skewed to the right

Solution:

D

The next three questions refer to the following information: The cost of unleaded gasoline in the Bay Area once followed an unknown distribution with a mean of $4.59 and a standard deviation of $0.10. Sixteen gas stations from the Bay Area are randomly chosen. We are interested in the average cost of gasoline for the 16 gas stations.

Exercise:

Problem:

The distribution to use for the average cost of gasoline for the 16 gas stations is

- **A** $X \sim N(4.59, 0.10)$
- **B** $X \sim N\left(4.59, \frac{0.10}{\sqrt{16}}\right)$
- **C** $X \sim N\left(4.59, \frac{0.10}{16}\right)$
- **D** $X \sim N\left(4.59, \frac{16}{0.10}\right)$

Solution:

B

Exercise:

Problem:

What is the probability that the average price for 16 gas stations is over \$4.69?

- **A** Almost zero
- **B** 0.1587
- **C** 0.0943
- **D** Unknown

Solution:

A

Exercise:

Problem:

Find the probability that the average price for 30 gas stations is less than \$4.55.

- **A**0.6554
- **B**0.3446
- **C**0.0142
- **D**0.9858
- **E**0

Solution:

C

Exercise:

Problem:

For the Charter School Problem (Example 6) in **Central Limit Theorem: Using the Central Limit Theorem**, calculate the following using the normal approximation to the binomial.

- **A** Find the probability that less than 100 favor a charter school for grades K - 5.
- **B** Find the probability that 170 or more favor a charter school for grades K - 5.
- **C** Find the probability that no more than 140 favor a charter school for grades K - 5.
- **D** Find the probability that there are fewer than 130 that favor a charter school for grades K - 5.
- **E** Find the probability that exactly 150 favor a charter school for grades K - 5.

If you either have access to an appropriate calculator or computer software, try calculating these probabilities using the technology. Try also using the suggestion that is at the bottom of **Central Limit Theorem: Using the Central Limit Theorem** for finding a website that calculates binomial probabilities.

Solution:

- **C** 0.0162
- **E** 0.0268

Exercise:

Problem:

Four friends, Janice, Barbara, Kathy and Roberta, decided to carpool together to get to school. Each day the driver would be chosen by randomly selecting one of the four names. They carpool to school for 96 days. Use the normal approximation to the binomial to calculate the following probabilities. Round the standard deviation to 4 decimal places.

- **A** Find the probability that Janice is the driver at most 20 days.
- **B** Find the probability that Roberta is the driver more than 16 days.
- **C** Find the probability that Barbara drives exactly 24 of those 96 days.

If you either have access to an appropriate calculator or computer software, try calculating these probabilities using the technology. Try also using the suggestion that is at the bottom of **Central Limit Theorem: Using the Central Limit Theorem** for finding a website that calculates binomial probabilities.

Solution:

- **A** 0.2047
- **B** 0.9615
- **C** 0.0938

**Exercise 24 contributed by Roberta Bloom

Review

The next three questions refer to the following information: Richard's Furniture Company delivers furniture from 10 A.M. to 2 P.M. continuously and uniformly. We are interested in how long (in hours) past the 10 A.M. start time that individuals wait for their delivery.

Exercise:

Problem: X ~

- **A** $U(0,4)$
- **B** $U(10,2)$
- **C** $N(2,1)$

Solution:

A

Exercise:

Problem: The average wait time is:

- **A** 1 hour
- **B** 2 hour
- **C** 2.5 hour
- **D** 4 hour

Solution:

B

Exercise:

Problem:

- 40% of full-time students took 4 years to graduate
- 30% of full-time students took 5 years to graduate

- 20% of full-time students took 6 years to graduate
- 10% of full-time students took 7 years to graduate

The expected time for full-time students to graduate is:

- **A** 4 years
- **B** 4.5 years
- **C** 5 years
- **D** 5.5 years

Solution:

C

Exercise:

Problem:

The length of time to brush one's teeth is generally thought to be exponentially distributed with a mean of $\frac{3}{4}$ minutes. Find the probability that a randomly selected person brushes his/her teeth less than $\frac{3}{4}$ minutes.

- **A** 0.5
- **B** $\frac{3}{4}$
- **C** 0.43
- **D** 0.63

Solution:

D

Exercise:

Problem:

Which distribution accurately describes the following situation?

The chance that a teenage boy regularly gives his mother a kiss goodnight (and he should!!) is about 20%. Fourteen teenage boys are randomly surveyed.

$X =$the number of teenage boys that regularly give their mother a kiss goodnight

- **A** $B(14,0.20)$
- **B** $N(2.8,2.24)$

Solution:

A

Confidence Intervals

Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Calculate and interpret confidence intervals for one population mean and one population proportion.
- Interpret the student-t probability distribution as the sample size changes.
- Discriminate between problems applying the normal and the student-t distributions.

Introduction

Suppose you are trying to determine the mean rent of a two-bedroom apartment in your town. You might look in the classified section of the newspaper, write down several rents listed, and average them together. You would have obtained a point estimate of the true mean. If you are trying to determine the percent of times you make a basket when shooting a basketball, you might count the number of shots you make and divide that by the number of shots you attempted. In this case, you would have obtained a point estimate for the true proportion.

We use sample data to make generalizations about an unknown population. This part of statistics is called **inferential statistics**. **The sample data help us to make an estimate of a population parameter**. We realize that the point estimate is most likely not the exact value of the population parameter, but close to it. After calculating point estimates, we construct confidence intervals in which we believe the parameter lies.

In this chapter, you will learn to construct and interpret confidence intervals. You will also learn a new distribution, the Student's-t, and how it is used with these intervals. Throughout the chapter, it is important to keep in mind that the confidence interval is a random variable. It is the parameter that is fixed.

If you worked in the marketing department of an entertainment company, you might be interested in the mean number of compact discs (CD's) a consumer buys per month. If so, you could conduct a survey and calculate the sample mean, $\overline{x}$, and the sample standard deviation, s. You would use $\overline{x}$ to estimate the population mean and s to estimate the population standard deviation. The sample mean, $\overline{x}$, is the **point estimate** for the population mean, μ. The sample standard deviation, s, is the point estimate for the population standard deviation, σ.

Each of $\overline{x}$ and s is also called a statistic.

A **confidence interval** is another type of estimate but, instead of being just one number, it is an interval of numbers. The interval of numbers is a range of values calculated from a given set of sample data. The confidence interval is likely to include an unknown population parameter.

Suppose for the CD example we do not know the population mean μ but we do know that the population standard deviation is $\sigma = 1$ and our sample size is 100. Then by the Central Limit Theorem, the standard deviation for the sample mean is

$$\frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1.$$

The **Empirical Rule**, which applies to bell-shaped distributions, says that in approximately 95% of the samples, the sample mean, $\overline{x}$, will be within two standard deviations of the population mean μ. For our CD example, two standard deviations is $(2)(0.1) = 0.2$. The sample mean $\overline{x}$ is likely to be within 0.2 units of μ.

Because $\overline{x}$ is within 0.2 units of μ, which is unknown, then μ is likely to be within 0.2 units of $\overline{x}$ in 95% of the samples. The population mean μ is contained in an interval whose lower number is calculated by taking the sample mean and subtracting two standard deviations ($(2)(0.1)$) and whose upper number is calculated by taking the sample mean and adding two standard deviations. In other words, μ is between $\overline{x} - 0.2$ and $\overline{x} + 0.2$ in 95% of all the samples.

For the CD example, suppose that a sample produced a sample mean $\overline{x} = 2$. Then the unknown population mean μ is between

$\overline{x} - 0.2 = 2 - 0.2 = 1.8$ and $\overline{x} + 0.2 = 2 + 0.2 = 2.2$

We say that we are **95% confident** that the unknown population mean number of CDs is between 1.8 and 2.2. **The 95% confidence interval is (1.8, 2.2).**

The 95% confidence interval implies two possibilities. Either the interval (1.8, 2.2) contains the true mean μ or our sample produced an $\overline{x}$ that is not within 0.2 units of the true mean μ. The second possibility happens for only 5% of all the samples (100% - 95%).

Remember that a confidence interval is created for an unknown population parameter like the population mean, μ. Confidence intervals for some parameters have the form

(point estimate - margin of error, point estimate + margin of error)

The margin of error depends on the confidence level or percentage of confidence.

When you read newspapers and journals, some reports will use the phrase "margin of error." Other reports will not use that phrase, but include a confidence interval as the point estimate + or - the margin of error. These are two ways of expressing the same concept.

Note: Although the text only covers symmetric confidence intervals, there are non-symmetric confidence intervals (for example, a confidence interval for the standard deviation).

Optional Collaborative Classroom Activity

Have your instructor record the number of meals each student in your class eats out in a week. Assume that the standard deviation is known to be 3 meals. Construct an approximate 95% confidence interval for the true mean number of meals students eat out each week.

1. Calculate the sample mean.
2. $\sigma = 3$ and $n =$ the number of students surveyed.
3. Construct the interval $\left(\overline{x} - 2 \cdot \frac{\sigma}{\sqrt{n}}, \overline{x} + 2 \cdot \frac{\sigma}{\sqrt{n}}\right)$

We say we are approximately 95% confident that the true average number of meals that students eat out in a week is between __________ and ___________.

Glossary

Confidence Interval (CI)
: An interval estimate for an unknown population parameter. This depends on:

- The desired confidence level.
- Information that is known about the distribution (for example, known standard deviation).
- The sample and its size.

Inferential Statistics
: Also called statistical inference or inductive statistics. This facet of statistics deals with estimating a population parameter based on a sample statistic. For example, if 4 out of the 100 calculators sampled are defective we might infer that 4 percent of the production is defective.

Parameter
: A numerical characteristic of the population.

Point Estimate
: A single number computed from a sample and used to estimate a population parameter.

Confidence Interval, Single Population Mean, Population Standard Deviation Known, Normal
Confidence Intervals: Confidence Interval, Single Population Mean, Population Standard Deviation Known, Normal is part of the collection col10555 written by Barbara Illowsky and Susan Dean with contributions from Roberta Bloom.

Calculating the Confidence Interval

To construct a confidence interval for a single unknown population mean μ **, where the population standard deviation is known,** we need $\overline{x}$ as an estimate for μ and we need the margin of error. Here, the margin of error is called the **error bound for a population mean** (abbreviated **EBM**). The sample mean $\overline{x}$ is the **point estimate** of the unknown population mean μ
The confidence interval estimate will have the form:

- (point estimate - error bound, point estimate + error bound) or, in symbols,$(\overline{x} - \text{EBM}, \overline{x} + \text{EBM})$

The margin of error depends on the **confidence level** (abbreviated **CL**). The confidence level is often considered the probability that the calculated confidence interval estimate will contain the true population parameter. However, it is more accurate to state that the confidence level is the percent of confidence intervals that contain the true population parameter when repeated samples are taken. Most often, it is the choice of the person constructing the confidence interval to choose a confidence level of 90% or higher because that person wants to be reasonably certain of his or her conclusions.

There is another probability called alpha (α). α is related to the confidence level CL. α is the probability that the interval does not contain the unknown population parameter.
Mathematically, α + CL = 1.

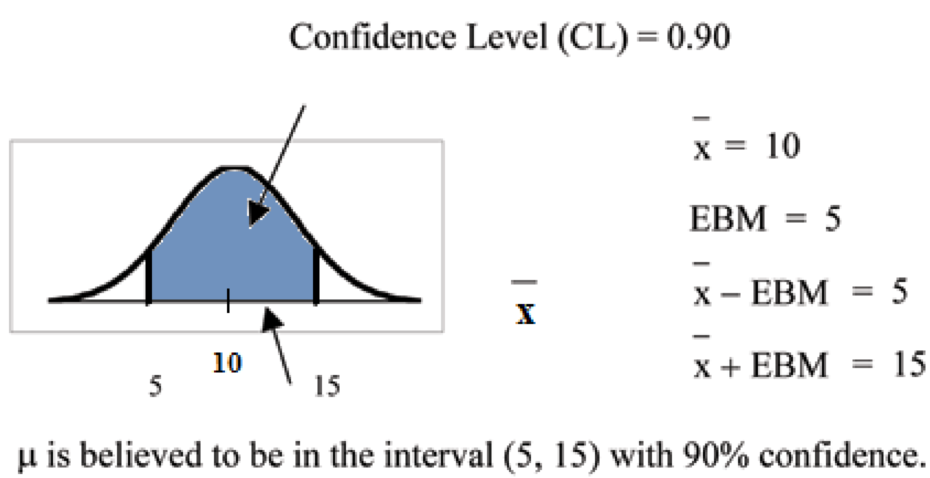
Example:

- Suppose we have collected data from a sample. We know the sample mean but we do not know the mean for the entire population.
- The sample mean is 7 and the error bound for the mean is 2.5.

$\overline{x} = 7$ and $\text{EBM} = 2.5$.
The confidence interval is $(7 - 2.5, 7 + 2.5)$; calculating the values gives $(4.5, 9.5)$.
If the confidence level (CL) is 95%, then we say that "We estimate with 95% confidence that the true value of the population mean is between 4.5 and 9.5."

A confidence interval for a population mean with a known standard deviation is based on the fact that the sample means follow an approximately normal distribution. Suppose that our sample has a mean of $\overline{x} = 10$ and we have constructed the 90% confidence interval (5, 15) where $\text{EBM} = 5$.

To get a 90% confidence interval, we must include the central 90% of the probability of the normal distribution. If we include the central 90%, we leave out a total of α = 10% in both tails, or 5% in each tail, of the normal distribution.



To capture the central 90%, we must go out 1.645 "standard deviations" on either side of the calculated sample mean. 1.645 is the z-score from a

Standard Normal probability distribution that puts an area of 0.90 in the center, an area of 0.05 in the far left tail, and an area of 0.05 in the far right tail.

It is important that the "standard deviation" used must be appropriate for the parameter we are estimating. So in this section, we need to use the standard deviation that applies to sample means, which is $\frac{\sigma}{\sqrt{n}}$. $\frac{\sigma}{\sqrt{n}}$ is commonly called the "standard error of the mean" in order to clearly distinguish the standard deviation for a mean from the population standard deviation σ.

In summary, as a result of the Central Limit Theorem:

- $\overline{X}$ is normally distributed, that is, $\overline{X} \sim \mathrm{N}\left(\mu_X, \frac{\sigma}{\sqrt{n}}\right)$.
- **When the population standard deviation σ is known, we use a Normal distribution to calculate the error bound.**

Calculating the Confidence Interval:

To construct a confidence interval estimate for an unknown population mean, we need data from a random sample. The steps to construct and interpret the confidence interval are:

- Calculate the sample mean $\overline{x}$ from the sample data. Remember, in this section, we already know the population standard deviation σ.
- Find the Z-score that corresponds to the confidence level.
- Calculate the error bound EBM
- Construct the confidence interval
- Write a sentence that interprets the estimate in the context of the situation in the problem. (Explain what the confidence interval means, in the words of the problem.)

We will first examine each step in more detail, and then illustrate the process with some examples.

Finding z for the stated Confidence Level

When we know the population standard deviation σ, we use a standard normal distribution to calculate the error bound EBM and construct the confidence interval. We need to find the value of z that puts an area equal to

the confidence level (in decimal form) in the middle of the standard normal distribution Z~N(0,1).

The confidence level, CL, is the area in the middle of the standard normal distribution. $\mathrm{CL} = 1 - \alpha$. So α is the area that is split equally between the two tails. Each of the tails contains an area equal to $\frac{\alpha}{2}$.

The z-score that has an area to the right of $\frac{\alpha}{2}$ is denoted by $z_{\frac{\alpha}{2}}$

For example, when $\mathrm{CL} = 0.95$ then $\alpha = 0.05$ and $\frac{\alpha}{2} = 0.025$; we write $z_{\frac{\alpha}{2}} = z_{.025}$

The area to the right of $z_{.025}$ is 0.025 and the area to the left of $z_{.025}$ is 1-0.025 = 0.975

$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$, using a calculator, computer or a Standard Normal probability table.

Using the TI83, TI83+ or TI84+ calculator: `invNorm`$(0.975, 0, 1) = 1.96$

CALCULATOR NOTE: Remember to use area to the LEFT of $z_{\frac{\alpha}{2}}$; in this chapter the last two inputs in the invNorm command are 0,1 because you are using a Standard Normal Distribution Z~N(0,1)

EBM: Error Bound
The error bound formula for an unknown population mean μ when the population standard deviation σ is known is

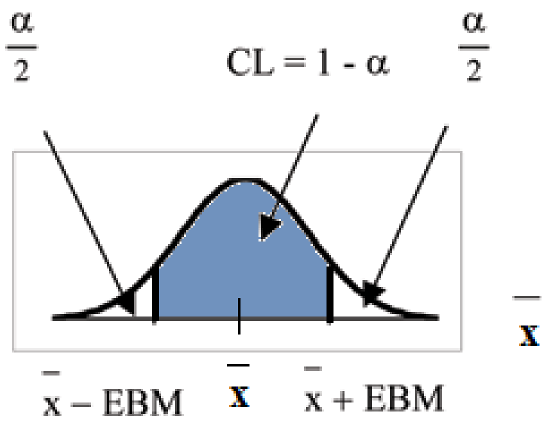
- $\mathrm{EBM} = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

Constructing the Confidence Interval

- The confidence interval estimate has the format $(\overline{x} - \mathrm{EBM}, \overline{x} + \mathrm{EBM})$.

The graph gives a picture of the entire situation.

$\mathrm{CL} + \frac{\alpha}{2} + \frac{\alpha}{2} = \mathrm{CL} + \alpha = 1.$



Writing the Interpretation
The interpretation should clearly state the confidence level (CL), explain what population parameter is being estimated (here, a **population mean**), and should state the confidence interval (both endpoints). "We estimate with ___% confidence that the true population mean (include context of the problem) is between ___ and ___ (include appropriate units)."

Example:
Suppose scores on exams in statistics are normally distributed with an unknown population mean and a population standard deviation of 3 points. A random sample of 36 scores is taken and gives a sample mean (sample mean score) of 68. Find a confidence interval estimate for the population mean exam score (the mean score on all exams).
Exercise:

Problem:

Find a 90% confidence interval for the true (population) mean of statistics exam scores.

Solution:

- You can use technology to directly calculate the confidence interval
- The first solution is shown step-by-step (Solution A).
- The second solution uses the TI-83, 83+ and 84+ calculators (Solution B).

Solution A
To find the confidence interval, you need the sample mean, $\overline{x}$, and the EBM.

- $\overline{x} = 68$
- $\text{EBM} = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$
- $\sigma = 3$; $n = 36$; The confidence level is 90% (CL=0.90)

$\text{CL} = 0.90$ so $\alpha = 1 - \text{CL} = 1 - 0.90 = 0.10$

$\frac{\alpha}{2} = 0.05 \qquad z_{\frac{\alpha}{2}} = z_{.05}$

The area to the right of $z_{.05}$ is 0.05 and the area to the left of $z_{.05}$ is 1−0.05=0.95

$z_{\frac{\alpha}{2}} = z_{.05} = 1.645$

using invNorm(0.95,0,1) on the TI-83,83+,84+ calculators. This can also be found using appropriate commands on other calculators, using a computer, or using a probability table for the Standard Normal distribution.

$\text{EBM} = 1.645 \cdot \left(\frac{3}{\sqrt{36}}\right) = 0.8225$

$\overline{x} - \text{EBM} = 68 - 0.8225 = 67.1775$

$\overline{x} + \text{EBM} = 68 + 0.8225 = 68.8225$

The 90% confidence interval is **(67.1775, 68.8225).**

Solution B

Using a function of the TI-83, TI-83+ or TI-84 calculators:

Press `STAT` and arrow over to `TESTS`.
Arrow down to `7:ZInterval`.
Press `ENTER`.
Arrow to `Stats` and press `ENTER`.
Arrow down and enter 3 for σ, 68 for $\overline{x}$, 36 for n, and .90 for `C-level`.
Arrow down to `Calculate` and press `ENTER`.
The confidence interval is (to 3 decimal places) (67.178, 68.822).

Interpretation

We estimate with 90% confidence that the true population mean exam score for all statistics students is between 67.18 and 68.82.

Explanation of 90% Confidence Level

90% of all confidence intervals constructed in this way contain the true mean statistics exam score. For example, if we constructed 100 of these confidence intervals, we would expect 90 of them to contain the true population mean exam score.

Changing the Confidence Level or Sample Size

Example: Changing the Confidence Level

Exercise:

Problem:

Suppose we change the original problem by using a 95% confidence level. Find a 95% confidence interval for the true (population) mean statistics exam score.

Solution:

To find the confidence interval, you need the sample mean, $\overline{x}$, and the EBM.

- $\overline{x} = 68$
- $\text{EBM} = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$
- $\sigma = 3$; $n = 36$; The confidence level is 95% (CL=0.95)

$\text{CL} = 0.95$ so $\alpha = 1 - \text{CL} = 1 - 0.95 = 0.05$

$\frac{\alpha}{2} = 0.025 \qquad z_{\frac{\alpha}{2}} = z_{.025}$

The area to the right of $z_{.025}$ is 0.025 and the area to the left of $z_{.025}$ is 1−0.025=0.975

$z_{\frac{\alpha}{2}} = z_{.025} = 1.96$

using invnorm(.975,0,1) on the TI-83,83+,84+ calculators. (This can also be found using appropriate commands on other calculators, using a computer, or using a probability table for the Standard Normal distribution.)

$\text{EBM} = 1.96 \cdot \left(\frac{3}{\sqrt{36}}\right) = 0.98$

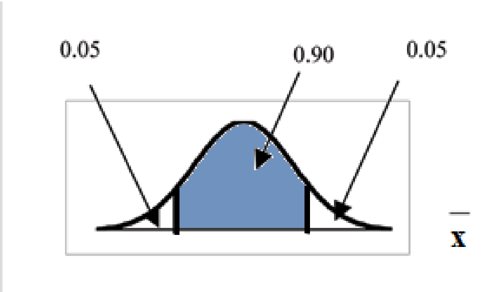
$\overline{x} - \text{EBM} = 68 - 0.98 = 67.02$

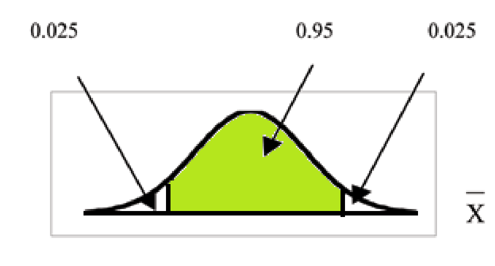
$\overline{x} + \text{EBM} = 68 + 0.98 = 68.98$

Interpretation
We estimate with 95 % confidence that the true population mean for all statistics exam scores is between 67.02 and 68.98.

Explanation of 95% Confidence Level
95% of all confidence intervals constructed in this way contain the true value of the population mean statistics exam score.

Comparing the results
The 90% confidence interval is (67.18, 68.82). The 95% confidence interval is (67.02, 68.98). The 95% confidence interval is wider. If you look at the graphs, because the area 0.95 is larger than the area 0.90, it makes sense that the 95% confidence interval is wider.





Summary: Effect of Changing the Confidence Level

- Increasing the confidence level increases the error bound, making the confidence interval wider.
- Decreasing the confidence level decreases the error bound, making the confidence interval narrower.

Example: Changing the Sample Size:
Suppose we change the original problem to see what happens to the error bound if the sample size is changed.
Exercise:

Problem:

Leave everything the same except the sample size. Use the original 90% confidence level. What happens to the error bound and the confidence interval if we increase the sample size and use n=100 instead of n=36? What happens if we decrease the sample size to n=25 instead of n=36?

- $\bar{x} = 68$
- $\text{EBM} = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$
- $\sigma = 3$; The confidence level is 90% (CL=0.90) ; $z_{\frac{\alpha}{2}} = z_{.05} = 1.645$

Solution:

If we **increase** the sample size n to 100, we **decrease** the error bound.

When $n = 100 : \text{EBM} = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right) = 1.645 \cdot \left(\frac{3}{\sqrt{100}}\right) = 0.4935$

Solution:

If we **decrease** the sample size n to 25, we **increase** the error bound.

When $n = 25 : \text{EBM} = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right) = 1.645 \cdot \left(\frac{3}{\sqrt{25}}\right) = 0.987$

Summary: Effect of Changing the Sample Size

- Increasing the sample size causes the error bound to decrease, making the confidence interval narrower.
- Decreasing the sample size causes the error bound to increase, making the confidence interval wider.

Working Backwards to Find the Error Bound or Sample Mean

Working Bacwards to find the Error Bound or the Sample Mean
When we calculate a confidence interval, we find the sample mean and calculate the error bound and use them to calculate the confidence interval. But sometimes when we read statistical studies, the study may state the confidence interval only. If we know the confidence interval, we can work backwards to find both the error bound and the sample mean.

Finding the Error Bound

- From the upper value for the interval, subtract the sample mean
- OR, From the upper value for the interval, subtract the lower value. Then divide the difference by 2.

Finding the Sample Mean

- Subtract the error bound from the upper value of the confidence interval
- OR, Average the upper and lower endpoints of the confidence interval

Notice that there are two methods to perform each calculation. You can choose the method that is easier to use with the information you know.

Example:
Suppose we know that a confidence interval is **(67.18, 68.82)** and we want to find the error bound. We may know that the sample mean is 68. Or perhaps our source only gave the confidence interval and did not tell us the value of the the sample mean.
Calculate the Error Bound:

- If we know that the sample mean is 68: $\text{EBM} = 68.82 - 68 = 0.82$
- If we don't know the sample mean: $\text{EBM} = \frac{(68.82-67.18)}{2} = 0.82$

Calculate the Sample Mean:

- If we know the error bound: $\overline{x} = 68.82 - 0.82 = 68$
- If we don't know the error bound: $\overline{x} = \frac{(67.18+68.82)}{2} = 68$

Calculating the Sample Size n

If researchers desire a specific margin of error, then they can use the error bound formula to calculate the required sample size.

The error bound formula for a population mean when the population standard deviation is known is $\text{EBM} = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$

The formula for sample size is $n = \frac{z^2\sigma^2}{\text{EBM}^2}$, found by solving the error bound formula for n

In this formula, z is $z_{\frac{\alpha}{2}}$, corresponding to the desired confidence level. A researcher planning a study who wants a specified confidence level and error bound can use this formula to calculate the size of the sample needed for the study.

Example:
The population standard deviation for the age of Foothill College students is 15 years. If we want to be 95% confident that the sample mean age is within 2 years of the true population mean age of Foothill College students , how many randomly selected Foothill College students must be surveyed?

- From the problem, we know that $\sigma = 15$ and EBM=2
- $z = z_{.025} = 1.96$, because the confidence level is 95%.
- $n = \frac{z^2\sigma^2}{\mathrm{EBM}^2} = \frac{1.96^2 15^2}{2^2}$ =216.09 using the sample size equation.
- Use n = 217: Always round the answer UP to the next higher integer to ensure that the sample size is large enough.

Therefore, 217 Foothill College students should be surveyed in order to be 95% confident that we are within 2 years of the true population mean age of Foothill College students.

**With contributions from Roberta Bloom

Glossary

Confidence Interval (CI)
: An interval estimate for an unknown population parameter. This depends on:

  - The desired confidence level.

- Information that is known about the distribution (for example, known standard deviation).
- The sample and its size.

Confidence Level (CL)
: The percent expression for the probability that the confidence interval contains the true population parameter. For example, if the $\mathrm{CL} = 90\%$, then in 90 out of 100 samples the interval estimate will enclose the true population parameter.

Error Bound for a Population Mean (EBM)
: The margin of error. Depends on the confidence level, sample size, and known or estimated population standard deviation.

Confidence Interval, Single Population Mean, Standard Deviation Unknown, Student-T

Confidence Interval, Single Population Mean, Population Standard Deviation Unknown, Student-t is part of the collection col10555 written by Barbara Illowsky and Susan Dean with contributions from Roberta Bloom.

In practice, we rarely know the population **standard deviation**. In the past, when the sample size was large, this did not present a problem to statisticians. They used the sample standard deviation s as an estimate for σ and proceeded as before to calculate a **confidence interval** with close enough results. However, statisticians ran into problems when the sample size was small. A small sample size caused inaccuracies in the confidence interval.

William S. Gossett (1876-1937) of the Guinness brewery in Dublin, Ireland ran into this problem. His experiments with hops and barley produced very few samples. Just replacing σ with s did not produce accurate results when he tried to calculate a confidence interval. He realized that he could not use a normal distribution for the calculation; he found that the actual distribution depends on the sample size. This problem led him to "discover" what is called the **Student's-t distribution**. The name comes from the fact that Gosset wrote under the pen name "Student."

Up until the mid 1970s, some statisticians used the **normal distribution** approximation for large sample sizes and only used the Student's-t distribution for sample sizes of at most 30. With the common use of graphing calculators and computers, the practice is to use the Student's-t distribution whenever s is used as an estimate for σ.

If you draw a simple random sample of size n from a population that has approximately a normal distribution with mean μ and unknown population standard deviation σ and calculate the t-score $\mathrm{t} = \frac{\bar{\mathrm{x}}-\mu}{\left(\frac{s}{\sqrt{n}}\right)}$, then the t-scores follow a **Student's-t distribution with** $n-1$ **degrees of freedom**. The t-score has the same interpretation as the **z-score**. It measures how far $\bar{x}$ is from its mean μ. For each sample size n, there is a different Student's-t distribution.

The **degrees of freedom**, $n-1$, come from the calculation of the sample standard deviation s. In Chapter 2, we used n deviations $(x-\overline{x}\ \text{values})$ to calculate s. Because the sum of the deviations is 0, we can find the last deviation once we know the other $n-1$ deviations. The other $n-1$ deviations can change or vary freely. **We call the number $n-1$ the degrees of freedom (df).**

Properties of the Student's-t Distribution

- The graph for the Student's-t distribution is similar to the Standard Normal curve.
- The mean for the Student's-t distribution is 0 and the distribution is symmetric about 0.
- The Student's-t distribution has more probability in its tails than the Standard Normal distribution because the spread of the t distribution is greater than the spread of the Standard Normal. So the graph of the Student's-t distribution will be thicker in the tails and shorter in the center than the graph of the Standard Normal distribution.
- The exact shape of the Student's-t distribution depends on the "degrees of freedom". As the degrees of freedom increases, the graph Student's-t distribution becomes more like the graph of the Standard Normal distribution.
- The underlying population of individual observations is assumed to be normally distributed with unknown population mean μ and unknown population standard deviation σ. The size of the underlying population is generally not relevant unless it is very small. If it is bell shaped (normal) then the assumption is met and doesn't need discussion. Random sampling is assumed but it is a completely separate assumption from normality.

Calculators and computers can easily calculate any Student's-t probabilities. The TI-83,83+,84+ have a tcdf function to find the probability for given values of t. The grammar for the tcdf command is tcdf(lower bound, upper bound, degrees of freedom). However for confidence intervals, we need to use **inverse** probability to find the value of t when we know the probability.

For the TI-84+ you can use the invT command on the DISTRibution menu. The invT command works similarly to the invnorm. The invT command

requires two inputs: **invT(area to the left, degrees of freedom)** The output is the t-score that corresponds to the area we specified.

The TI-83 and 83+ do not have the invT command. (The TI-89 has an inverse T command.)

A probability table for the Student's-t distribution can also be used. The table gives t-scores that correspond to the confidence level (column) and degrees of freedom (row). (The TI-86 does not have an invT program or command, so if you are using that calculator, you need to use a probability table for the Student's-t distribution.) When using t-table, note that some tables are formatted to show the confidence level in the column headings, while the column headings in some tables may show only corresponding area in one or both tails.

A Student's-t table (See the Table of Contents **15. Tables**) gives t-scores given the degrees of freedom and the right-tailed probability. The table is very limited. **Calculators and computers can easily calculate any Student's-t probabilities.**
The notation for the Student's-t distribution is (using T as the random variable) is

- $T \sim t_{\text{df}}$ where $\text{df} = n - 1$.
- For example, if we have a sample of size n=20 items, then we calculate the degrees of freedom as df=n−1=20−1=19 and we write the distribution as $T \sim t_{19}$

If the population standard deviation is not known, the **error bound for a population mean** is:

- $\text{EBM} = t_{\frac{\alpha}{2}} \cdot \left(\frac{s}{\sqrt{n}}\right)$
- $t_{\frac{\alpha}{2}}$ is the t-score with area to the right equal to $\frac{\alpha}{2}$
- use $\text{df} = n - 1$ degrees of freedom
- s = sample standard deviation

The format for the confidence interval is:

$(\overline{x} - \text{EBM}, \overline{x} + \text{EBM})$.

The TI-83, 83+ and 84 calculators have a function that calculates the confidence interval directly. To get to it,
Press `STAT`
Arrow over to `TESTS`.
Arrow down to `8:TInterval` and press `ENTER` (or just press `8`).

Example:
Exercise:

Problem:

Suppose you do a study of acupuncture to determine how effective it is in relieving pain. You measure sensory rates for 15 subjects with the results given below. Use the sample data to construct a 95% confidence interval for the mean sensory rate for the population (assumed normal) from which you took the data.

The solution is shown step-by-step and by using the TI-83, 83+ and 84+ calculators.
8.6 9.4 7.9 6.8 8.3 7.3 9.2 9.6 8.7 11.4 10.3 5.4 8.1 5.5 6.9

Solution:

- You can use technology to directly calculate the confidence interval.
- The first solution is step-by-step (Solution A).
- The second solution uses the Ti-83+ and Ti-84 calculators (Solution B).

Solution A
To find the confidence interval, you need the sample mean, $\overline{x}$, and the EBM.

$$\overline{x} = 8.2267 \qquad s = 1.6722 \qquad n = 15$$

$\text{df} = 15 - 1 = 14$

$\text{CL} = 0.95 \quad \text{so} \quad \alpha = 1 - \text{CL} = 1 - 0.95 = 0.05$

$\frac{\alpha}{2} = 0.025 \qquad t_{\frac{\alpha}{2}} = t_{.025}$

The area to the right of $t_{.025}$ is 0.025 and the area to the left of $t_{.025}$ is 1−0.025=0.975

$t_{\frac{\alpha}{2}} = t_{.025} = 2.14$ using invT(.975,14) on the TI-84+ calculator.

$\text{EBM} = t_{\frac{\alpha}{2}} \cdot \left(\frac{s}{\sqrt{n}}\right)$

$\text{EBM} = 2.14 \cdot \left(\frac{1.6722}{\sqrt{15}}\right) = 0.924$

$\overline{x} - \text{EBM} = 8.2267 - 0.9240 = 7.3$

$\overline{x} + \text{EBM} = 8.2267 + 0.9240 = 9.15$

The 95% confidence interval is **(7.30, 9.15)**.

We estimate with 95% confidence that the true population mean sensory rate is between 7.30 and 9.15.

Solution B
Using a function of the TI-83, TI-83+ or TI-84 calculators:

Press `STAT` and arrow over to `TESTS`.
Arrow down to `8:TInterval` and press `ENTER` (or you can just press `8`). Arrow to `Data` and press `ENTER`.
Arrow down to `List` and enter the list name where you put the data.
Arrow down to `Freq` and enter 1.
Arrow down to `C-level` and enter .95
Arrow down to `Calculate` and press `ENTER`.
The 95% confidence interval is (7.3006, 9.1527)

Note: When calculating the error bound, a probability table for the Student's-t distribution can also be used to find the value of t. The table gives t-scores that correspond to the confidence level (column) and degrees of freedom (row); the t-score is found where the row and column intersect in the table.

**With contributions from Roberta Bloom

Glossary

Confidence Interval (CI)
: An interval estimate for an unknown population parameter. This depends on:

- The desired confidence level.
- Information that is known about the distribution (for example, known standard deviation).
- The sample and its size.

Confidence Level (CL)
: The percent expression for the probability that the confidence interval contains the true population parameter. For example, if the $\mathrm{CL} = 90\%$, then in 90 out of 100 samples the interval estimate will enclose the true population parameter.

Degrees of Freedom (df)
: The number of objects in a sample that are free to vary.

Error Bound for a Population Mean (EBM)
: The margin of error. Depends on the confidence level, sample size, and known or estimated population standard deviation.

Normal Distribution

A continuous random variable (RV) with pdf
$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean of the distribution and σ is the standard deviation. Notation: $X \sim N(\mu, \sigma)$. If $\mu = 0$ and $\sigma = 1$, the RV is called **the standard normal distribution**.

Standard Deviation

A number that is equal to the square root of the variance and measures how far data values are from their mean. Notation: s for sample standard deviation and σ for population standard deviation.

Student's-**t** Distribution

Investigated and reported by William S. Gossett in 1908 and published under the pseudonym Student. The major characteristics of the random variable (RV) are:

- It is continuous and assumes any real values.
- The pdf is symmetrical about its mean of zero. However, it is more spread out and flatter at the apex than the normal distribution.
- It approaches the standard normal distribution as n gets larger.
- There is a "family" of t distributions: every representative of the family is completely defined by the number of degrees of freedom which is one less than the number of data.

Confidence Interval for a Population Proportion
Confidence Interval for a Population Proportion is part of the collection col10555 written by Barbara Illowsky and Susan Dean with contributions from Roberta Bloom.

During an election year, we see articles in the newspaper that state **confidence intervals** in terms of proportions or percentages. For example, a poll for a particular candidate running for president might show that the candidate has 40% of the vote within 3 percentage points. Often, election polls are calculated with 95% confidence. So, the pollsters would be 95% confident that the true proportion of voters who favored the candidate would be between 0.37 and 0.43 : $(0.40-0.03, 0.40+0.03)$.

Investors in the stock market are interested in the true proportion of stocks that go up and down each week. Businesses that sell personal computers are interested in the proportion of households in the United States that own personal computers. Confidence intervals can be calculated for the true proportion of stocks that go up or down each week and for the true proportion of households in the United States that own personal computers.

The procedure to find the confidence interval, the sample size, the **error bound,** and the **confidence level** for a proportion is similar to that for the population mean. The formulas are different.

How do you know you are dealing with a proportion problem? First, the underlying **distribution is binomial**. (There is no mention of a mean or average.) If X is a binomial random variable, then $X \sim B(n, p)$ where n = the number of trials and p = the probability of a success. To form a proportion, take X, the random variable for the number of successes and divide it by n, the number of trials (or the sample size). The random variable P' (read "P prime") is that proportion,

$$P' = \frac{X}{n}$$

(Sometimes the random variable is denoted as $\widehat{P}$, read "P hat".)

When n is large and p is not close to 0 or 1, we can use the **normal distribution** to approximate the binomial.

$$X \sim N\left(n \cdot p, \sqrt{n \cdot p \cdot q}\right)$$

If we divide the random variable by n, the mean by n, and the standard deviation by n, we get a normal distribution of proportions with $P\prime$, called the estimated proportion, as the random variable. (Recall that a proportion = the number of successes divided by n.)

$$\frac{X}{n} = P\prime \sim N\left(\frac{n \cdot p}{n}, \frac{\sqrt{n \cdot p \cdot q}}{n}\right)$$

Using algebra to simplify : $\frac{\sqrt{n \cdot p \cdot q}}{n} = \sqrt{\frac{p \cdot q}{n}}$

$P\prime$ **follows a normal distribution for proportions**: $P\prime \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$

The confidence interval has the form $(p\prime - \text{EBP}, p\prime + \text{EBP})$.

$p\prime = \frac{x}{n}$

$p\prime$ = the **estimated proportion** of successes ($p\prime$ is a **point estimate** for p, the true proportion)

x = the **number** of successes.

n = the size of the sample

The error bound for a proportion is

$$\text{EBP} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p\prime \cdot q\prime}{n}} \qquad \text{where } q\prime = 1 - p\prime$$

This formula is similar to the error bound formula for a mean, except that the "appropriate standard deviation" is different. For a mean, when the population standard deviation is known, the appropriate standard deviation

that we use is $\frac{\sigma}{\sqrt{n}}$. For a proportion, the appropriate standard deviation is $\sqrt{\frac{p \cdot q}{n}}$.

However, in the error bound formula, we use $\sqrt{\frac{p\prime \cdot q\prime}{n}}$ as the standard deviation, instead of $\sqrt{\frac{p \cdot q}{n}}$

However, in the error bound formula, the standard deviation is $\sqrt{\frac{p\prime \cdot q\prime}{n}}$.

In the error bound formula, the **sample proportions** $p\prime$ **and** $q\prime$ **are estimates of the unknown population proportions** p **and** q. The estimated proportions $p\prime$ and $q\prime$ are used because p and q are not known. $p\prime$ and $q\prime$ are calculated from the data. $p\prime$ is the estimated proportion of successes. $q\prime$ is the estimated proportion of failures.

The confidence interval can only be used if the number of successes $np\prime$ and the number of failures $nq\prime$ are both larger than 5.

Note: For the normal distribution of proportions, the z-score formula is as follows.

If $P\prime \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$ then the z-score formula is $z = \frac{p\prime - p}{\sqrt{\frac{p \cdot q}{n}}}$

Example:
Exercise:

Problem:

Suppose that a market research firm is hired to estimate the percent of adults living in a large city who have cell phones. 500 randomly selected adult residents in this city are surveyed to determine whether they have cell phones. Of the 500 people surveyed, 421 responded yes - they own cell phones. Using a 95% confidence level, compute a confidence interval estimate for the true proportion of adults residents of this city who have cell phones.

Solution

- You can use technology to directly calculate the confidence interval.
- The first solution is step-by-step (Solution A).
- The second solution uses a function of the TI-83, 83+ or 84 calculators (Solution B).

Solution:

Let X = the number of people in the sample who have cell phones. X is binomial. X ~ $B\left(500, \frac{421}{500}\right)$.

To calculate the confidence interval, you must find $p\prime$, $q\prime$, and EBP.

$n = 500$ x = the number of successes $= 421$

$p\prime = \frac{x}{n} = \frac{421}{500} = 0.842$

$p\prime = 0.842$ is the sample proportion; this is the point estimate of the population proportion.

$q\prime = 1 - p\prime = 1 - 0.842 = 0.158$

Since $\text{CL} = 0.95$, then
$\alpha = 1 - \text{CL} = 1 - 0.95 = 0.05$ $\frac{\alpha}{2} = 0.025$.

Then $z_{\frac{\alpha}{2}} = z_{.025} = 1.96$

Use the TI-83, 83+ or 84+ calculator command invNorm(0.975,0,1) to find $z_{.025}$. Remember that the area to the right of $z_{.025}$ is 0.025 and the area to the left of $z_{0.025}$ is 0.975. This can also be found using appropriate commands on other calculators, using a computer, or using a Standard Normal probability table.

$$\text{EBP} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p\prime \cdot q\prime}{n}} = 1.96 \cdot \sqrt{\frac{(0.842)\cdot(0.158)}{500}} = 0.032$$

$$p\prime - \text{EBP} = 0.842 - 0.032 = 0.81$$

$$p\prime + \text{EBP} = 0.842 + 0.032 = 0.874$$

The confidence interval for the true binomial population proportion is $(p\prime - \text{EBP}, p\prime + \text{EBP}) = (0.810, 0.874)$.

Interpretation
We estimate with 95% confidence that between 81% and 87.4% of all adult residents of this city have cell phones.

Explanation of 95% Confidence Level
95% of the confidence intervals constructed in this way would contain the true value for the population proportion of all adult residents of this city who have cell phones.

Solution:

Using a function of the TI-83, 83+ or 84 calculators:

Press `STAT` and arrow over to `TESTS`.
Arrow down to `A:1-PropZint`. Press `ENTER`.
Arrow down to x and enter 421.
Arrow down to n and enter 500.
Arrow down to `C-Level` and enter .95.
Arrow down to `Calculate` and press `ENTER`.
The confidence interval is (0.81003, 0.87397).

Example:
Exercise:

Problem:

For a class project, a political science student at a large university wants to estimate the percent of students that are registered voters. He surveys 500 students and finds that 300 are registered voters. Compute a 90% confidence interval for the true percent of students that are registered voters and interpret the confidence interval.

Solution:

- You can use technology to directly calculate the confidence interval.
- The first solution is step-by-step (Solution A).
- The second solution uses a function of the TI-83, 83+ or 84 calculators (Solution B).

Solution A
$x = 300$ and $n = 500$.

$p\prime = \frac{x}{n} = \frac{300}{500} = 0.600$

$q\prime = 1 - p\prime = 1 - 0.600 = 0.400$

Since $\text{CL} = 0.90$, then
$\alpha = 1 - \text{CL} = 1 - 0.90 = 0.10 \qquad \frac{\alpha}{2} = 0.05$.

$z_{\frac{\alpha}{2}} = z_{.05} = 1.645$

Use the TI-83, 83+ or 84+ calculator command invNorm(0.95,0,1) to find $z_{.05}$. Remember that the area to the right of $z_{.05}$ is 0.05 and the area to the left of $z_{.05}$ is 0.95. This can also be found using

appropriate commands on other calculators, using a computer, or using a Standard Normal probability table.

$$\text{EBP} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p\prime \cdot q\prime}{n}} = 1.645 \cdot \sqrt{\frac{(0.60)\cdot(0.40)}{500}} = 0.036$$

$$p\prime - \text{EBP} = 0.60 - 0.036 = 0.564$$

$$p\prime + \text{EBP} = 0.60 + 0.036 = 0.636$$

The confidence interval for the true binomial population proportion is $(p\prime - \text{EBP}, p\prime + \text{EBP}) = (0.564, 0.636)$.

Interpretation:

- We estimate with 90% confidence that the true percent of all students that are registered voters is between 56.4% and 63.6%.
- Alternate Wording: We estimate with 90% confidence that between 56.4% and 63.6% of ALL students are registered voters.

Explanation of 90% Confidence Level
90% of all confidence intervals constructed in this way contain the true value for the population percent of students that are registered voters.

Solution B
Using a function of the TI-83, 83+ or 84 calculators:

Press `STAT` and arrow over to `TESTS`.
Arrow down to `A:1-PropZint`. Press `ENTER`.
Arrow down to x and enter 300.
Arrow down to n and enter 500.
Arrow down to `C-Level` and enter .90.
Arrow down to `Calculate` and press `ENTER`.
The confidence interval is (0.564, 0.636).

Calculating the Sample Size n

If researchers desire a specific margin of error, then they can use the error bound formula to calculate the required sample size.

The error bound formula for a population proportion is

- $\text{EBP} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\text{p'q'}}{n}}$
- Solving for n gives you an equation for the sample size.
- $n = \frac{z_{\frac{\alpha}{2}}{}^2 \cdot \text{p'q'}}{\text{EBP}^2}$

Example:
Suppose a mobile phone company wants to determine the current percentage of customers aged 50+ that use text messaging on their cell phone. How many customers aged 50+ should the company survey in order to be 90% confident that the estimated (sample) proportion is within 3 percentage points of the true population proportion of customers aged 50+ that use text messaging on their cell phone.

Solution
From the problem, we know that **EBP=0.03** (3%=0.03) and

$z_{\frac{\alpha}{2}} = z_{.05} = 1.645$ because the confidence level is 90%
However, in order to find n , we need to know the estimated (sample) proportion p'. Remember that q'=1-p'. But, we do not know p' yet. Since we multiply p' and q' together, we make them both equal to 0.5 because p'q'=(.5)(.5)=.25 results in the largest possible product. (Try other products: (.6)(.4)=.24; (.3)(.7)=.21; (.2)(.8)=.16 and so on). The largest possible product gives us the largest n. This gives us a large enough sample so that we can be 90% confident that we are within 3 percentage points of the true population proportion. To calculate the sample size n, use the formula and make the substitutions.
$n = \frac{z^2\text{p'q'}}{\text{EBP}^2}$ gives $n = \frac{1.645^2(.5)(.5)}{.03^2}$ =751.7
Round the answer to the next higher value. The sample size should be 752 cell phone customers aged 50+ in order to be 90% confident that the

estimated (sample) proportion is within 3 percentage points of the true population proportion of all customers aged 50+ that use text messaging on their cell phone.
**With contributions from Roberta Bloom.

Glossary

Binomial Distribution
: A discrete random variable (RV) which arises from Bernoulli trials. There are a fixed number, n, of independent trials. "Independent" means that the result of any trial (for example, trial 1) does not affect the results of the following trials, and all trials are conducted under the same conditions. Under these circumstances the binomial RV X is defined as the number of successes in n trials. The notation is: X~ $B(n,p)$. The mean is $\mu = \text{np}$ and the standard deviation is $\sigma = \sqrt{\text{npq}}$. The probability of exactly x successes in n trials is $P\left(X = x\right) = \binom{n}{x}p^x q^{n-x}$.

Confidence Interval (CI)
: An interval estimate for an unknown population parameter. This depends on:

- The desired confidence level.
- Information that is known about the distribution (for example, known standard deviation).
- The sample and its size.

Confidence Level (CL)
: The percent expression for the probability that the confidence interval contains the true population parameter. For example, if the $\text{CL} = 90\%$, then in 90 out of 100 samples the interval estimate will enclose the true population parameter.

Error Bound for a Population Proportion(EBP)
: The margin of error. Depends on the confidence level, sample size, and the estimated (from the sample) proportion of successes.

Normal Distribution

A continuous random variable (RV) with pdf $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean of the distribution and σ is the standard deviation. Notation: $X \sim N(\mu, \sigma)$. If $\mu = 0$ and $\sigma = 1$, the RV is called **the standard normal distribution**.

Summary of Formulas

Formula General form of a confidence interval

$$(\text{lower value}, \text{upper value}) = (\text{point estimate} - \text{error bound}, \text{point estimate} + \text{error bound})$$

FormulaTo find the error bound when you know the confidence interval

$\text{error bound} = \text{upper value} - \text{point estimate}$ OR $\text{error bound} = \frac{\text{upper value} - \text{lower value}}{2}$

FormulaSingle Population Mean, Known Standard Deviation, Normal Distribution

Use the Normal Distribution for Means $\text{EBM} = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

The confidence interval has the format $(\overline{x} - \text{EBM}, \overline{x} + \text{EBM})$.

FormulaSingle Population Mean, Unknown Standard Deviation, Student's-t Distribution

Use the Student's-t Distribution with degrees of freedom $\text{df} = n - 1$. $\text{EBM} = t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$

FormulaSingle Population Proportion, Normal Distribution

Use the Normal Distribution for a single population proportion $p\prime = \frac{x}{n}$

$\text{EBP} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p\prime \cdot q\prime}{n}}$ $p\prime + q\prime = 1$

The confidence interval has the format $(p\prime - \text{EBP}, p\prime + \text{EBP})$.

FormulaPoint Estimates

$\overline{x}$ is a point estimate for μ

$p\prime$ is a point estimate for ρ

s is a point estimate for σ

Practice 1: Confidence Intervals for Averages, Known Population Standard Deviation

Student Learning Outcomes

- The student will calculate confidence intervals for means when the population standard deviation is known.

Given

The mean age for all Foothill College students for a recent Fall term was 33.2. The population standard deviation has been pretty consistent at 15. Suppose that twenty-five Winter students were randomly selected. The mean age for the sample was 30.4. We are interested in the true mean age for Winter Foothill College students. (http://research.fhda.edu/factbook/FH_Demo_Trends/FoothillDemographicTrends.htm)

Let $X =$ the age of a Winter Foothill College student

Calculating the Confidence Interval

Exercise:

Problem: $\overline{x} =$

Solution:

30.4

Exercise:

Problem: n=

Solution:

25

Exercise:

Problem: 15=(insert symbol here)

Solution:

σ

Exercise:

Problem: Define the Random Variable, $\overline{X}$, in words.

$\overline{X}$ =

Solution:

the mean age of 25 randomly selected Winter Foothill students

Exercise:

Problem: What is $\overline{x}$ estimating?

Solution:

μ

Exercise:

Problem: Is σ_x known?

Solution:

yes

Exercise:

Problem:

As a result of your answer to (4), state the exact distribution to use when calculating the Confidence Interval.

Solution:

Normal

Explaining the Confidence Interval

Construct a 95% Confidence Interval for the true mean age of Winter Foothill College students.

Exercise:

Problem: How much area is in both tails (combined)? $\alpha =$ ________

Solution:

0.05

Exercise:

Problem: How much area is in each tail? $\frac{\alpha}{2} =$ ________

Solution:

0.025

Exercise:

Problem: Identify the following specifications:

- **a** lower limit =
- **b** upper limit =
- **c** error bound =

Solution:

- **a**24.52
- **b**36.28
- **c**5.88

Exercise:

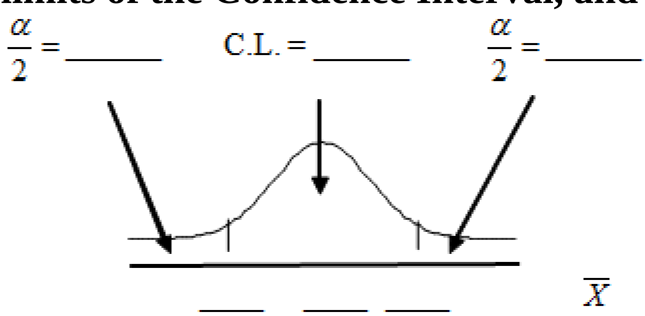
Problem: The 95% Confidence Interval is:____________________

Solution:

$(24.52, 36.28)$

Exercise:

Problem:
Fill in the blanks on the graph with the areas, upper and lower limits of the Confidence Interval, and the sample mean.



Exercise:

Problem: In one complete sentence, explain what the interval means.

Discussion Questions

Exercise:

Problem:

Using the same mean, standard deviation and level of confidence, suppose that n were 69 instead of 25. Would the error bound become larger or smaller? How do you know?

Exercise:

Problem:

Using the same mean, standard deviation and sample size, how would the error bound change if the confidence level were reduced to 90%? Why?

Practice 2: Confidence Intervals for Averages, Unknown Population Standard Deviation

Student Learning Outcomes

- The student will calculate confidence intervals for means when the population standard deviation is unknown.

Given

The following real data are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true mean number of colors on a national flag. Let $X =$ the number of colors on a national flag.

X	Freq.
1	1
2	7
3	18
4	7
5	6

Calculating the Confidence Interval

Exercise:

Problem: Calculate the following:

- **a** $\overline{x} =$
- **b**$s_x =$
- **c**$n =$

Solution:

- **a**3.26
- **b**1.02
- **c**39

Exercise:

Problem:

Define the Random Variable, $\overline{X}$, in words. $\overline{X} =$ ______________________________

Solution:

the mean number of colors of 39 flags

Exercise:

Problem: What is $\overline{x}$ estimating?

Solution:

μ

Exercise:

Problem: Is σ_x known?

Solution:

No

Exercise:

Problem:

As a result of your answer to (4), state the exact distribution to use when calculating the Confidence Interval.

Solution:

t_{38}

Confidence Interval for the True Mean Number

Construct a 95% Confidence Interval for the true mean number of colors on national flags.

Exercise:

Problem: How much area is in both tails (combined)? $\alpha =$

Solution:

0.05

Exercise:

Problem: How much area is in each tail? $\frac{\alpha}{2} =$

Solution:

0.025

Exercise:

Problem: Calculate the following:

- **a**lower limit =

- **b**upper limit =
- **c**error bound =

Solution:

- **a** 2.93
- **b**3.59
- **c**0.33

Exercise:

Problem: The 95% Confidence Interval is:

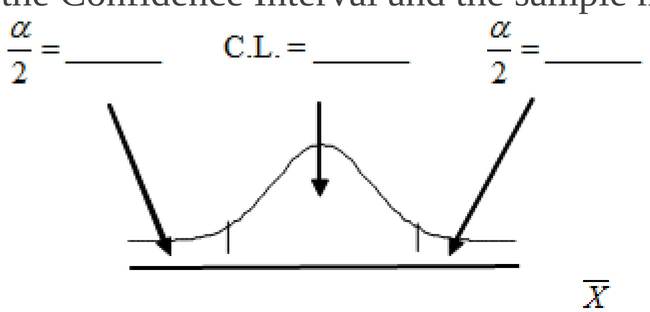
Solution:

2.93; 3.59

Exercise:

Problem:

Fill in the blanks on the graph with the areas, upper and lower limits of the Confidence Interval and the sample mean.



Exercise:

Problem: In one complete sentence, explain what the interval means.

Discussion Questions

Exercise:

Problem:

Using the same $\overline{x}$, s_x, and level of confidence, suppose that n were 69 instead of 39. Would the error bound become larger or smaller? How do you know?

Exercise:

Problem:

Using the same $\overline{x}$, s_x, and $n = 39$, how would the error bound change if the confidence level were reduced to 90%? Why?

Practice 3: Confidence Intervals for Proportions

Student Learning Outcomes

- The student will calculate confidence intervals for proportions.

Given

The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 - 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 - 12, in all beginning ice-skating classes at the Ice Chalet. Assume that the children in the selected class is a random sample of the population.

Estimated Distribution

Exercise:

Problem: What is being counted?

Exercise:

Problem: In words, define the Random Variable X. $X =$

Solution:

The number of girls, age 8-12, in the beginning ice skating class

Exercise:

Problem: Calculate the following:

- **a** $x =$
- **b** $n =$
- **c** $p\prime =$

Solution:

- **a**64
- **b**80
- **c**0.8

Exercise:

Problem: State the estimated distribution of X. X ~

Solution:

$B(80, 0.80)$

Exercise:

Problem: Define a new Random Variable $P\prime$. What is $p\prime$ estimating?

Solution:

p

Exercise:

Problem: In words, define the Random Variable $P\prime$. $P\prime=$

Solution:

The proportion of girls, age 8-12, in the beginning ice skating class.

Exercise:

Problem: State the estimated distribution of $P\prime$. $P\prime$ ~

Explaining the Confidence Interval

Construct a 92% Confidence Interval for the true proportion of girls in the age 8 - 12 beginning ice-skating classes at the Ice Chalet.

Exercise:

Problem: How much area is in both tails (combined)? $\alpha =$

Solution:

1 - 0.92 = 0.08

Exercise:

Problem: How much area is in each tail? $\frac{\alpha}{2} =$

Solution:

0.04

Exercise:

Problem: Calculate the following:

- **a**lower limit =
- **b**upper limit =
- **c**error bound =

Solution:

- **a**0.72
- **b**0.88
- **c**0.08

Exercise:

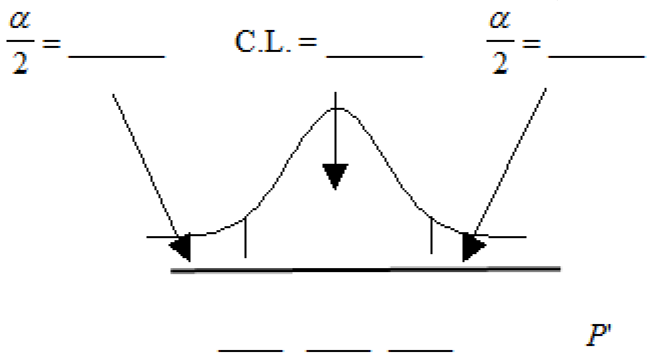
Problem: The 92% Confidence Interval is:

Solution:

(0.72; 0.88)

Exercise:

Problem:
Fill in the blanks on the graph with the areas, upper and lower limits of the Confidence Interval, and the sample proportion.



Exercise:

Problem: In one complete sentence, explain what the interval means.

Discussion Questions

Exercise:

Problem:

Using the same p' and level of confidence, suppose that n were increased to 100. Would the error bound become larger or smaller? How do you know?

Exercise:

Problem:

Using the same p' and $n = 80$, how would the error bound change if the confidence level were increased to 98%? Why?

Exercise:

Problem:

If you decreased the allowable error bound, why would the minimum sample size increase (keeping the same level of confidence)?

Exercises

> **Note:** If you are using a student's-t distribution for a homework problem below, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, though.)

Exercise:

Problem:

Among various ethnic groups, the standard deviation of heights is known to be approximately 3 inches. We wish to construct a 95% confidence interval for the mean height of male Swedes. 48 male Swedes are surveyed. The sample mean is 71 inches. The sample standard deviation is 2.8 inches.

- **a**
  - **i** $x =$________
  - **ii** $\sigma =$ ________
  - **iii** $s_x =$________
  - **iv** $n =$________
  - **v** $n - 1 =$________
- **b** Define the Random Variables X and X, in words.
- **c** Which distribution should you use for this problem? Explain your choice.
- **d** Construct a 95% confidence interval for the population mean height of male Swedes.
  - **i** State the confidence interval.
  - **ii** Sketch the graph.
  - **iii** Calculate the error bound.

- **e**What will happen to the level of confidence obtained if 1000 male Swedes are surveyed instead of 48? Why?

Solution:

- **a**
  - **i**71
  - **ii**3
  - **iii**2.8
  - **iv**48
  - **v**47
- **c**$N\left(71,\frac{3}{\sqrt{48}}\right)$
- **d**
  - **i**CI: (70.15,71.85)
  - **iii**EB = 0.85

Exercise:

Problem:

In six packages of "The Flintstones® Real Fruit Snacks" there were 5 Bam-Bam snack pieces. The total number of snack pieces in the six bags was 68. We wish to calculate a 96% confidence interval for the population proportion of Bam-Bam snack pieces.

- **a**Define the Random Variables X and P', in words.
- **b**Which distribution should you use for this problem? Explain your choice
- **c**Calculate p'.
- **d**Construct a 96% confidence interval for the population proportion of Bam-Bam snack pieces per bag.
  - **i** State the confidence interval.
  - **ii**Sketch the graph.

- **iii**Calculate the error bound.

- **e**Do you think that six packages of fruit snacks yield enough data to give accurate results? Why or why not?

Exercise:

Problem:

A random survey of enrollment at 35 community colleges across the United States yielded the following figures (source: Microsoft Bookshelf): 6414; 1550; 2109; 9350; 21828; 4300; 5944; 5722; 2825; 2044; 5481; 5200; 5853; 2750; 10012; 6357; 27000; 9414; 7681; 3200; 17500; 9200; 7380; 18314; 6557; 13713; 17768; 7493; 2771; 2861; 1263; 7285; 28165; 5080; 11622. Assume the underlying population is normal.

- **a**
  - **i**$x =$
  - **ii** $s_x =$ ________
  - **iii**$n =$ ________
  - **iv**$n - 1 =$________
- **b**Define the Random Variables X and X, in words.
- **c**Which distribution should you use for this problem? Explain your choice.
- **d**Construct a 95% confidence interval for the population mean enrollment at community colleges in the United States.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.
- **e**What will happen to the error bound and confidence interval if 500 community colleges were surveyed? Why?

Solution:

- **a**
  - **i**8629
  - **ii**6944
  - **iii**35
  - **iv**34
- **c** t_{34}
- **d**
  - **i**CI: (6244, 11,014)
  - **iii**EB = 2385
- **e**It will become smaller

Exercise:

Problem:

From a stack of IEEE Spectrum magazines, announcements for 84 upcoming engineering conferences were randomly picked. The mean length of the conferences was 3.94 days, with a standard deviation of 1.28 days. Assume the underlying population is normal.

- **a**Define the Random Variables X and X, in words.
- **b**Which distribution should you use for this problem? Explain your choice.
- **c**Construct a 95% confidence interval for the population mean length of engineering conferences.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.

Exercise:

Problem:

Suppose that a committee is studying whether or not there is waste of time in our judicial system. It is interested in the mean amount of time individuals waste at the courthouse waiting to be called for service. The committee randomly surveyed 81 people. The sample mean was 8 hours with a sample standard deviation of 4 hours.

- **a**
  - **i**$x =$_________
  - **ii** $s_x =$ _________
  - **iii**$n =$_________
  - **iv**$n - 1 =$_________
- **b**Define the Random Variables X and X, in words.
- **c**Which distribution should you use for this problem? Explain your choice.
- **d**Construct a 95% confidence interval for the population mean time wasted.
  - **a**State the confidence interval.
  - **b**Sketch the graph.
  - **c**Calculate the error bound.
- **e**Explain in a complete sentence what the confidence interval means.

Solution:

- **a**
  - **i**8
  - **ii**4
  - **iii**81
  - **iv**80

- **c** t_{80}
- **d**

    - **i**CI: (7.12, 8.88)
    - **iii**EB = 0.88

Exercise:

Problem:

Suppose that an accounting firm does a study to determine the time needed to complete one person's tax forms. It randomly surveys 100 people. The sample mean is 23.6 hours. There is a known standard deviation of 7.0 hours. The population distribution is assumed to be normal.

- **a**

    - **i**$x =$ _________
    - **ii**$\sigma =$_________
    - **iii** $s_x =$_________
    - **iv**$n =$ _________
    - **v**$n - 1 =$_________

- **b**Define the Random Variables X and X, in words.
- **c**Which distribution should you use for this problem? Explain your choice.
- **d**Construct a 90% confidence interval for the population mean time to complete the tax forms.

    - **i**State the confidence interval.
    - **ii**Sketch the graph.
    - **iii**Calculate the error bound.

- **e**If the firm wished to increase its level of confidence and keep the error bound the same by taking another survey, what changes should it make?

- **f**If the firm did another survey, kept the error bound the same, and only surveyed 49 people, what would happen to the level of confidence? Why?
- **g**Suppose that the firm decided that it needed to be at least 96% confident of the population mean length of time to within 1 hour. How would the number of people the firm surveys change? Why?

Exercise:

Problem:

A sample of 16 small bags of the same brand of candies was selected. Assume that the population distribution of bag weights is normal. The weight of each bag was then recorded. The mean weight was 2 ounces with a standard deviation of 0.12 ounces. The population standard deviation is known to be 0.1 ounce.

- **a**
  - **i**$x =$ _________
  - **ii**$\sigma =$ _________
  - **iii** $s_x =$_________
  - **iv**$n =$ _________
  - **v**$n - 1 =$ _________
- **b**Define the Random Variable X, in words.
- **c**Define the Random Variable X, in words.
- **d**Which distribution should you use for this problem? Explain your choice.
- **e**Construct a 90% confidence interval for the population mean weight of the candies.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.
- **f**Construct a 98% confidence interval for the population mean weight of the candies.

    - **i**State the confidence interval.
    - **ii**Sketch the graph.
    - **iii**Calculate the error bound.

- **g**In complete sentences, explain why the confidence interval in (f) is larger than the confidence interval in (e).
- **h**In complete sentences, give an interpretation of what the interval in (f) means.

Solution:

- **a**

    - **i**2
    - **ii**0.1
    - **iii** 0.12
    - **iv**16
    - **v**15

- **b**the weight of 1 small bag of candies
- **c**the mean weight of 16 small bags of candies
- **d**$N\left(2, \frac{0.1}{\sqrt{16}}\right)$
- **e**

    - **i** CI: (1.96, 2.04)
    - **iii** EB = 0.04

- **f**

    - **i** CI: (1.94, 2.06)
    - **iii** EB = 0.06

Exercise:

Problem:

A pharmaceutical company makes tranquilizers. It is assumed that the distribution for the length of time they last is approximately normal. Researchers in a hospital used the drug on a random sample of 9 patients. The effective period of the tranquilizer for each patient (in hours) was as follows: 2.7; 2.8; 3.0; 2.3; 2.3; 2.2; 2.8; 2.1; and 2.4 .

- **a**
    - **i**$x =$ __________
    - **ii** $s_x =$ __________
    - **iii**$n =$ __________
    - **iv**$n - 1 =$ __________
- **b**Define the Random Variable X, in words.
- **c**Define the Random Variable X, in words.
- **d**Which distribution should you use for this problem? Explain your choice.
- **e**Construct a 95% confidence interval for the population mean length of time.
    - **i**State the confidence interval.
    - **ii**Sketch the graph.
    - **iii**Calculate the error bound.
- **f**What does it mean to be “95% confident” in this problem?

Exercise:

Problem:

Suppose that 14 children were surveyed to determine how long they had to use training wheels. It was revealed that they used them an average of 6 months with a sample standard deviation of 3 months. Assume that the underlying population distribution is normal.

- **a**

- **i** $x =$ ________
- **ii** $s_x =$ ________
- **iii** $n =$ ________
- **iv** $n - 1 =$ ________

- **b** Define the Random Variable X, in words.
- **c** Define the Random Variable X, in words.
- **d** Which distribution should you use for this problem? Explain your choice.
- **e** Construct a 99% confidence interval for the population mean length of time using training wheels.
  - **i** State the confidence interval.
  - **ii** Sketch the graph.
  - **iii** Calculate the error bound.
- **f** Why would the error bound change if the confidence level was lowered to 90%?

Solution:

- **a**
  - **i** 6
  - **ii** 3
  - **iii** 14
  - **iv** 13
- **b** the time for a child to remove his training wheels
- **c** the mean time for 14 children to remove their training wheels.
- **d** t_{13}
- **e**
  - **i** CI: (3.58, 8.42)
  - **iii** EB = 2.42

Exercise:

Problem:

Insurance companies are interested in knowing the population percent of drivers who always buckle up before riding in a car.

- **a**When designing a study to determine this population proportion, what is the minimum number you would need to survey to be 95% confident that the population proportion is estimated to within 0.03?
- **b**If it was later determined that it was important to be more than 95% confident and a new survey was commissioned, how would that affect the minimum number you would need to survey? Why?

Exercise:

Problem:

Suppose that the insurance companies did do a survey. They randomly surveyed 400 drivers and found that 320 claimed to always buckle up. We are interested in the population proportion of drivers who claim to always buckle up.

- **a**
  - **i**$x =$ ________
  - **ii**$n =$ ________
  - **iii**p' $=$ ________
- **b**Define the Random Variables X and P', in words.
- **c**Which distribution should you use for this problem? Explain your choice.
- **d**Construct a 95% confidence interval for the population proportion that claim to always buckle up.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.

- **e**If this survey were done by telephone, list 3 difficulties the companies might have in obtaining random results.

Solution:

- **a**
  - **i**320
  - **ii** 400
  - **iii**0.80
- **c** $N\left(0.80, \sqrt{\frac{(0.80)(0.20)}{400}}\right)$
- **d**
  - **i** CI: (0.76, 0.84)
  - **iii** EB = 0.04

Exercise:

Problem:

Unoccupied seats on flights cause airlines to lose revenue. Suppose a large airline wants to estimate its mean number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected and the number of unoccupied seats is noted for each of the sampled flights. The sample mean is 11.6 seats and the sample standard deviation is 4.1 seats.

- **a**
  - **i**$x =$ ________
  - **ii** $s_x =$ ________
  - **iii**$n =$ ________
  - **iv**$n - 1 =$ ________
- **b**Define the Random Variables X and X, in words.

- **c**Which distribution should you use for this problem? Explain your choice.
- **d**Construct a 92% confidence interval for the population mean number of unoccupied seats per flight.

  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii** Calculate the error bound.

Exercise:

Problem:

According to a recent survey of 1200 people, 61% feel that the president is doing an acceptable job. We are interested in the population proportion of people who feel the president is doing an acceptable job.

- **a**Define the Random Variables X and P', in words.
- **b**Which distribution should you use for this problem? Explain your choice.
- **c**Construct a 90% confidence interval for the population proportion of people who feel the president is doing an acceptable job.

  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.

Solution:

- **b** $N\left(0.61, \sqrt{\frac{(0.61)(0.39)}{1200}}\right)$
- **c**

  - **i**CI: (0.59, 0.63)
  - **iii** EB = 0.02

Exercise:

Problem:

A survey of the mean amount of cents off that coupons give was done by randomly surveying one coupon per page from the coupon sections of a recent San Jose Mercury News. The following data were collected: 20¢; 75¢; 50¢; 65¢; 30¢; 55¢; 40¢; 40¢; 30¢; 55¢; $1.50; 40¢; 65¢; 40¢. Assume the underlying distribution is approximately normal.

- **a**
  - **i**$x =$ _________
  - **ii** $s_x =$ _________
  - **iii**$n =$ _________
  - **iv**$n - 1 =$ _________
- **b**Define the Random Variables X and X, in words.
- **c**Which distribution should you use for this problem? Explain your choice.
- **d**Construct a 95% confidence interval for the population mean worth of coupons.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.
- **e**If many random samples were taken of size 14, what percent of the confident intervals constructed should contain the population mean worth of coupons? Explain why.

Exercise:

Problem:

An article regarding interracial dating and marriage recently appeared in the Washington Post. Of the 1709 randomly selected adults, 315 identified themselves as Latinos, 323 identified themselves as blacks, 254 identified themselves as Asians, and 779 identified themselves as whites. In this survey, 86% of blacks said that their families would welcome a white person into their families. Among Asians, 77% would welcome a white person into their families, 71% would welcome a Latino, and 66% would welcome a black person.

- **a**We are interested in finding the 95% confidence interval for the percent of all black families that would welcome a white person into their families. Define the Random Variables X and P', in words.
- **b**Which distribution should you use for this problem? Explain your choice.
- **c**Construct a 95% confidence interval
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.

Solution:

- **b** $N\left(0.86, \sqrt{\frac{(0.86)(0.14)}{323}}\right)$
- **c**
  - **i**CI: (0.823, 0.898)
  - **iii** EB = 0.038

Exercise:

Problem: Refer to the problem above.

- **a**Construct three 95% confidence intervals.
  - **i**Percent of all Asians that would welcome a white person into their families.
  - **ii**Percent of all Asians that would welcome a Latino into their families.
  - **iii**Percent of all Asians that would welcome a black person into their families.
- **b**Even though the three point estimates are different, do any of the confidence intervals overlap? Which?
- **c**For any intervals that do overlap, in words, what does this imply about the significance of the differences in the true proportions?
- **d**For any intervals that do not overlap, in words, what does this imply about the significance of the differences in the true proportions?

Exercise:

Problem:

A camp director is interested in the mean number of letters each child sends during his/her camp session. The population standard deviation is known to be 2.5. A survey of 20 campers is taken. The mean from the sample is 7.9 with a sample standard deviation of 2.8.

- **a**
  - **i**$x =$ __________
  - **ii**$\sigma =$ __________
  - **iii** $s_x =$ __________
  - **iv**$n =$ __________
  - **v**$n - 1 =$ __________
- **b**Define the Random Variables X and X, in words.
- **c**Which distribution should you use for this problem? Explain your choice.

- **d**Construct a 90% confidence interval for the population mean number of letters campers send home.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.
- **e**What will happen to the error bound and confidence interval if 500 campers are surveyed? Why?

Solution:

- **a**
  - **i** 7.9
  - **ii** 2.5
  - **iii** 2.8
  - **iv** 20
  - **v** 19
- **c** $N(7.9, \frac{2.5}{\sqrt{20}})$
- **d**
  - **i** CI: (6.98, 8.82)
  - **iii** EB: 0.92

Exercise:

Problem:

Stanford University conducted a study of whether running is healthy for men and women over age 50. During the first eight years of the study, 1.5% of the 451 members of the 50-Plus Fitness Association died. We are interested in the proportion of people over 50 who ran and died in the same eight–year period.

- **a**Define the Random Variables X and P', in words.

- **b**Which distribution should you use for this problem? Explain your choice.
- **c**Construct a 97% confidence interval for the population proportion of people over 50 who ran and died in the same eight–year period.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.
- **d** Explain what a "97% confidence interval" means for this study.

Exercise:

Problem:

In a recent sample of 84 used cars sales costs, the sample mean was $6425 with a standard deviation of $3156. Assume the underlying distribution is approximately normal.

- **a**Which distribution should you use for this problem? Explain your choice.
- **b**Define the Random Variable X, in words.
- **c**Construct a 95% confidence interval for the population mean cost of a used car.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.
- **d**Explain what a "95% confidence interval" means for this study.

Solution:

- **a** t_{83}
- **b**mean cost of 84 used cars
- **c**

- **i**CI: (5740.10, 7109.90)
- **iii** EB = 684.90

Exercise:

Problem:

A telephone poll of 1000 adult Americans was reported in an issue of Time Magazine. One of the questions asked was “What is the main problem facing the country?” 20% answered “crime”. We are interested in the population proportion of adult Americans who feel that crime is the main problem.

- **a**Define the Random Variables X and P', in words.
- **b**Which distribution should you use for this problem? Explain your choice.
- **c**Construct a 95% confidence interval for the population proportion of adult Americans who feel that crime is the main problem.
  - **i**State the confidence interval.
  - **ii**Sketch the graph.
  - **iii**Calculate the error bound.
- **d**Suppose we want to lower the sampling error. What is one way to accomplish that?
- **e**The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is ± 3%. In 1-3 complete sentences, explain what the ± 3% represents.

Exercise:

Problem:

Refer to the above problem. Another question in the poll was “[How much are] you worried about the quality of education in our schools?” 63% responded “a lot”. We are interested in the population proportion of adult Americans who are worried a lot about the quality of education in our schools.

1. Define the Random Variables X and P', in words.
2. Which distribution should you use for this problem? Explain your choice.
3. Construct a 95% confidence interval for the population proportion of adult Americans worried a lot about the quality of education in our schools.

    - **i**State the confidence interval.
    - **ii**Sketch the graph.
    - **iii**Calculate the error bound.

4. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is ± 3%. In 1-3 complete sentences, explain what the ± 3% represents.

Solution:

- **b** $N\left(0.63, \sqrt{\frac{(0.63)(0.37)}{1000}}\right)$
- **c**

    - **i**CI: (0.60, 0.66)
    - **iii** EB = 0.03

Exercise:

Problem:

Six different national brands of chocolate chip cookies were randomly selected at the supermarket. The grams of fat per serving are as follows: 8; 8; 10; 7; 9; 9. Assume the underlying distribution is approximately normal.

- **a**Calculate a 90% confidence interval for the population mean grams of fat per serving of chocolate chip cookies sold in supermarkets.
    - **i**State the confidence interval.
    - **ii**Sketch the graph.
    - **iii**Calculate the error bound.
- **b**If you wanted a smaller error bound while keeping the same level of confidence, what should have been changed in the study before it was done?
- **c**Go to the store and record the grams of fat per serving of six brands of chocolate chip cookies.
- **d**Calculate the mean.
- **e**Is the mean within the interval you calculated in part (a)? Did you expect it to be? Why or why not?

Exercise:

Problem:

A confidence interval for a proportion is given to be (– 0.22, 0.34). Why doesn't the lower limit of the confidence interval make practical sense? How should it be changed? Why?

Try these multiple choice questions.

The next three problems refer to the following: According to a Field Poll, 79% of California adults (actual results are 400 out of 506 surveyed) feel that "education and our schools" is one of the top issues facing

California. We wish to construct a 90% confidence interval for the true proportion of California adults who feel that education and the schools is one of the top issues facing California. (Source: http://field.com/fieldpollonline/subscribers/)

Exercise:

Problem:A point estimate for the true population proportion is:

- **A**0.90
- **B**1.27
- **C**0.79
- **D**400

Solution:

C

Exercise:

Problem:A 90% confidence interval for the population proportion is:

- **A**(0.761, 0.820)
- **B**(0.125, 0.188)
- **C**(0.755, 0.826)
- **D**(0.130, 0.183)

Solution:

A

Exercise:

Problem: The error bound is approximately

- **A**1.581
- **B**0.791

- **C**0.059
- **D**0.030

Solution:

D

The next two problems refer to the following:

A quality control specialist for a restaurant chain takes a random sample of size 12 to check the amount of soda served in the 16 oz. serving size. The sample mean is 13.30 with a sample standard deviation of 1.55. Assume the underlying population is normally distributed.

Exercise:

Problem:

Find the 95% Confidence Interval for the true population mean for the amount of soda served.

- **A**(12.42, 14.18)
- **B**(12.32, 14.29)
- **C**(12.50, 14.10)
- **D**Impossible to determine

Solution:

B

Exercise:

Problem: What is the error bound?

- **A**0.87
- **B**1.98
- **C**0.99
- **D**1.74

Solution:

C

Exercise:

Problem:

What is meant by the term “90% confident” when constructing a confidence interval for a mean?

- **A**If we took repeated samples, approximately 90% of the samples would produce the same confidence interval.
- **B**If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the sample mean.
- **C**If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the true value of the population mean.
- **D**If we took repeated samples, the sample mean would equal the population mean in approximately 90% of the samples.

Solution:

C

The next two problems refer to the following:

Five hundred and eleven (511) homes in a certain southern California community are randomly surveyed to determine if they meet minimal earthquake preparedness recommendations. One hundred seventy-three (173) of the homes surveyed met the minimum recommendations for earthquake preparedness and 338 did not.

Exercise:

Problem:

Find the Confidence Interval at the 90% Confidence Level for the true population proportion of southern California community homes meeting at least the minimum recommendations for earthquake preparedness.

- **A**(0.2975, 0.3796)
- **B**(0.6270, 6959)
- **C**(0.3041, 0.3730)
- **D**(0.6204, 0.7025)

Solution:

C

Exercise:

Problem:

The point estimate for the population proportion of homes that do not meet the minimum recommendations for earthquake preparedness is:

- **A**0.6614
- **B**0.3386
- **C**173
- **D**338

Solution:

A

Review

The next three problems refer to the following situation: Suppose that a sample of 15 randomly chosen people were put on a special weight loss diet. The amount of weight lost, in pounds, follows an unknown distribution with mean equal to 12 pounds and standard deviation equal to 3 pounds.

Exercise:

Problem:

To find the probability that the average of the 15 people lose no more than 14 pounds, the random variable should be:

- **A** The number of people who lost weight on the special weight loss diet
- **B**The number of people who were on the diet
- **C**The average amount of weight lost by 15 people on the special weight loss diet
- **D**The total amount of weight lost by 15 people on the special weight loss diet

---

Solution:

C

Exercise:

Problem: Find the probability asked for in the previous problem.

Solution:

0.9951

Exercise:

Problem:

Find the 90th percentile for the average amount of weight lost by 15 people.

Solution:

12.99

The next five problems refer to the following study: Twenty percent of the students at a local community college live in within five miles of the campus. Thirty percent of the students at the same community college receive some kind of financial aid. Of those who live within five miles of the campus, 75% receive some kind of financial aid.

Exercise:

Problem:

Find the probability that a randomly chosen student at the local community college does not live within five miles of the campus.

- **A** 80%
- **B** 20%
- **C** 30%
- **D** Cannot be determined

Solution:

A

Exercise:

Problem:

Find the probability that a randomly chosen student at the local community college lives within five miles of the campus or receives some kind of financial aid.

- **A** 50%
- **B** 35%
- **C** 27.5%
- **D** 75%

Solution:

B

Exercise:

Problem:

Based upon the above information, are living in student housing within five miles of the campus and receiving some kind of financial aid mutually exclusive?

- **A** Yes
- **B** No
- **C** Cannot be determined

Solution:

B

Exercise:

Problem:

The interest rate charged on the financial aid is ________ data.

- **A** quantitative discrete
- **B** quantitative continuous
- **C**qualitative discrete
- **D** qualitative

Solution:

B

Exercise:

Problem:

What follows is information about the students who receive financial aid at the local community college.

- 1st quartile = $250
- 2nd quartile = $700
- 3rd quartile = $1200

(These amounts are for the school year.) If a sample of 200 students is taken, how many are expected to receive $250 or more?

- **A** 50
- **B** 250
- **C** 150
- **D** Cannot be determined

Solution:

- **C** 150

The next two problems refer to the following information: $P(A) = 0.2$, $P(B) = 0.3$, A and B are independent events.
Exercise:

Problem: $P(A \text{ AND } B) =$

- **A** 0.5
- **B** 0.6
- **C**0
- **D**0.06

Solution:

D

Exercise:

Problem: $P(A \text{ OR } B) =$

- **A** 0.56
- **B** 0.5
- **C** 0.44
- **D** 1

Solution:

C

Exercise:

Problem:

If H and D are mutually exclusive events, $P(H) = 0.25$, $P(D) = 0.15$, then $P(H|D)$

- **A** 1
- **B** 0
- **C** 0.40
- **D** 0.0375

Solution:

B

Hypothesis Testing: Single Mean and Single Proportion

Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Differentiate between Type I and Type II Errors
- Describe hypothesis testing in general and in practice
- Conduct and interpret hypothesis tests for a single population mean, population standard deviation known.
- Conduct and interpret hypothesis tests for a single population mean, population standard deviation unknown.
- Conduct and interpret hypothesis tests for a single population proportion.

Introduction

One job of a statistician is to make statistical inferences about populations based on samples taken from the population. **Confidence intervals** are one way to estimate a population parameter. Another way to make a statistical inference is to make a decision about a parameter. For instance, a car dealer advertises that its new small truck gets 35 miles per gallon, on the average. A tutoring service claims that its method of tutoring helps 90% of its students get an A or a B. A company says that women managers in their company earn an average of $60,000 per year.

A statistician will make a decision about these claims. This process is called **"hypothesis testing."** A hypothesis test involves collecting data from a sample and evaluating the data. Then, the statistician makes a decision as to whether or not there is sufficient evidence based upon analyses of the data, to reject the null hypothesis.

In this chapter, you will conduct hypothesis tests on single means and single proportions. You will also learn about the errors associated with these tests.

Hypothesis testing consists of two contradictory hypotheses or statements, a decision based on the data, and a conclusion. To perform a hypothesis test, a

statistician will:

1. Set up two contradictory hypotheses.
2. Collect sample data (in homework problems, the data or summary statistics will be given to you).
3. Determine the correct distribution to perform the hypothesis test.
4. Analyze sample data by performing the calculations that ultimately will allow you to reject or fail to reject the null hypothesis.
5. Make a decision and write a meaningful conclusion.

Note: To do the hypothesis test homework problems for this chapter and later chapters, make copies of the appropriate special solution sheets. See the Table of Contents topic "Solution Sheets".

Glossary

Confidence Interval (CI)
: An interval estimate for an unknown population parameter. This depends on:

    - The desired confidence level.
    - Information that is known about the distribution (for example, known standard deviation).
    - The sample and its size.

Hypothesis Testing
: Based on sample evidence, a procedure to determine whether the hypothesis stated is a reasonable statement and cannot be rejected, or is unreasonable and should be rejected.

Null and Alternate Hypotheses

The actual test begins by considering two **hypotheses**. They are called the **null hypothesis** and the **alternate hypothesis**. These hypotheses contain opposing viewpoints.

H_o: **The null hypothesis:** It is a statement about the population that will be assumed to be true unless it can be shown to be incorrect beyond a reasonable doubt.

H_a: **The alternate hypothesis:** It is a claim about the population that is contradictory to H_o and what we conclude when we reject H_o.

Example:
H_o: No more than 30% of the registered voters in Santa Clara County voted in the primary election.
H_a: More than 30% of the registered voters in Santa Clara County voted in the primary election.

Example:
We want to test whether the mean grade point average in American colleges is different from 2.0 (out of 4.0).
H_o: $\mu = 2.0$ H_a: $\mu \neq 2.0$

Example:
We want to test if college students take less than five years to graduate from college, on the average.
H_o: $\mu \geq 5$ H_a: $\mu < 5$

Example:

In an issue of **U. S. News and World Report**, an article on school standards stated that about half of all students in France, Germany, and Israel take advanced placement exams and a third pass. The same article stated that 6.6% of U. S. students take advanced placement exams and 4.4 % pass. Test if the percentage of U. S. students who take advanced placement exams is more than 6.6%.
H_o: p= 0.066 H_a: p > 0.066

Since the null and alternate hypotheses are contradictory, you must examine evidence to decide if you have enough evidence to reject the null hypothesis or not. The evidence is in the form of sample data.

After you have determined which hypothesis the sample supports, you make a **decision.** There are two options for a decision. They are "reject H_o" if the sample information favors the alternate hypothesis or "do not reject H_o" or "fail to reject H_o" if the sample information is insufficient to reject the null hypothesis.

Mathematical Symbols Used in H_o and H_a:

H_o	H_a
equal (=)	not equal (≠) **or** greater than (>) **or** less than (<)
greater than or equal to (≥)	less than (<)
less than or equal to (≤)	more than (>)

Note: H_o always has a symbol with an equal in it. H_a never has a symbol with an equal in it. The choice of symbol depends on the wording of the hypothesis test. However, be aware that many researchers (including one of the co-authors in research work) use $=$ in the Null Hypothesis, even with $>$ or $<$ as the symbol in the Alternate Hypothesis. This practice is acceptable because we only make the decision to reject or not reject the Null Hypothesis.

Optional Collaborative Classroom Activity

Bring to class a newspaper, some news magazines, and some Internet articles . In groups, find articles from which your group can write a null and alternate hypotheses. Discuss your hypotheses with the rest of the class.

Glossary

Hypothesis
: A statement about the value of a population parameter. In case of two hypotheses, the statement assumed to be true is called the null hypothesis (notation H_0) and the contradictory statement is called the alternate hypothesis (notation H_a).

Outcomes and the Type I and Type II Errors

When you perform a hypothesis test, there are four possible outcomes depending on the actual truth (or falseness) of the null hypothesis H_o and the decision to reject or not. The outcomes are summarized in the following table:

ACTION	**H_o IS ACTUALLY**	**...**
	True	False
Do not reject H_o	Correct Outcome	Type II error
Reject H_o	Type I Error	Correct Outcome

The four possible outcomes in the table are:

- The decision is to **not reject** H_o when, in fact, H_o **is true (correct decision).**
- The decision is to **reject** H_o when, in fact, H_o **is true** (incorrect decision known as a **Type I error**).
- The decision is to **not reject** H_o when, in fact, H_o **is false** (incorrect decision known as a **Type II error**).
- The decision is to **reject** H_o when, in fact, H_o **is false** (**correct decision** whose probability is called the **Power of the Test**).

Each of the errors occurs with a particular probability. The Greek letters α and β represent the probabilities.

α = probability of a Type I error = **P(Type I error)** = probability of rejecting the null hypothesis when the null hypothesis is true.

β = probability of a Type II error = **P(Type II error)** = probability of not rejecting the null hypothesis when the null hypothesis is false.

α and β should be as small as possible because they are probabilities of errors. They are rarely 0.

The Power of the Test is $1 - \beta$. Ideally, we want a high power that is as close to 1 as possible. Increasing the sample size can increase the Power of the Test.

The following are examples of Type I and Type II errors.

Example:
Suppose the null hypothesis, H_o, is: Frank's rock climbing equipment is safe.
Type I error: Frank thinks that his rock climbing equipment may not be safe when, in fact, it really is safe. **Type II error**: Frank thinks that his rock climbing equipment may be safe when, in fact, it is not safe.
α **= probability** that Frank thinks his rock climbing equipment may not be safe when, in fact, it really is safe. β **= probability** that Frank thinks his rock climbing equipment may be safe when, in fact, it is not safe.
Notice that, in this case, the error with the greater consequence is the Type II error. (If Frank thinks his rock climbing equipment is safe, he will go ahead and use it.)

Example:
Suppose the null hypothesis, H_o, is: The victim of an automobile accident is alive when he arrives at the emergency room of a hospital.
Type I error: The emergency crew thinks that the victim is dead when, in fact, the victim is alive. **Type II error**: The emergency crew does not know if the victim is alive when, in fact, the victim is dead.
α **= probability** that the emergency crew thinks the victim is dead when, in fact, he is really alive = $\mathrm{P(Type\ I\ error)}$. β **= probability** that the

emergency crew does not know if the victim is alive when, in fact, the victim is dead = $P(\text{Type II error})$.
The error with the greater consequence is the Type I error. (If the emergency crew thinks the victim is dead, they will not treat him.)

Glossary

Type 1 Error
: The decision is to reject the Null hypothesis when, in fact, the Null hypothesis is true.

Type 2 Error
: The decision is to not reject the Null hypothesis when, in fact, the Null hypothesis is false.

Distribution Needed for Hypothesis Testing

Earlier in the course, we discussed sampling distributions. **Particular distributions are associated with hypothesis testing.** Perform tests of a population mean using a **normal distribution** or a **student's-t distribution.** (Remember, use a student's-t distribution when the population **standard deviation** is unknown and the distribution of the sample mean is approximately normal.) In this chapter we perform tests of a population proportion using a normal distribution (usually n is large or the sample size is large).

If you are testing a **single population mean**, the distribution for the test is for **means**:

$$X \sim N\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right) \qquad \text{or} \qquad t_{\text{df}}$$

The population parameter is μ. The estimated value (point estimate) for μ is x, the sample mean.

If you are testing a **single population proportion**, the distribution for the test is for proportions or percentages:

$$\text{P'} \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$$

The population parameter is p. The estimated value (point estimate) for p is p'. p' $= \frac{x}{n}$ where x is the number of successes and n is the sample size.

Glossary

Normal Distribution

A continuous random variable (RV) with pdf $\mathrm{f(x)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean of the distribution and σ is the standard deviation. Notation: $X \sim N(\mu, \sigma)$. If $\mu = 0$ and $\sigma = 1$, the RV is called **the standard normal distribution**.

Standard Deviation

A number that is equal to the square root of the variance and measures how far data values are from their mean. Notation: s for sample standard deviation and σ for population standard deviation.

Student's-**t** Distribution

Investigated and reported by William S. Gossett in 1908 and published under the pseudonym Student. The major characteristics of the random variable (RV) are:

- It is continuous and assumes any real values.
- The pdf is symmetrical about its mean of zero. However, it is more spread out and flatter at the apex than the normal distribution.
- It approaches the standard normal distribution as n gets larger.
- There is a "family" of t distributions: every representative of the family is completely defined by the number of degrees of freedom which is one less than the number of data.

Assumption

When you perform a **hypothesis test of a single population mean** μ using a **Student's-t distribution** (often called a t-test), there are fundamental assumptions that need to be met in order for the test to work properly. Your data should be a **simple random sample** that comes from a population that is approximately **normally distributed**. You use the sample **standard deviation** to approximate the population standard deviation. (Note that if the sample size is sufficiently large, a t-test will work even if the population is not approximately normally distributed).

When you perform a **hypothesis test of a single population mean** μ using a normal distribution (often called a z-test), you take a simple random sample from the population. The population you are testing is normally distributed or your sample size is sufficiently large. You know the value of the population standard deviation.

When you perform a **hypothesis test of a single population proportion** p, you take a simple random sample from the population. You must meet the conditions for a **binomial distribution** which are there are a certain number n of independent trials, the outcomes of any trial are success or failure, and each trial has the same probability of a success p. The shape of the binomial distribution needs to be similar to the shape of the normal distribution. To ensure this, the quantities np and nq must both be greater than five ($np > 5$ and $nq > 5$). Then the binomial distribution of sample (estimated) proportion can be approximated by the normal distribution with $\mu = p$ and $\sigma = \sqrt{\frac{p \cdot q}{n}}$. Remember that $q = 1 - p$.

Glossary

Binomial Distribution

A discrete random variable (RV) which arises from Bernoulli trials. There are a fixed number, n, of independent trials. “Independent” means that the result of any trial (for example, trial 1) does not affect the results of the following trials, and all trials are conducted under the same conditions. Under these circumstances the binomial RV X is

defined as the number of successes in n trials. The notation is: $X \sim B(n, p)$. The mean is $\mu = \mathrm{np}$ and the standard deviation is $\sigma = \sqrt{\mathrm{npq}}$. The probability of exactly x successes in n trials is $P(X = x) = \binom{n}{x} p^x q^{n-x}$.

Normal Distribution

A continuous random variable (RV) with pdf $\mathrm{f(x)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, where μ is the mean of the distribution and σ is the standard deviation. Notation: $X \sim N(\mu, \sigma)$. If $\mu = 0$ and $\sigma = 1$, the RV is called **the standard normal distribution**.

Standard Deviation

A number that is equal to the square root of the variance and measures how far data values are from their mean. Notation: s for sample standard deviation and σ for population standard deviation.

Student-**t** Distribution

Investigated and reported by William S. Gossett in 1908 and published under the pseudonym Student. The major characteristics of the random variable (RV) are:

- It is continuous and assumes any real values.
- The pdf is symmetrical about its mean of zero. However, it is more spread out and flatter at the apex than the normal distribution.
- It approaches the standard normal distribution as n gets larger.
- There is a "family" of t distributions: every representative of the family is completely defined by the number of degrees of freedom which is one less than the number of data.

Rare Events

Suppose you make an assumption about a property of the population (this assumption is the **null hypothesis**). Then you gather sample data randomly. If the sample has properties that would be very **unlikely** to occur if the assumption is true, then you would conclude that your assumption about the population is probably incorrect. (Remember that your assumption is just an **assumption** - it is not a fact and it may or may not be true. But your sample data are real and the data are showing you a fact that seems to contradict your assumption.)

For example, Didi and Ali are at a birthday party of a very wealthy friend. They hurry to be first in line to grab a prize from a tall basket that they cannot see inside because they will be blindfolded. There are 200 plastic bubbles in the basket and Didi and Ali have been told that there is only one with a $100 bill. Didi is the first person to reach into the basket and pull out a bubble. Her bubble contains a $100 bill. The probability of this happening is $\frac{1}{200} = 0.005$. Because this is so unlikely, Ali is hoping that what the two of them were told is wrong and there are more $100 bills in the basket. A "rare event" has occurred (Didi getting the $100 bill) so Ali doubts the assumption about only one $100 bill being in the basket.

Glossary

Hypothesis
: A statement about the value of a population parameter. In case of two hypotheses, the statement assumed to be true is called the null hypothesis (notation H_0) and the contradictory statement is called the alternate hypothesis (notation H_a).

Using the Sample to Support One of the Hypotheses

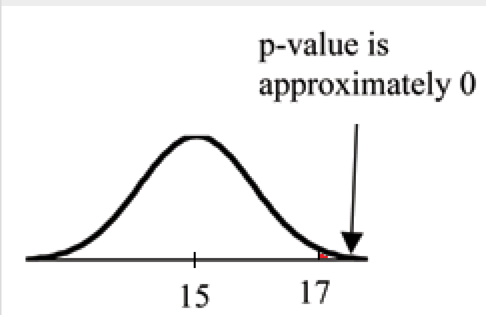
Use the sample data to calculate the actual probability of getting the test result, called the **p-value**. The p-value is the **probability that, if the null hypothesis is true, the results from another randomly selected sample will be as extreme or more extreme as the results obtained from the given sample.**

A large p-value calculated from the data indicates that we should fail to reject the **null hypothesis**. The smaller the p-value, the more unlikely the outcome, and the stronger the evidence is against the null hypothesis. We would reject the null hypothesis if the evidence is strongly against it.

Draw a graph that shows the p-value. The hypothesis test is easier to perform if you use a graph because you see the problem more clearly.

Example:
(to illustrate the p-value)
Suppose a baker claims that his bread height is more than 15 cm, on the average. Several of his customers do not believe him. To persuade his customers that he is right, the baker decides to do a hypothesis test. He bakes 10 loaves of bread. The mean height of the sample loaves is 17 cm. The baker knows from baking hundreds of loaves of bread that the **standard deviation** for the height is 0.5 cm. and the distribution of heights is normal.
The null hypothesis could be H_o: $\mu \leq 15$ The alternate hypothesis is H_a: $\mu > 15$
The words **"is more than"** translates as a "$>$" so "$\mu > 15$" goes into the alternate hypothesis. The null hypothesis must contradict the alternate hypothesis.
Since σ **is known** ($\sigma = 0.5$ cm.), the distribution for the population is known to be normal with mean μ= 15 and standard deviation $\frac{\sigma}{\sqrt{n}}$ $= \frac{0.5}{\sqrt{10}} = 0.16$.
Suppose the null hypothesis is true (the mean height of the loaves is no more than 15 cm). Then is the mean height (17 cm) calculated from the

sample unexpectedly large? The hypothesis test works by asking the question how **unlikely** the sample mean would be if the null hypothesis were true. The graph shows how far out the sample mean is on the normal curve. The p-value is the probability that, if we were to take other samples, any other sample mean would fall at least as far out as 17 cm.
The p-value, then, is the probability that a sample mean is the same or greater than 17 cm. when the population mean is, in fact, 15 cm. We can calculate this probability using the normal distribution for means from Chapter 7.



p-value $= P(x > 17)$ which is approximately 0.
A p-value of approximately 0 tells us that it is highly unlikely that a loaf of bread rises no more than 15 cm, on the average. That is, almost 0% of all loaves of bread would be at least as high as 17 cm. **purely by CHANCE** had the population mean height really been 15 cm. Because the outcome of 17 cm. is so **unlikely (meaning it is happening NOT by chance alone)**, we conclude that the evidence is strongly against the null hypothesis (the mean height is at most 15 cm.). There is sufficient evidence that the true mean height for the population of the baker's loaves of bread is greater than 15 cm.

Glossary

Hypothesis
: A statement about the value of a population parameter. In case of two hypotheses, the statement assumed to be true is called the null hypothesis (notation H_0) and the contradictory statement is called the alternate hypothesis (notation H_a).

p-value

The probability that an event will happen purely by chance assuming the null hypothesis is true. The smaller the p-value, the stronger the evidence is against the null hypothesis.

Standard Deviation

A number that is equal to the square root of the variance and measures how far data values are from their mean. Notation: s for sample standard deviation and σ for population standard deviation.

Decision and Conclusion

A systematic way to make a decision of whether to reject or not reject the **null hypothesis** is to compare the **p-value** and a **preset or preconceived** α **(also called a "significance level")**. A preset α is the probability of a **Type I error** (rejecting the null hypothesis when the null hypothesis is true). It may or may not be given to you at the beginning of the problem.

When you make a **decision** to reject or not reject H_o, do as follows:

- If $\alpha >$ p-value, reject H_o. The results of the sample data are significant. There is sufficient evidence to conclude that H_o is an incorrect belief and that the **alternative hypothesis**, H_a, may be correct.
- If $\alpha \leq$ p-value, do not reject H_o. The results of the sample data are not significant. There is not sufficient evidence to conclude that the alternative hypothesis, H_a, may be correct.
- When you "do not reject H_o", it does not mean that you should believe that H_o is true. It simply means that the sample data have **failed** to provide sufficient evidence to cast serious doubt about the truthfulness of H_o.

Conclusion: After you make your decision, write a thoughtful **conclusion** about the hypotheses in terms of the given problem.

Glossary

Hypothesis
: A statement about the value of a population parameter. In case of two hypotheses, the statement assumed to be true is called the null hypothesis (notation H_0) and the contradictory statement is called the alternate hypothesis (notation H_a).

Level of Significance of the Test
: Probability of a Type I error (reject the null hypothesis when it is true). Notation: α. In hypothesis testing, the Level of Significance is called the preconceived α or the preset α.

p-value

The probability that an event will happen purely by chance assuming the null hypothesis is true. The smaller the p-value, the stronger the evidence is against the null hypothesis.

Type 1 Error

The decision is to reject the Null hypothesis when, in fact, the Null hypothesis is true.

Additional Information

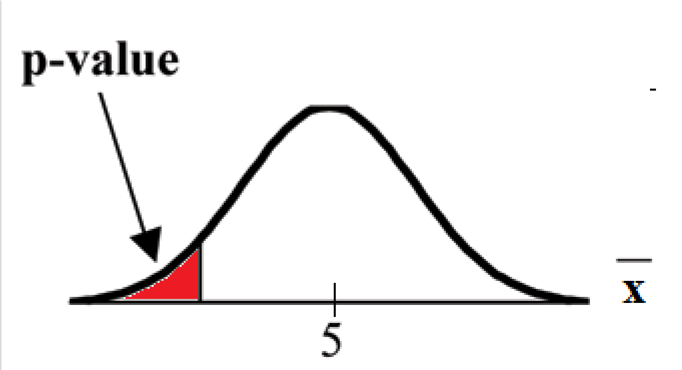
- In a **hypothesis test** problem, you may see words such as "the level of significance is 1%." The "1%" is the preconceived or preset α.
- The statistician setting up the hypothesis test selects the value of α to use **before** collecting the sample data.
- **If no level of significance is given, the accepted standard is to use $\alpha = 0.05$.**
- When you calculate the **p-value** and draw the picture, the p-value is the area in the left tail, the right tail, or split evenly between the two tails. For this reason, we call the hypothesis test left, right, or two tailed.
- The **alternate hypothesis**, H_a, tells you if the test is left, right, or two-tailed. It is the **key** to conducting the appropriate test.
- H_a **never** has a symbol that contains an equal sign.
- **Thinking about the meaning of the p-value**: A data analyst (and anyone else) should have more confidence that he made the correct decision to reject the null hypothesis with a smaller p-value (for example, 0.001 as opposed to 0.04) even if using the 0.05 level for alpha. Similarly, for a large p-value like 0.4, as opposed to a p-value of 0.056 (alpha = 0.05 is less than either number), a data analyst should have more confidence that she made the correct decision in failing to reject the null hypothesis. This makes the data analyst use judgment rather than mindlessly applying rules.

The following examples illustrate a left, right, and two-tailed test.

Example:

H_o: $\mu = 5$ H_a: $\mu < 5$

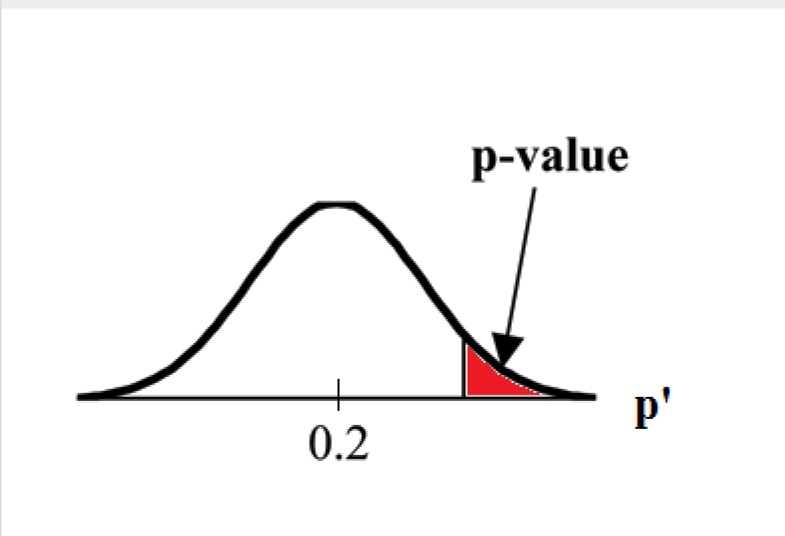
Test of a single population mean. H_a tells you the test is left-tailed. The picture of the p-value is as follows:



Example:

H_o: $p \leq 0.2$ H_a: $p > 0.2$

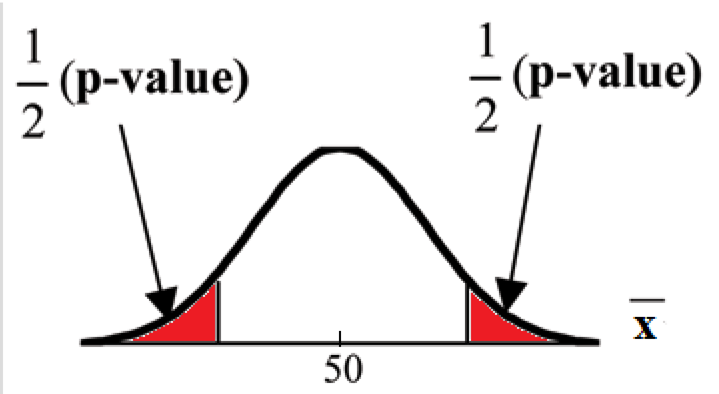
This is a test of a single population proportion. H_a tells you the test is **right-tailed**. The picture of the p-value is as follows:



Example:

H_o: $\mu = 50$ H_a: $\mu \neq 50$

This is a test of a single population mean. H_a tells you the test is **two-tailed**. The picture of the p-value is as follows.



Glossary

Hypothesis Testing

Based on sample evidence, a procedure to determine whether the hypothesis stated is a reasonable statement and cannot be rejected, or is unreasonable and should be rejected.

p-value

The probability that an event will happen purely by chance assuming the null hypothesis is true. The smaller the p-value, the stronger the evidence is against the null hypothesis.

Summary of the Hypothesis Test

The **hypothesis test** itself has an established process. This can be summarized as follows:

1. Determine H_o and H_a. Remember, they are contradictory.
2. Determine the random variable.
3. Determine the distribution for the test.
4. Draw a graph, calculate the test statistic, and use the test statistic to calculate the **p-value**. (A z-score and a t-score are examples of test statistics.)
5. Compare the preconceived α with the p-value, make a decision (reject or do not reject H_o), and write a clear conclusion using English sentences.

Notice that in performing the hypothesis test, you use α and not β. β is needed to help determine the sample size of the data that is used in calculating the p-value. Remember that the quantity $1 - \beta$ is called the **Power of the Test**. A high power is desirable. If the power is too low, statisticians typically increase the sample size while keeping α the same. If the power is low, the null hypothesis might not be rejected when it should be.

Glossary

Hypothesis Testing
: Based on sample evidence, a procedure to determine whether the hypothesis stated is a reasonable statement and cannot be rejected, or is unreasonable and should be rejected.

p-value
: The probability that an event will happen purely by chance assuming the null hypothesis is true. The smaller the p-value, the stronger the evidence is against the null hypothesis.

Examples

This module provides examples of Hypothesis Testing of a Single Mean and a Single Proportion as a part of the Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Example:

Exercise:

Problem:

Jeffrey, as an eight-year old, **established a mean time of 16.43 seconds** for swimming the 25-yard freestyle, with a **standard deviation of 0.8 seconds**. His dad, Frank, thought that Jeffrey could swim the 25-yard freestyle faster by using goggles. Frank bought Jeffrey a new pair of expensive goggles and timed Jeffrey for **15 25-yard freestyle swims**. For the 15 swims, **Jeffrey's mean time was 16 seconds. Frank thought that the goggles helped Jeffrey to swim faster than the 16.43 seconds.** Conduct a hypothesis test using a preset $\alpha = 0.05$. Assume that the swim times for the 25-yard freestyle are normal.

Solution:

Set up the Hypothesis Test:

Since the problem is about a mean, this is a **test of a single population mean**.

H_{o}: $\mu = 16.43$ H_a: $\mu < 16.43$

For Jeffrey to swim faster, his time will be less than 16.43 seconds. The "$<$" tells you this is left-tailed.

Determine the distribution needed:

Random variable: X = the mean time to swim the 25-yard freestyle.

Distribution for the test: X is normal (population **standard deviation** is known: $\sigma = 0.8$)

$X \sim N\left(\mu, \frac{\sigma_X}{\sqrt{n}}\right)$ Therefore, $X \sim N\left(16.43, \frac{0.8}{\sqrt{15}}\right)$

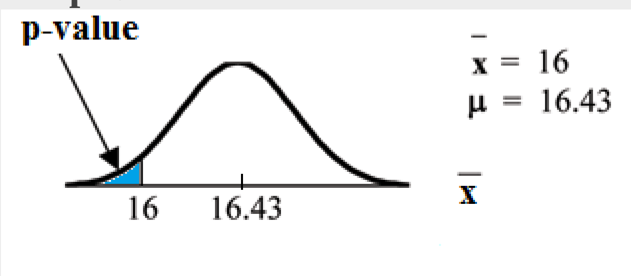
$\mu = 16.43$ comes from H_0 and not the data. $\sigma = 0.8$, and $n = 15$.

Calculate the p-value using the normal distribution for a mean:

p-value $= P\left(\bar{x} < 16\right) = 0.0187$ where the sample mean in the problem is given as 16.

p-value $= 0.0187$ (This is called the **actual level of significance**.) The p-value is the area to the left of the sample mean is given as 16.

Graph:



$\mu = 16.43$ comes from H_o. Our assumption is $\mu = 16.43$.

Interpretation of the p-value: If H_o is true, there is a 0.0187 probability (1.87%) that Jeffrey's mean time to swim the 25-yard freestyle is 16 seconds or less. Because a 1.87% chance is small, the mean time of 16 seconds or less is unlikely to have happened randomly. It is a rare event.

Compare α and the p-value:

$\alpha = 0.05$ p-value $= 0.0187$ $\alpha >$ p-value

Make a decision: Since $\alpha >$ p-value, reject H_o.

This means that you reject $\mu = 16.43$. In other words, you do not think Jeffrey swims the 25-yard freestyle in 16.43 seconds but faster with the new goggles.

Conclusion: At the 5% significance level, we conclude that Jeffrey swims faster using the new goggles. The sample data show there is sufficient evidence that Jeffrey's mean time to swim the 25-yard freestyle is less than 16.43 seconds.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Press `STAT` and arrow over to `TESTS`. Press `1:Z-Test`. Arrow over to `Stats` and press `ENTER`. Arrow down and enter 16.43 for μ_0 (null hypothesis), .8 for σ, 16 for the sample mean, and 15 for n. Arrow down to μ: (alternate hypothesis) and arrow over to $<\mu_0$. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The calculator not only calculates the p-value ($p = 0.0187$) but it also calculates the test statistic (z-score) for the sample mean. $\mu < 16.43$ is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $z = -2.08$ (test statistic) and $p = 0.0187$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

When the calculator does a Z-Test, the `Z-Test` function finds the p-value by doing a normal probability calculation using the **Central Limit Theorem**:

$P(x < 16\ ) =$ `2nd DISTR normcdf` $\left(-10\ \hat{}\ 99, 16, 16.43, 0.8/\sqrt{15}\right)$.

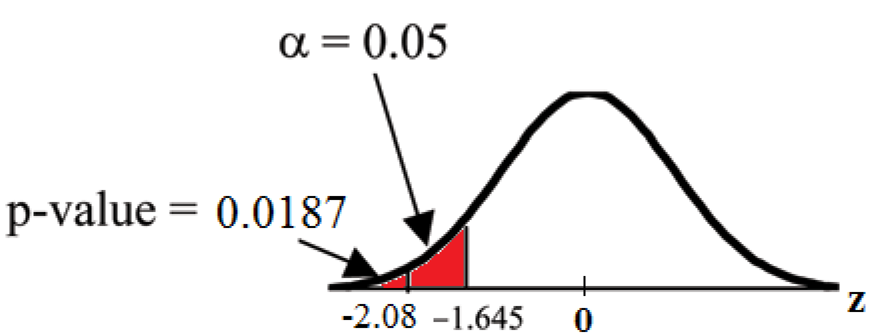
The Type I and Type II errors for this problem are as follows:

The Type I error is to conclude that Jeffrey swims the 25-yard freestyle, on average, in less than 16.43 seconds when, in fact, he

actually swims the 25-yard freestyle, on average, in 16.43 seconds. (Reject the null hypothesis when the null hypothesis is true.)

The Type II error is that there is not evidence to conclude that Jeffrey swims the 25-yard free-style, on average, in less than 16.43 seconds when, in fact, he actually does swim the 25-yard free-style, on average, in less than 16.43 seconds. (Do not reject the null hypothesis when the null hypothesis is false.)

Historical Note: The traditional way to compare the two probabilities, α and the p-value, is to compare the critical value (z-score from α) to the test statistic (z-score from data). The calculated test statistic for the p-value is -2.08. (From the Central Limit Theorem, the test statistic formula is $z = \frac{x - \mu_X}{\left(\frac{\sigma_X}{\sqrt{n}}\right)}$. For this problem, $x = 16$, $\mu_X = 16.43$ from the null hypothesis, $\sigma_X = 0.8$, and $n = 15$.) You can find the critical value for $\alpha = 0.05$ in the normal table (see **15.Tables** in the Table of Contents). The z-score for an area to the left equal to 0.05 is midway between -1.65 and -1.64 (0.05 is midway between 0.0505 and 0.0495). The z-score is -1.645. Since $-1.645 > -2.08$ (which demonstrates that $\alpha >$ p-value), reject H_o. Traditionally, the decision to reject or not reject was done in this way. Today, comparing the two probabilities α and the p-value is very common. For this problem, the p-value, 0.0187 is considerably smaller than α, 0.05. You can be confident about your decision to reject. The graph shows α, the p-value, and the test statistics and the critical value.



Example:
Exercise:

Problem:

A college football coach thought that his players could bench press a **mean weight of 275 pounds**. It is known that the **standard deviation is 55 pounds**. Three of his players thought that the mean weight was **more than** that amount. They asked **30** of their teammates for their estimated maximum lift on the bench press exercise. The data ranged from 205 pounds to 385 pounds. The actual different weights were (frequencies are in parentheses) 205(3) 215(3) 225(1) 241(2) 252(2) 265(2) 275(2) 313(2) 316(5) 338(2) 341(1) 345(2) 368(2) 385(1). (Source: data from Reuben Davis, Kraig Evans, and Scott Gunderson.)

Conduct a hypothesis test using a 2.5% level of significance to determine if the bench press mean is **more than 275 pounds**.

Solution:

Set up the Hypothesis Test:

Since the problem is about a mean weight, this is a **test of a single population mean**.

H_o: $\mu = 275$ H_a: $\mu > 275$ This is a right-tailed test.

Calculating the distribution needed:

Random variable: X = the mean weight, in pounds, lifted by the football players.

Distribution for the test: It is normal because σ is known.

X ~ $N\left(275, \frac{55}{\sqrt{30}}\right)$

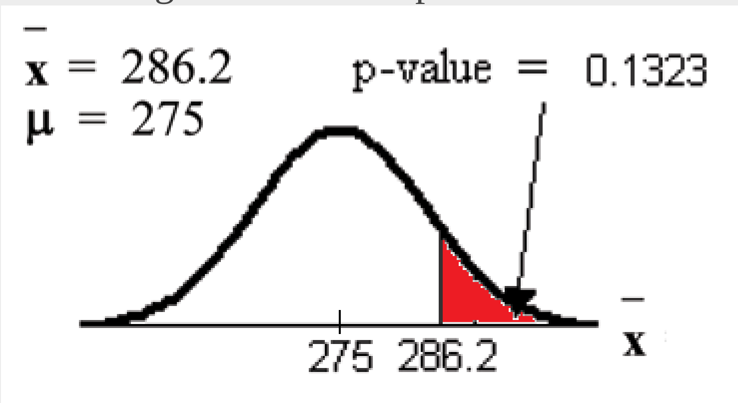
$x = 286.2$ pounds (from the data).

$\sigma = 55$ pounds **(Always use σ if you know it.)** We assume $\mu = 275$ pounds unless our data shows us otherwise.

Calculate the p-value using the normal distribution for a mean and using the sample mean as input (see the calculator instructions below for using the data as input):

p-value $= P(\ x > 286.2) = 0.1323.$

Interpretation of the p-value: If H_o is true, then there is a 0.1331 probability (13.23%) that the football players can lift a mean weight of 286.2 pounds or more. Because a 13.23% chance is large enough, a mean weight lift of 286.2 pounds or more is not a rare event.



Compare α and the p-value:

$\alpha = 0.025$ p-value $= 0.1323$

Make a decision: Since α<p-value, do not reject H_o.

Conclusion: At the 2.5% level of significance, from the sample data, there is not sufficient evidence to conclude that the true mean weight lifted is more than 275 pounds.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Put the data and frequencies into lists. Press `STAT` and arrow over to `TESTS`. Press `1:Z-Test`. Arrow over to `Data` and press `ENTER`.

Arrow down and enter 275 for μ_0, 55 for σ, the name of the list where you put the data, and the name of the list where you put the frequencies. Arrow down to μ : and arrow over to $> \mu_0$. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The calculator not only calculates the p-value ($p = 0.1331$, a little different from the above calculation - in it we used the sample mean rounded to one decimal place instead of the data) but it also calculates the test statistic (z-score) for the sample mean, the sample mean, and the sample standard deviation. $\mu > 275$ is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $z = 1.112$ (test statistic) and $p = 0.1331$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

Example:
Exercise:

Problem:

Statistics students believe that the mean score on the first statistics test is 65. A statistics instructor thinks the mean score is higher than 65. He samples ten statistics students and obtains the scores 65 65 70 67 66 63 63 68 72 71. He performs a hypothesis test using a 5% level of significance. The data are from a normal distribution.

Solution:

Set up the Hypothesis Test:

A 5% level of significance means that $\alpha = 0.05$. This is a test of a **single population mean**.

H_o: $\mu = 65$ $\qquad$ H_a: $\mu > 65$

Since the instructor thinks the average score is higher, use a "$>$". The "$>$" means the test is right-tailed.

Determine the distribution needed:

Random variable: X = average score on the first statistics test.

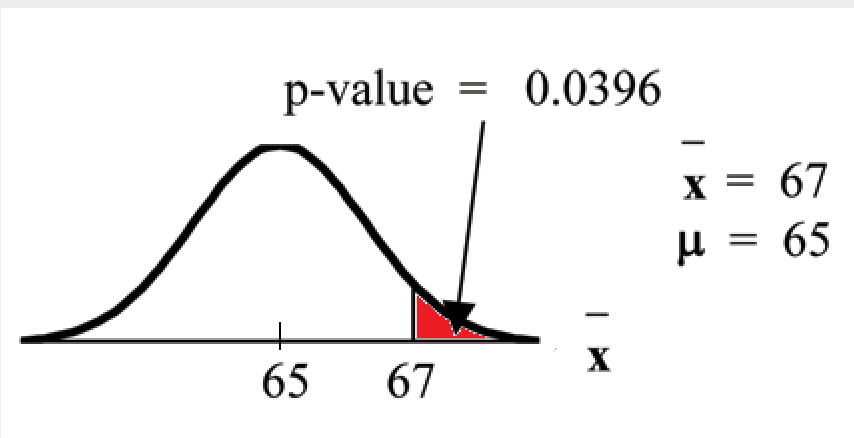
Distribution for the test: If you read the problem carefully, you will notice that there is **no population standard deviation given**. You are only given $n = 10$ sample data values. Notice also that the data come from a normal distribution. This means that the distribution for the test is a student's-t.

Use t_{df}. Therefore, the distribution for the test is t_9 where $n = 10$ and $\text{df} = 10 - 1 = 9$.

Calculate the p-value using the Student's-t distribution:

p-value $= P(\ x > 67\ ) = 0.0396$ where the sample mean and sample standard deviation are calculated as 67 and 3.1972 from the data.

Interpretation of the p-value: If the null hypothesis is true, then there is a 0.0396 probability (3.96%) that the sample mean is 67 or more.



Compare α and the p-value:

Since $\alpha = .05$ and p-value $= 0.0396$. Therefore, $\alpha >$ p-value.

Make a decision: Since $\alpha >$ p-value, reject H_o.

This means you reject $\mu = 65$. In other words, you believe the average test score is more than 65.

Conclusion: At a 5% level of significance, the sample data show sufficient evidence that the mean (average) test score is more than 65, just as the math instructor thinks.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Put the data into a list. Press `STAT` and arrow over to `TESTS`. Press `2:T-Test`. Arrow over to `Data` and press `ENTER`. Arrow down and enter 65 for μ_0, the name of the list where you put the data, and 1 for `Freq:`. Arrow down to μ : and arrow over to $> \mu_0$. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The calculator not only calculates the p-value ($p = 0.0396$) but it also calculates the test statistic (t-score) for the sample mean, the sample mean, and the sample standard deviation. $\mu > 65$ is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $t = 1.9781$ (test statistic) and $p = 0.0396$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

Example:
Exercise:

Problem:

Joon believes that 50% of first-time brides in the United States are younger than their grooms. She performs a hypothesis test to determine if the percentage is **the same or different from 50%**. Joon samples **100 first-time brides** and **53** reply that they are younger than their grooms. For the hypothesis test, she uses a 1% level of significance.

Solution:

Set up the Hypothesis Test:

The 1% level of significance means that $\alpha = 0.01$. This is a **test of a single population proportion**.

$H_o: p = 0.50$ $\quad\quad$ $H_a: p \neq 0.50$

The words **"is the same or different from"** tell you this is a two-tailed test.

Calculate the distribution needed:

Random variable: $P\prime$ = the percent of of first-time brides who are younger than their grooms.

Distribution for the test: The problem contains no mention of a mean. The information is given in terms of percentages. Use the distribution for $P\prime$, the estimated proportion.

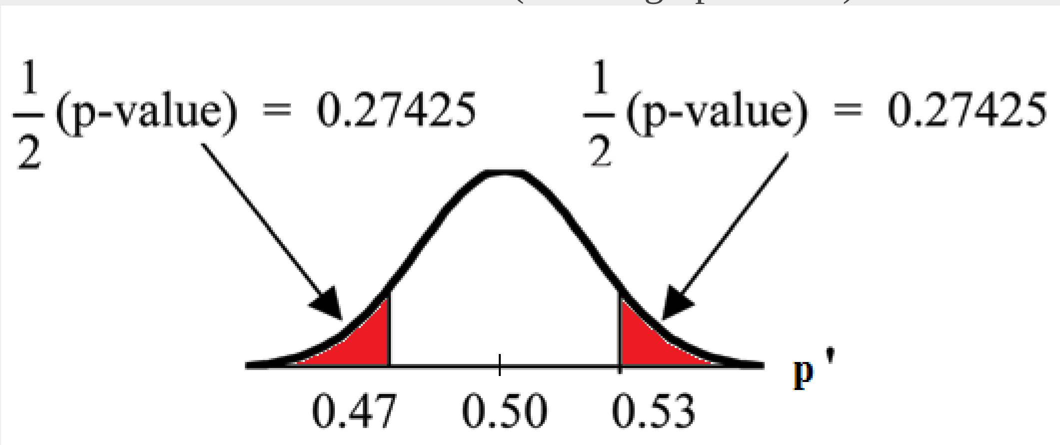
$P\prime \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$ $\quad\quad$ Therefore, $P\prime \sim N\left(0.5, \sqrt{\frac{0.5 \cdot 0.5}{100}}\right)$ where $p = 0.50$, $q = 1 - p = 0.50$, and $n = 100$.

Calculate the p-value using the normal distribution for proportions:

p-value $= P($p'< 0.47 or p' > 0.53 $) = 0.5485$

where $x = 53$, $p\prime = \frac{x}{n} = \frac{53}{100} = 0.53$.

Interpretation of the p-value: If the null hypothesis is true, there is 0.5485 probability (54.85%) that the sample (estimated) proportion $p\prime$ is 0.53 or more OR 0.47 or less (see the graph below).



$\mu = p = 0.50$ comes from H_o, the null hypothesis.

$p\prime = 0.53$. Since the curve is symmetrical and the test is two-tailed, the $p\prime$ for the left tail is equal to $0.50 - 0.03 = 0.47$ where $\mu = p = 0.50$. (0.03 is the difference between 0.53 and 0.50.)

Compare α and the p-value:

Since $\alpha = 0.01$ and $\text{p-value} = 0.5485$. Therefore, $\alpha <$ p-value.

Make a decision: Since α<p-value, you cannot reject H_o.

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of first-time brides that are younger than their grooms is different from 50%.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Press `STAT` and arrow over to `TESTS`. Press `5:1-PropZTest`. Enter .5 for p_0, 53 for x and 100 for n. Arrow down to `Prop` and arrow to `not equals` p_0. Press `ENTER`. Arrow down to

`Calculate` and press `ENTER`. The calculator calculates the p-value ($p = 0.5485$) and the test statistic (z-score). `Prop not equals` .5 is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $z = 0.6$ (test statistic) and $p = 0.5485$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

The Type I and Type II errors are as follows:

The Type I error is to conclude that the proportion of first-time brides that are younger than their grooms is different from 50% when, in fact, the proportion is actually 50%. (Reject the null hypothesis when the null hypothesis is true).

The Type II error is there is not enough evidence to conclude that the proportion of first time brides that are younger than their grooms differs from 50% when, in fact, the proportion does differ from 50%. (Do not reject the null hypothesis when the null hypothesis is false.)

Example:
Exercise:

Problem:

Suppose a consumer group suspects that the proportion of households that have three cell phones is 30%. A cell phone company has reason to believe that the proportion is 30%. Before they start a big advertising campaign, they conduct a hypothesis test. Their marketing people survey 150 households with the result that 43 of the households have three cell phones.

Solution:

Set up the Hypothesis Test:

H_o: $p = 0.30$ H_a: $p \neq 0.30$

Determine the distribution needed:

The **random variable** is $P\prime$ = proportion of households that have three cell phones.

The **distribution** for the hypothesis test is $P\prime \sim N\left(0.30, \sqrt{\frac{(0.30)\cdot(0.70)}{150}}\right)$

Exercise:

Problem:

The value that helps determine the p-value is $p\prime$. Calculate $p\prime$.

Solution:

$p\prime = \frac{x}{n}$ where x is the number of successes and n is the total number in the sample.

$x = 43, n = 150$

p' = $\frac{43}{150}$

Exercise:

Problem: What is a **success** for this problem?

Solution:

A success is having three cell phones in a household.

Exercise:

Problem: What is the level of significance?

Solution:

The level of significance is the preset α. Since α is not given, assume that $\alpha = 0.05$.

Draw the graph for this problem. Draw the horizontal axis. Label and shade appropriately.

Exercise:

Problem: Calculate the p-value.

Solution:

p-value = 0.7216

Exercise:

Problem:

Make a decision. _____________(Reject/Do not reject) H_0 because____________.

Solution:

Assuming that α = 0.05, $\alpha <$ p-value. The Decision is do not reject H_0 because there is not sufficient evidence to conclude that the proportion of households that have three cell phones is not 30%.

The next example is a poem written by a statistics student named Nicole Hart. The solution to the problem follows the poem. Notice that the hypothesis test is for a single population proportion. This means that the null and alternate hypotheses use the parameter p. The distribution for the test is normal. The estimated proportion $p\prime$ is the proportion of fleas killed to the total fleas found on Fido. This is sample information. The problem gives a preconceived $\alpha = 0.01$, for comparison, and a 95% confidence interval computation. The poem is clever and humorous, so please enjoy it!

Note: Hypothesis testing problems consist of multiple steps. To help you do the problems, solution sheets are provided for your use. Look in the Table of Contents Appendix for the topic "Solution Sheets." If you like, use copies of the appropriate solution sheet for homework problems.

Example:

Exercise:

Problem:

```
My dog has so many fleas, They do not come off
with ease. As for shampoo, I have tried many
types Even one called Bubble Hype, Which only
killed 25% of the fleas, Unfortunately I was
not pleased. I've used all kinds of soap, Until
I had give up hope Until one day I saw An ad
that put me in awe. A shampoo used for dogs
Called GOOD ENOUGH to Clean a Hog Guaranteed to
kill more fleas. I gave Fido a bath And after
doing the math His number of fleas Started
dropping by 3's! Before his shampoo I counted
42. At the end of his bath, I redid the math
And the new shampoo had killed 17 fleas. So now
I was pleased. Now it is time for you to have
some fun With the level of significance being
.01, You must help me figure out Use the new
shampoo or go without?
```

Solution:

Set up the Hypothesis Test:

H_o: $p = 0.25$ H_a: $p > 0.25$

Determine the distribution needed:

In words, CLEARLY state what your random variable X or P' represents.

P' = The proportion of fleas that are killed by the new shampoo

State the distribution to use for the test.

Normal: $N\left(0.25, \sqrt{\frac{(0.25)(1-0.25)}{42}}\right)$

Test Statistic: $z = 2.3163$

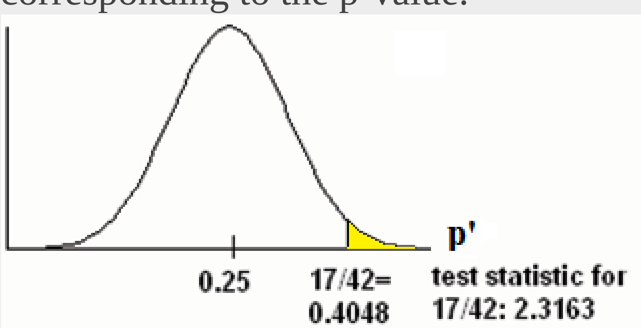
Calculate the p-value using the normal distribution for proportions:

p-value $=$0.0103

In 1 – 2 complete sentences, explain what the p-value means for this problem.

If the null hypothesis is true (the proportion is 0.25), then there is a 0.0103 probability that the sample (estimated) proportion is 0.4048 $\left(\frac{17}{42}\right)$ or more.

Use the previous information to sketch a picture of this situation. CLEARLY, label and scale the horizontal axis and shade the region(s) corresponding to the p-value.



Compare α and the p-value:

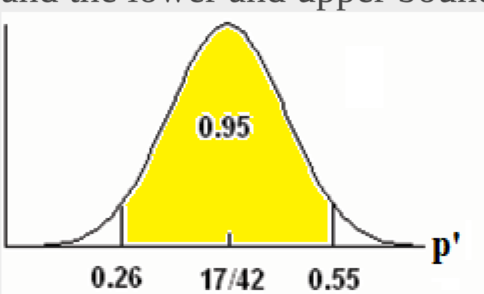
Indicate the correct decision ("reject" or "do not reject" the null hypothesis), the reason for it, and write an appropriate conclusion,

using COMPLETE SENTENCES.

alpha	**decision**	**reason for decision**
0.01	Do not reject H_o	α<p-value

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of fleas that are killed by the new shampoo is more than 25%.

Construct a 95% Confidence Interval for the true mean or proportion. Include a sketch of the graph of the situation. Label the point estimate and the lower and upper bounds of the Confidence Interval.



Confidence Interval: $(0.26, 0.55)$ We are 95% confident that the true population proportion p of fleas that are killed by the new shampoo is between 26% and 55%.

Note: This test result is not very definitive since the p-value is very close to alpha. In reality, one would probably do more tests by giving the dog another bath after the fleas have had a chance to return.

Glossary

Central Limit Theorem

Given a random variable (RV) with known mean μ and known standard deviation σ. We are sampling with size n and we are interested in two new RVs - the sample mean, $\bar{X}$, and the sample sum, ΣX. If the size n of the sample is sufficiently large, then $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\Sigma X \sim N\left(\mathrm{n}\mu, \sqrt{n}\sigma\right)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

Standard Deviation

A number that is equal to the square root of the variance and measures how far data values are from their mean. Notation: s for sample standard deviation and σ for population standard deviation.

Summary of Formulas

H_o and H_a are contradictory.

If H_o has:	equal ($=$)	greater than or equal to ($\geq$)	less than or equal to ($\leq$)
then H_a has:	not equal ($\neq$) **or** greater than ($>$) **or** less than ($<$)	less than ($<$)	greater than ($>$)

If $\alpha \leq$ p-value, then do not reject H_o.

If $\alpha >$ p-value, then reject H_o .

α is preconceived. Its value is set before the hypothesis test starts. The p-value is calculated from the data.

α = probability of a Type I error = P(Type I error) = probability of rejecting the null hypothesis when the null hypothesis is true.

β = probability of a Type II error = P(Type II error) = probability of not rejecting the null hypothesis when the null hypothesis is false.

If there is no given preconceived α, then use $\alpha = 0.05$.

Types of Hypothesis Tests

- Single population mean, **known** population variance (or standard deviation): **Normal test**.
- Single population mean, **unknown** population variance (or standard deviation): **Student's-t test**.
- Single population proportion: **Normal test**.

Practice 1: Single Mean, Known Population Standard Deviation
This module provides a practice of Hypothesis Testing of Single Mean and Single Proportion as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcomes

- The student will conduct a hypothesis test of a single mean with known population standard deviation.

Given

Suppose that a recent article stated that the mean time spent in jail by a first–time convicted burglar is 2.5 years. A study was then done to see if the mean time has increased in the new century. A random sample of 26 first–time convicted burglars in a recent year was picked. The mean length of time in jail from the survey was 3 years with a standard deviation of 1.8 years. Suppose that it is somehow known that the population standard deviation is 1.5. Conduct a hypothesis test to determine if the mean length of jail time has increased. The distribution of the population is normal.

Hypothesis Testing: Single Mean

Exercise:

Problem: Is this a test of means or proportions?

Solution:

Means

Exercise:

Problem: State the null and alternative hypotheses.

- **a**H_o:

- **b** H_a:

Solution:

- **a** H_o:$\mu = 2.5$ (or, H_o:$\mu \leq 2.5$)
- **b** H_a:$\mu > 2.5$

Exercise:

Problem:

Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Solution:

right-tailed

Exercise:

Problem: What symbol represents the Random Variable for this test?

Solution:

X

Exercise:

Problem: In words, define the Random Variable for this test.

Solution:

The mean time spent in jail for 26 first time convicted burglars

Exercise:

Problem:

Is the population standard deviation known and, if so, what is it?

Solution:

Yes, 1.5

Exercise:

Problem: Calculate the following:

- **a** $x =$
- **b** $\sigma =$
- **c** $s_x =$
- **d** $n =$

Solution:

- **a**3
- **b**1.5
- **c**1.8
- **d**26

Exercise:

Problem:

Since both σ and s_x are given, which should be used? In 1 -2 complete sentences, explain why.

Solution:

σ

Exercise:

Problem: State the distribution to use for the hypothesis test.

Solution:

$X \sim N(2.5, \frac{1.5}{\sqrt{26}})$

Exercise:

Problem:

Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized mean and the sample mean x. Shade the area corresponding to the p-value.

Exercise:

Problem: Find the p-value.

Solution:

0.0446

Exercise:

Problem: At a pre-conceived $\alpha = 0.05$, what is your:

- **a** Decision:
- **b** Reason for the decision:
- **c** Conclusion (write out in a complete sentence):

Solution:

- **a**Reject the null hypothesis

Discussion Questions

Exercise:

Problem:

Does it appear that the mean jail time spent for first time convicted burglars has increased? Why or why not?

Practice 2: Single Mean, Unknown Population Standard Deviation
This module provides a practice of Hypothesis Testing of Single Mean and Single Proportion as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcomes

- The student will conduct a hypothesis test of a single mean with unknown population standard deviation.

Given

A random survey of 75 death row inmates revealed that the mean length of time on death row is 17.4 years with a standard deviation of 6.3 years. Conduct a hypothesis test to determine if the population mean time on death row could likely be 15 years.

Hypothesis Testing: Single Mean

Exercise:

Problem: Is this a test of means or proportions?

Solution:

averages

Exercise:

Problem: State the null and alternative hypotheses.

- **a** H_o:
- **b** H_a:

Solution:

- **a** H_o:$\mu = 15$
- **b** H_a:$\mu \neq 15$

Exercise:

Problem:

Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Solution:

two-tailed

Exercise:

Problem: What symbol represents the Random Variable for this test?

Solution:

X

Exercise:

Problem: In words, define the Random Variable for this test.

Solution:

the mean time spent on death row for the 75 inmates

Exercise:

Problem:

Is the population standard deviation known and, if so, what is it?

Solution:

No

Exercise:

Problem: Calculate the following:

- **a** $x =$
- **b** $6.3 =$
- **c** $n =$

Solution:

- **a**17.4
- **b** s
- **c**75

Exercise:

Problem:

Which test should be used? In 1 -2 complete sentences, explain why.

Solution:

t −test

Exercise:

Problem: State the distribution to use for the hypothesis test.

Solution:

t_{74}

Exercise:

Problem:

Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized mean and the sample mean, x. Shade the area corresponding to the p-value.

Exercise:

Problem: Find the p-value.

Solution:

0.0015

Exercise:

Problem: At a pre-conceived $\alpha = 0.05$, what is your:

- **a** Decision:
- **b** Reason for the decision:
- **c** Conclusion (write out in a complete sentence):

Solution:

- **a**Reject the null hypothesis

Discussion Question

Does it appear that the mean time on death row could be 15 years? Why or why not?

Practice 3: Single Proportion
This module provides a practice of Hypothesis Testing of Single Mean and Single Proportion as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcomes

- The student will conduct a hypothesis test of a single population proportion.

Given

The National Institute of Mental Health published an article stating that in any one-year period, approximately 9.5 percent of American adults suffer from depression or a depressive illness. (http://www.nimh.nih.gov/publicat/depression.cfm) Suppose that in a survey of 100 people in a certain town, seven of them suffered from depression or a depressive illness. Conduct a hypothesis test to determine if the true proportion of people in that town suffering from depression or a depressive illness is lower than the percent in the general adult American population.

Hypothesis Testing: Single Proportion

Exercise:

Problem: Is this a test of means or proportions?

Solution:

Proportions

Exercise:

Problem: State the null and alternative hypotheses.

- **a** H_o:
- **b** H_a:

Solution:

- **a** H_o:$p = 0.095$
- **b** H_a:$p < 0.095$

Exercise:

Problem:

Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Solution:

left-tailed

Exercise:

Problem: What symbol represents the Random Variable for this test?

Solution:

P′

Exercise:

Problem: In words, define the Random Variable for this test.

Solution:

the proportion of people in that town surveyed suffering from depression or a depressive illness

Exercise:

Problem: Calculate the following:

- **a** $x =$
- **b** $n =$
- **c** p′=

Solution:

- **a**7
- **b**100
- **c**0.07

Exercise:

Problem:

Calculate $\sigma_{p'}$. Make sure to show how you set up the formula.

Solution:

0.0293

Exercise:

Problem: State the distribution to use for the hypothesis test.

Solution:

Normal

Exercise:

Problem:

Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized mean and the sample proportion, p-hat. Shade the area corresponding to the p-value.

Exercise:

Problem: Find the p-value

Solution:

0.1969

Exercise:

Problem: At a pre-conceived $\alpha = 0.05$, what is your:

- **a** Decision:
- **b** Reason for the decision:
- **c** Conclusion (write out in a complete sentence):

Solution:

- **a**Do not reject the null hypothesis

Discusion Question

Exercise:

Problem:

Does it appear that the proportion of people in that town with depression or a depressive illness is lower than general adult American population? Why or why not?

Exercises
This module provides a homework of Hypothesis Testing of Single Mean and Single Proportion as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Exercise:

Problem:

Some of the statements below refer to the null hypothesis, some to the alternate hypothesis.

State the null hypothesis, H_o, and the alternative hypothesis, H_a, in terms of the appropriate parameter (μ or p).

- **a**The mean number of years Americans work before retiring is 34.
- **b**At most 60% of Americans vote in presidential elections.
- **c**The mean starting salary for San Jose State University graduates is at least $100,000 per year.
- **d**29% of high school seniors get drunk each month.
- **e**Fewer than 5% of adults ride the bus to work in Los Angeles.
- **f**The mean number of cars a person owns in her lifetime is not more than 10.
- **g**About half of Americans prefer to live away from cities, given the choice.
- **h**Europeans have a mean paid vacation each year of six weeks.
- **i**The chance of developing breast cancer is under 11% for women.
- **j**Private universities mean tuition cost is more than $20,000 per year.

Solution:

- **a** H_o:$\mu = 34$; H_a:$\mu \neq 34$
- **c** H_o:$\mu \geq 100{,}000$; H_a:$\mu < 100{,}000$
- **d** H_o:$p = 0.29$; H_a:$p \neq 0.29$
- **g** H_o:$p = 0.50$; H_a:$p \neq 0.50$
- **i** H_o:$p \geq 0.11$; H_a:$p < 0.11$

Exercise:

Problem:

For (a) - (j) above, state the Type I and Type II errors in complete sentences.

Solution:

- **a**Type I error: We conclude that the mean is not 34 years, when it really is 34 years. Type II error: We do not conclude that the mean is not 34 years, when it is not really 34 years.
- **c**Type I error: We conclude that the mean is less than $100,000, when it really is at least $100,000. Type II error: We do not conclude that the mean is less than $100,000, when it is really less than $100,000.
- **d**Type I error: We conclude that the proportion of h.s. seniors who get drunk each month is not 29%, when it really is 29%. Type II error: We do not conclude that the proportion of h.s. seniors that get drunk each month is not 29%, when it is really not 29%.
- **i**Type I error: We conclude that the proportion is less than 11%, when it is really at least 11%. Type II error: We do not conclude that the proportion is less than 11%, when it really is less than 11%.

Exercise:

Problem: For (a) - (j) above, in complete sentences:

- **a**State a consequence of committing a Type I error.
- **b**State a consequence of committing a Type II error.

Note: For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in 14. Appendix (online book

version: the link is "Solution Sheets"; PDF book version: look under 14.5 Solution Sheets). Please feel free to make copies of the solution sheets. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.

Note: If you are using a student's-t distribution for a homework problem below, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, though.)

Exercise:

Problem:

A particular brand of tires claims that its deluxe tire averages at least 50,000 miles before it needs to be replaced. From past studies of this tire, the standard deviation is known to be 8000. A survey of owners of that tire design is conducted. From the 28 tires surveyed, the mean lifespan was 46,500 miles with a standard deviation of 9800 miles. Do the data support the claim at the 5% level?

Exercise:

Problem:

From generation to generation, the mean age when smokers first start to smoke varies. However, the standard deviation of that age remains constant of around 2.1 years. A survey of 40 smokers of this generation was done to see if the mean starting age is at least 19. The sample mean was 18.1 with a sample standard deviation of 1.3. Do the data support the claim at the 5% level?

Solution:

- **e** $z = -2.71$
- **f**0.0034
- **h**Decision: Reject null; Conclusion: $\mu < 19$

- **i** $(17.449, 18.757)$

Exercise:

Problem:

The cost of a daily newspaper varies from city to city. However, the variation among prices remains steady with a standard deviation of 20¢. A study was done to test the claim that the mean cost of a daily newspaper is $1.00. Twelve costs yield a mean cost of 95¢ with a standard deviation of 18¢. Do the data support the claim at the 1% level?

Exercise:

Problem:

An article in the **San Jose Mercury News** stated that students in the California state university system take 4.5 years, on average, to finish their undergraduate degrees. Suppose you believe that the mean time is longer. You conduct a survey of 49 students and obtain a sample mean of 5.1 with a sample standard deviation of 1.2. Do the data support your claim at the 1% level?

Solution:

- **e**3.5
- **f**0.0005
- **h**Decision: Reject null; Conclusion: $\mu > 4.5$
- **i** $(4.7553, 5.4447)$

Exercise:

Problem:

The mean number of sick days an employee takes per year is believed to be about 10. Members of a personnel department do not believe this figure. They randomly survey 8 employees. The number of sick days they took for the past year are as follows: 12; 4; 15; 3; 11; 8; 6; 8. Let x = the number of sick days they took for the past year. Should the personnel team believe that the mean number is about 10?

Exercise:

Problem:

In 1955, **Life Magazine** reported that the 25 year-old mother of three worked, on average, an 80 hour week. Recently, many groups have been studying whether or not the women's movement has, in fact, resulted in an increase in the average work week for women (combining employment and at-home work). Suppose a study was done to determine if the mean work week has increased. 81 women were surveyed with the following results. The sample mean was 83; the sample standard deviation was 10. Does it appear that the mean work week has increased for women at the 5% level?

Solution:

- **e**2.7
- **f**0.0042
- **h**Decision: Reject Null
- **i** $(80.789, 85.211)$

Exercise:

Problem:

Your statistics instructor claims that 60 percent of the students who take her Elementary Statistics class go through life feeling more enriched. For some reason that she can't quite figure out, most people don't believe her. You decide to check this out on your own. You randomly survey 64 of her past Elementary Statistics students and find that 34 feel more enriched as a result of her class. Now, what do you think?

Exercise:

Problem:

A Nissan Motor Corporation advertisement read, “The average man’s I.Q. is 107. The average brown trout’s I.Q. is 4. So why can’t man catch brown trout?” Suppose you believe that the brown trout’s mean I.Q. is greater than 4. You catch 12 brown trout. A fish psychologist determines the I.Q.s as follows: 5; 4; 7; 3; 6; 4; 5; 3; 6; 3; 8; 5. Conduct a hypothesis test of your belief.

Solution:

- **d** t_{11}
- **e**1.96
- **f**0.0380
- **h**Decision: Reject null when $a = 0.05$; do not reject null when $a = 0.01$
- **i** $(3.8865,5.9468)$

Exercise:

Problem:

Refer to the previous problem. Conduct a hypothesis test to see if your decision and conclusion would change if your belief were that the brown trout’s mean I.Q. is **not** 4.

Exercise:

Problem:

According to an article in **Newsweek**, the natural ratio of girls to boys is 100:105. In China, the birth ratio is 100: 114 (46.7% girls). Suppose you don't believe the reported figures of the percent of girls born in China. You conduct a study. In this study, you count the number of girls and boys born in 150 randomly chosen recent births. There are 60 girls and 90 boys born of the 150. Based on your study, do you believe that the percent of girls born in China is 46.7?

Solution:

- **e**-1.64
- **f**0.1000
- **h**Decision: Do not reject null
- **i** $(0.3216, 0.4784)$

Exercise:

Problem:

A poll done for **Newsweek** found that 13% of Americans have seen or sensed the presence of an angel. A contingent doubts that the percent is really that high. It conducts its own survey. Out of 76 Americans surveyed, only 2 had seen or sensed the presence of an angel. As a result of the contingent's survey, would you agree with the **Newsweek** poll? In complete sentences, also give three reasons why the two polls might give different results.

Exercise:

Problem:

The mean work week for engineers in a start-up company is believed to be about 60 hours. A newly hired engineer hopes that it's shorter. She asks 10 engineering friends in start-ups for the lengths of their mean work weeks. Based on the results that follow, should she count on the mean work week to be shorter than 60 hours?

Data (length of mean work week): 70; 45; 55; 60; 65; 55; 55; 60; 50; 55.

Solution:

- **d** t_9
- **e**-1.33
- **f**0.1086
- **h**Decision: Do not reject null
- **i** $(51.886, 62.114)$

Exercise:

Problem:

Use the “Lap time” data for Lap 4 (see Table of Contents) to test the claim that Terri finishes Lap 4, on average, in less than 129 seconds. Use all twenty races given.

Exercise:

Problem:

Use the “Initial Public Offering” data (see Table of Contents) to test the claim that the mean offer price was $18 per share. Do not use all the data. Use your random number generator to randomly survey 15 prices.

Note: The following questions were written by past students. They are excellent problems!

Exercise:

Problem:18. "Asian Family Reunion" by Chau Nguyen

Every two years it comes around
We all get together from different towns.
In my honest opinion
It's not a typical family reunion
Not forty, or fifty, or sixty,
But how about seventy companions!
The kids would play, scream, and shout
One minute they're happy, another they'll pout.
The teenagers would look, stare, and compare
From how they look to what they wear.
The men would chat about their business
That they make more, but never less.
Money is always their subject
And there's always talk of more new projects.
The women get tired from all of the chats
They head to the kitchen to set out the mats.
Some would sit and some would stand
Eating and talking with plates in their hands.
Then come the games and the songs
And suddenly, everyone gets along!
With all that laughter, it's sad to say
That it always ends in the same old way.
They hug and kiss and say "good-bye"
And then they all begin to cry!
I say that 60 percent shed their tears
But my mom counted 35 people this year.
She said that boys and men will always have their pride,
So we won't ever see them cry.
I myself don't think she's correct,
So could you please try this problem to see if you object?

Exercise:

Problem:"The Problem with Angels" by Cyndy Dowling

```
Although this problem is wholly mine,
The catalyst came from the magazine, Time.
On the magazine cover I did find
The realm of angels tickling my mind.

Inside, 69% I found to be
In angels, Americans do believe.

Then, it was time to rise to the task,
Ninety-five high school and college students I
did ask.
Viewing all as one group,
Random sampling to get the scoop.

So, I asked each to be true,
"Do you believe in angels?"  Tell me, do!

Hypothesizing at the start,
Totally believing in my heart
That the proportion who said yes
Would be equal on this test.

Lo and behold, seventy-three did arrive,
Out of the sample of ninety-five.
Now your job has just begun,
Solve this problem and have some fun.
```

Solution:

- **e**1.65
- **f**0.0984
- **h**Decision: Do not reject null
- **i** $(0.6836, 0.8533)$

Exercise:

Problem:"Blowing Bubbles" by Sondra Prull

```
Studying stats just made me tense,
I had to find some sane defense.
Some light and lifting simple play
To float my math anxiety away.

Blowing bubbles lifts me high
Takes my troubles to the sky.
POIK! They're gone, with all my stress
Bubble therapy is the best.

The label said each time I blew
The average number of bubbles would be at least
22.
I blew and blew and this I found
From 64 blows, they all are round!

But the number of bubbles in 64 blows
Varied widely, this I know.
20 per blow became the mean
They deviated by 6, and not 16.

From counting bubbles, I sure did relax
But now I give to you your task.
Was 22 a reasonable guess?
Find the answer and pass this test!
```

Exercise:

Problem:21. "Dalmatian Darnation" by Kathy Sparling

```
A greedy dog breeder named Spreckles
Bred puppies with numerous freckles
The Dalmatians he sought
```

Possessed spot upon spot
The more spots, he thought, the more shekels.

His competitors did not agree
That freckles would increase the fee.
They said, “Spots are quite nice
But they don't affect price;
One should breed for improved pedigree.”

The breeders decided to prove
This strategy was a wrong move.
Breeding only for spots
Would wreak havoc, they thought.
His theory they want to disprove.

They proposed a contest to Spreckles
Comparing dog prices to freckles.
In records they looked up
One hundred one pups:
Dalmatians that fetched the most shekels.

They asked Mr. Spreckles to name
An average spot count he'd claim
To bring in big bucks.
Said Spreckles, “Well, shucks,
It's for one hundred one that I aim.”

Said an amateur statistician
Who wanted to help with this mission.
“Twenty-one for the sample
Standard deviation's ample:

They examined one hundred and one
Dalmatians that fetched a good sum.
They counted each spot,
Mark, freckle and dot
And tallied up every one.

```
Instead of one hundred one spots
They averaged ninety six dots
Can they muzzle Spreckles'
Obsession with freckles
Based on all the dog data they've got?
```

Solution:

- **e**-2.39
- **f**0.0093
- **h**Decision: Reject null
- **i** $(91.854,100.15)$

Exercise:

Problem:

"Macaroni and Cheese, please!!" by Nedda Misherghi and Rachelle Hall

As a poor starving student I don't have much money to spend for even the bare necessities. So my favorite and main staple food is macaroni and cheese. It's high in taste and low in cost and nutritional value.

One day, as I sat down to determine the meaning of life, I got a serious craving for this, oh, so important, food of my life. So I went down the street to Greatway to get a box of macaroni and cheese, but it was SO expensive! $2.02 !!! Can you believe it? It made me stop and think. The world is changing fast. I had thought that the mean cost of a box (the normal size, not some super-gigantic-family-value-pack) was at most $1, but now I wasn't so sure. However, I was determined to find out. I went to 53 of the closest grocery stores and surveyed the prices of macaroni and cheese. Here are the data I wrote in my notebook:
Price per box of Mac and Cheese:

- 5 stores @ $2.02
- 15 stores @ $0.25

- 3 stores @ $1.29
- 6 stores @ $0.35
- 4 stores @ $2.27
- 7 stores @ $1.50
- 5 stores @ $1.89
- 8 stores @ 0.75.

I could see that the costs varied but I had to sit down to figure out whether or not I was right. If it does turn out that this mouth-watering dish is at most $1, then I'll throw a big cheesy party in our next statistics lab, with enough macaroni and cheese for just me. (After all, as a poor starving student I can't be expected to feed our class of animals!)

Exercise:

Problem:

"William Shakespeare: The Tragedy of Hamlet, Prince of Denmark" by Jacqueline Ghodsi
THE CHARACTERS (in order of appearance):

- HAMLET, Prince of Denmark and student of Statistics
- POLONIUS, Hamlet’s tutor
- HOROTIO, friend to Hamlet and fellow student

Scene: The great library of the castle, in which Hamlet does his lessons

Act I

(The day is fair, but the face of Hamlet is clouded. He paces the large room. His tutor, Polonius, is reprimanding Hamlet regarding the latter’s recent experience. Horatio is seated at the large table at right stage.)

POLONIUS: My Lord, how cans’t thou admit that thou hast seen a ghost! It is but a figment of your imagination!

HAMLET: I beg to differ; I know of a certainty that five-and-seventy in one hundred of us, condemned to the whips and scorns of time as we are, have gazed upon a spirit of health, or goblin damn'd, be their intents wicked or charitable.

POLONIUS If thou doest insist upon thy wretched vision then let me invest your time; be true to thy work and speak to me through the reason of the null and alternate hypotheses. (He turns to Horatio.) Did not Hamlet himself say, "What piece of work is man, how noble in reason, how infinite in faculties? Then let not this foolishness persist. Go, Horatio, make a survey of three-and-sixty and discover what the true proportion be. For my part, I will never succumb to this fantasy, but deem man to be devoid of all reason should thy proposal of at least five-and-seventy in one hundred hold true.

HORATIO (to Hamlet): What should we do, my Lord?

HAMLET: Go to thy purpose, Horatio.

HORATIO: To what end, my Lord?

HAMLET: That you must teach me. But let me conjure you by the rights of our fellowship, by the consonance of our youth, but the obligation of our ever-preserved love, be even and direct with me, whether I am right or no.

(Horatio exits, followed by Polonius, leaving Hamlet to ponder alone.)

Act II

(The next day, Hamlet awaits anxiously the presence of his friend, Horatio. Polonius enters and places some books upon the table just a moment before Horatio enters.)

POLONIUS: So, Horatio, what is it thou didst reveal through thy deliberations?

HORATIO: In a random survey, for which purpose thou thyself sent me forth, I did discover that one-and-forty believe fervently that the spirits of the dead walk with us. Before my God, I might not this believe, without the sensible and true avouch of mine own eyes.

POLONIUS: Give thine own thoughts no tongue, Horatio. (Polonius turns to Hamlet.) But look to't I charge you, my Lord. Come Horatio, let us go together, for this is not our test. (Horatio and Polonius leave together.)

HAMLET: To reject, or not reject, that is the question: whether 'tis nobler in the mind to suffer the slings and arrows of outrageous statistics, or to take arms against a sea of data, and, by opposing, end them. (Hamlet resignedly attends to his task.)

(Curtain falls)

Solution:

- **e**-1.82
- **f**0.0345
- **h**Decision: Do not reject null
- **i** $(0.5331, 0.7685)$

Exercise:

Problem:"Untitled" by Stephen Chen

I've often wondered how software is released and sold to the public. Ironically, I work for a company that sells products with known problems. Unfortunately, most of the problems are difficult to create, which makes them difficult to fix. I usually use the test program X, which tests the product, to try to create a specific problem. When the test program is run to make an error occur, the likelihood of generating an error is 1%.

So, armed with this knowledge, I wrote a new test program Y that will generate the same error that test program X creates, but more often. To find out if my test program is better than the original, so that I can convince the management that I'm right, I ran my test program to find out how often I can generate the same error. When I ran my test program 50 times, I generated the error twice. While this may not seem much better, I think that I can convince the management to use my test program instead of the original test program. Am I right?

Exercise:

Problem: Japanese Girls' Names

by Kumi Furuichi

It used to be very typical for Japanese girls' names to end with "ko." (The trend might have started around my grandmothers' generation and its peak might have been around my mother's generation.) "Ko" means "child" in Chinese character. Parents would name their daughters with "ko" attaching to other Chinese characters which have meanings that they want their daughters to become, such as Sachiko – a happy child, Yoshiko – a good child, Yasuko – a healthy child, and so on.

However, I noticed recently that only two out of nine of my Japanese girlfriends at this school have names which end with "ko." More and more, parents seem to have become creative, modernized, and, sometimes, westernized in naming their children.

I have a feeling that, while 70 percent or more of my mother's generation would have names with "ko" at the end, the proportion has dropped among my peers. I wrote down all my Japanese friends', ex-classmates', co-workers, and acquaintances' names that I could remember. Below are the names. (Some are repeats.) Test to see if the proportion has dropped for this generation.

Ai, Akemi, Akiko, Ayumi, Chiaki, Chie, Eiko, Eri, Eriko, Fumiko, Harumi, Hitomi, Hiroko, Hiroko, Hidemi, Hisako, Hinako, Izumi,

Izumi, Junko, Junko, Kana, Kanako, Kanayo, Kayo, Kayoko, Kazumi, Keiko, Keiko, Kei, Kumi, Kumiko, Kyoko, Kyoko, Madoka, Maho, Mai, Maiko, Maki, Miki, Miki, Mikiko, Mina, Minako, Miyako, Momoko, Nana, Naoko, Naoko, Naoko, Noriko, Rieko, Rika, Rika, Rumiko, Rei, Reiko, Reiko, Sachiko, Sachiko, Sachiyo, Saki, Sayaka, Sayoko, Sayuri, Seiko, Shiho, Shizuka, Sumiko, Takako, Takako, Tomoe, Tomoe, Tomoko, Touko, Yasuko, Yasuko, Yasuyo, Yoko, Yoko, Yoko, Yoshiko, Yoshiko, Yoshiko, Yuka, Yuki, Yuki, Yukiko, Yuko, Yuko.

Solution:

- **e** $z = -2.99$
- **f**0.0014
- **h**Decision: Reject null; Conclusion: $p < .70$
- **i** $(0.4529, 0.6582)$

Exercise:

Problem: Phillip’s Wish by Suzanne Osorio

```
My nephew likes to play
Chasing the girls makes his day.
He asked his mother
If it is okay
To get his ear pierced.
She said, “No way!”
To poke a hole through your ear,
Is not what I want for you, dear.
He argued his point quite well,
Says even my macho pal,  Mel,
Has gotten this done.
It’s all just for fun.
C’mon please, mom, please, what the hell.
Again Phillip complained to his mother,
Saying half his friends (including their
brothers)
```

Are piercing their ears
And they have no fears
He wants to be like the others.
She said, “I think it’s much less.
We must do a hypothesis test.
And if you are right,
I won’t put up a fight.
But, if not, then my case will rest.”
We proceeded to call fifty guys
To see whose prediction would fly.
Nineteen of the fifty
Said piercing was nifty
And earrings they’d occasionally buy.
Then there’s the other thirty-one,
Who said they’d never have this done.
So now this poem’s finished.
Will his hopes be diminished,
Or will my nephew have his fun?

Exercise:

Problem: The Craven by Mark Salangsang

Once upon a morning dreary
In stats class I was weak and weary.
Pondering over last night’s homework
Whose answers were now on the board
This I did and nothing more.

While I nodded nearly napping
Suddenly, there came a tapping.
As someone gently rapping,
Rapping my head as I snore.
Quoth the teacher, “Sleep no more.”

“In every class you fall asleep,”
The teacher said, his voice was deep.

```
“So a tally I’ve begun to keep
Of every class you nap and snore.
The percentage being forty-four.”

“My dear teacher I must confess,
While sleeping is what I do best.
The percentage, I think, must be less,
A percentage less than forty-four.”
This I said and nothing more.

“We’ll see,” he said and walked away,
And fifty classes from that day
He counted till the month of May
The classes in which I napped and snored.
The number he found was twenty-four.

At a significance level of 0.05,
Please tell me am I still alive?
Or did my grade just take a dive
Plunging down beneath the floor?
Upon thee I hereby implore.
```

Solution:

- **e**0.57
- **f**0.7156
- **h**Decision: Do not reject null
- **i** $(0.3415, 0.6185)$

Exercise:

Problem:

Toastmasters International cites a report by Gallop Poll that 40% of Americans fear public speaking. A student believes that less than 40% of students at her school fear public speaking. She randomly surveys 361 schoolmates and finds that 135 report they fear public speaking. Conduct a hypothesis test to determine if the percent at her school is less than 40%. (*Source: http://toastmasters.org/artisan/detail.asp?CategoryID=1&SubCategoryID=10&ArticleID=429&Page=1*)

Exercise:

Problem:

68% of online courses taught at community colleges nationwide were taught by full-time faculty. To test if 68% also represents California's percent for full-time faculty teaching the online classes, Long Beach City College (LBCC), CA, was randomly selected for comparison. In the same year, 34 of the 44 online courses LBCC offered were taught by full-time faculty. Conduct a hypothesis test to determine if 68% represents CA. NOTE: For more accurate results, use more CA community colleges and this past year's data. (Sources: **Growing by Degrees** by Allen and Seaman; Amit Schitai, Director of Instructional Technology and Distance Learning, LBCC).

Solution:

- **e**1.32
- **f**0.1873
- **h**Decision: Do not reject null
- **i** (0.65,0.90)

Exercise:

Problem:

According to an article in **Bloomberg Businessweek**, New York City's most recent adult smoking rate is 14%. Suppose that a survey is conducted to determine this year's rate. Nine out of 70 randomly chosen N.Y. City residents reply that they smoke. Conduct a hypothesis test to determine if the rate is still 14% or if it has decreased. (*Source: http://www.businessweek.com/news/2011-09-15/nyc-smoking-rate-falls-to-record-low-of-14-bloomberg-says.html*)

Exercise:

Problem:

The mean age of De Anza College students in a previous term was 26.6 years old. An instructor thinks the mean age for online students is older than 26.6. She randomly surveys 56 online students and finds that the sample mean is 29.4 with a standard deviation of 2.1. Conduct a hypothesis test. (*Source: http://research.fhda.edu/factbook/DAdemofs/Fact_sheet_da_2006w.pdf*)

Solution:

- **e**9.98
- **f**0.0000
- **h**Decision: Reject null
- **i** (28.8,30.0)

Exercise:

Problem:

Registered nurses earned an average annual salary of $69,110. For that same year, a survey was conducted of 41 California registered nurses to determine if the annual salary is higher than $69,110 for California nurses. The sample average was $71,121 with a sample standard deviation of $7,489. Conduct a hypothesis test. (*Source: http://www.bls.gov/oes/current/oes291111.htm*)

Exercise:

Problem:

La Leche League International reports that the mean age of weaning a child from breastfeeding is age 4 to 5 worldwide. In America, most nursing mothers wean their children much earlier. Suppose a random survey is conducted of 21 U.S. mothers who recently weaned their children. The mean weaning age was 9 months (3/4 year) with a standard deviation of 4 months. Conduct a hypothesis test to determine if the mean weaning age in the U.S. is less than 4 years old. (*Source: http://www.lalecheleague.org/Law/BAFeb01.html*)

Solution:

- **e**-44.7
- **f**0.0000
- **h**Decision: Reject null
- **i** (0.60,0.90) - in years

Try these multiple choice questions.

Exercise:

Problem:

When a new drug is created, the pharmaceutical company must subject it to testing before receiving the necessary permission from the Food and Drug Administration (FDA) to market the drug. Suppose the null hypothesis is “the drug is unsafe.” What is the Type II Error?

- **A**To conclude the drug is safe when in, fact, it is unsafe
- **B**To not conclude the drug is safe when, in fact, it is safe.
- **C**To conclude the drug is safe when, in fact, it is safe.
- **D**To not conclude the drug is unsafe when, in fact, it is unsafe

Solution:

B

The next two questions refer to the following information: Over the past few decades, public health officials have examined the link between weight concerns and teen girls smoking. Researchers surveyed a group of 273 randomly selected teen girls living in Massachusetts (between 12 and 15 years old). After four years the girls were surveyed again. Sixty-three (63) said they smoked to stay thin. Is there good evidence that more than thirty percent of the teen girls smoke to stay thin?

Exercise:

Problem: The alternate hypothesis is

- **A** $p < 0.30$
- **B** $p \leq 0.30$
- **C** $p \geq 0.30$
- **D** $p > 0.30$

Solution:

D

Exercise:

Problem: After conducting the test, your decision and conclusion are

- **A**Reject H_o: There is sufficient evidence to conclude that more than 30% of teen girls smoke to stay thin.
- **B**Do not reject H_o: There is not sufficient evidence to conclude that less than 30% of teen girls smoke to stay thin.
- **C**Do not reject H_o: There is not sufficient evidence to conclude that more than 30% of teen girls smoke to stay thin.
- **D**Reject H_o: There is sufficient evidence to conclude that less than 30% of teen girls smoke to stay thin.

Solution:

C

The next three questions refer to the following information: A statistics instructor believes that fewer than 20% of Evergreen Valley College (EVC) students attended the opening night midnight showing of the latest Harry Potter movie. She surveys 84 of her students and finds that 11 of attended the midnight showing.

Exercise:

Problem: An appropriate alternative hypothesis is

- **A** $p = 0.20$
- **B** $p > 0.20$
- **C** $p < 0.20$
- **D** $p \leq 0.20$

Solution:

C

Exercise:

Problem: At a 1% level of significance, an appropriate conclusion is:

- **A**There is insufficient evidence to conclude that the percent of EVC students that attended the midnight showing of Harry Potter is less than 20%.
- **B**There is sufficient evidence to conclude that the percent of EVC students that attended the midnight showing of Harry Potter is more than 20%.
- **C**There is sufficient evidence to conclude that the percent of EVC students that attended the midnight showing of Harry Potter is less than 20%.

- **D**There is insufficient evidence to conclude that the percent of EVC students that attended the midnight showing of Harry Potter is at least 20%.

Solution:

A

Exercise:

Problem:

The Type I error is to conclude that the percent of EVC students who attended is

- **A**at least 20%, when in fact, it is less than 20%.
- **B**20%, when in fact, it is 20%.
- **C**less than 20%, when in fact, it is at least 20%.
- **D**less than 20%, when in fact, it is less than 20%.

Solution:

C

The next two questions refer to the following information:

It is believed that Lake Tahoe Community College (LTCC) Intermediate Algebra students get less than 7 hours of sleep per night, on average. A survey of 22 LTCC Intermediate Algebra students generated a mean of 7.24 hours with a standard deviation of 1.93 hours. At a level of significance of 5%, do LTCC Intermediate Algebra students get less than 7 hours of sleep per night, on average?

Exercise:

Problem: The distribution to be used for this test is X ~

- **A** $N(7.24, \frac{1.93}{\sqrt{22}})$
- **B** $N(7.24, 1.93)$
- **C** t_{22}
- **D** t_{21}

Solution:

D

Exercise:

Problem:

The Type II error is to not reject that the mean number of hours of sleep LTCC students get per night is at least 7 when, in fact, the mean number of hours

- **A**is more than 7 hours.
- **B**is at most 7 hours.
- **C**is at least 7 hours.
- **D**is less than 7 hours.

Solution:

D

The next three questions refer to the following information: Previously, an organization reported that teenagers spent 4.5 hours per week, on average, on the phone. The organization thinks that, currently, the mean is higher. Fifteen (15) randomly chosen teenagers were asked how many hours per week they spend on the phone. The sample mean was 4.75 hours with a sample standard deviation of 2.0. Conduct a hypothesis test.

Exercise:

Problem: The null and alternate hypotheses are:

- **A**H_o:$x = 4.5$, H_a:$x > 4.5$
- **B** H_o:$\mu \geq 4.5$ H_a:$\mu < 4.5$
- **C** H_o:$\mu = 4.75$ $H_{a:}\mu > 4.75$
- **D** H_o:$\mu = 4.5$ H_a:$\mu > 4.5$

Solution:

D

Exercise:

Problem:

At a significance level of $a = 0.05$, what is the correct conclusion?

- **A**There is enough evidence to conclude that the mean number of hours is more than 4.75
- **B**There is enough evidence to conclude that the mean number of hours is more than 4.5
- **C**There is not enough evidence to conclude that the mean number of hours is more than 4.5
- **D**There is not enough evidence to conclude that the mean number of hours is more than 4.75

Solution:

C

Exercise:

Problem: The Type I error is:

- **A**To conclude that the current mean hours per week is higher than 4.5, when in fact, it is higher.
- **B**To conclude that the current mean hours per week is higher than 4.5, when in fact, it is the same.

- **C**To conclude that the mean hours per week currently is 4.5, when in fact, it is higher.
- **D**To conclude that the mean hours per week currently is no higher than 4.5, when in fact, it is not higher.

Solution:

B

Review
This module provides an overview of Hypothesis Testing of Single Mean and Single Proportion as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Exercise:

Problem:

Rebecca and Matt are 14 year old twins. Matt's height is 2 standard deviations below the mean for 14 year old boys' height. Rebecca's height is 0.10 standard deviations above the mean for 14 year old girls' height. Interpret this.

- **A**Matt is 2.1 inches shorter than Rebecca
- **B**Rebecca is very tall compared to other 14 year old girls.
- **C**Rebecca is taller than Matt.
- **D**Matt is shorter than the average 14 year old boy.

Solution:

D

Exercise:

Problem:

Construct a histogram of the IPO data (see Table of Contents, 14. Appendix, Data Sets). Use 5 intervals.

Solution:

No solution provided. There are several ways in which the histogram could be constructed.

The next three exercises refer to the following information: Ninety homeowners were asked the number of estimates they obtained before having their homes fumigated. X = the number of estimates.

x	**Rel. Freq.**	**Cumulative Rel. Freq.**
1	0.3	
2	0.2	
4	0.4	
5	0.1	

Complete the cumulative relative frequency column.

Exercise:

Problem:

Calculate the sample mean (a), the sample standard deviation (b) and the percent of the estimates that fall at or below 4 (c).

Solution:

- **a**2.8
- **b** 1.48
- **c**90%

Exercise:

Problem:

Calculate the median, M, the first quartile, Q1, the third quartile, Q3. Then construct a boxplot of the data.

Solution:

$M = 3$; $Q1 = 1$; $Q3 = 4$

Exercise:

Problem: The middle 50% of the data are between _____ and _____.

Solution:

1 and 4

The next three questions refer to the following table: Seventy 5th and 6th graders were asked their favorite dinner.

	Pizza	**Hamburgers**	**Spaghetti**	**Fried shrimp**
5th grader	15	6	9	0
6th grader	15	7	10	8

Exercise:

Problem:

Find the probability that one randomly chosen child is in the 6th grade and prefers fried shrimp.

- **A** $\frac{32}{70}$
- **B** $\frac{8}{32}$
- **C** $\frac{8}{8}$
- **D** $\frac{8}{70}$

Solution:

D

Exercise:

Problem: Find the probability that a child does not prefer pizza.

- **A** $\frac{30}{70}$
- **B** $\frac{30}{40}$
- **C** $\frac{40}{70}$
- **D**1

Solution:

C

Exercise:

Problem:

Find the probability a child is in the 5th grade given that the child prefers spaghetti.

- **A** $\frac{9}{19}$
- **B** $\frac{9}{70}$
- **C** $\frac{9}{30}$
- **D** $\frac{19}{70}$

Solution:

A

Exercise:

Problem: A sample of convenience is a random sample.

- **A**true

- **B**false

Solution:

B

Exercise:

Problem: A statistic is a number that is a property of the population.

- **A**true
- **B**false

Solution:

B

Exercise:

Problem: You should always throw out any data that are outliers.

- **A**true
- **B**false

Solution:

B

Exercise:

Problem:

Lee bakes pies for a small restaurant in Felton, CA. She generally bakes 20 pies in a day, on the average. Of interest is the num.ber of pies she bakes each day

- **a**Define the Random Variable X.

- **b**State the distribution for X.
- **c**Find the probability that Lee bakes more than 25 pies in any given day.

Solution:

- **b** $P(20)$
- **c**0.1122

Exercise:

Problem:

Six different brands of Italian salad dressing were randomly selected at a supermarket. The grams of fat per serving are 7, 7, 9, 6, 8, 5. Assume that the underlying distribution is normal. Calculate a 95% confidence interval for the population mean grams of fat per serving of Italian salad dressing sold in supermarkets.

Solution:

CI: $(5.52, 8.48)$

Exercise:

Problem:

Given: uniform, exponential, normal distributions. Match each to a statement below.

- **a**mean = median ≠ mode
- **b**mean > median > mode
- **c**mean = median = mode

Solution:

- **a**uniform

- **b**exponential
- **c**normal

Lab: Hypothesis Testing of a Single Mean and Single Proportion

Class Time:

Names:

Student Learning Outcomes:

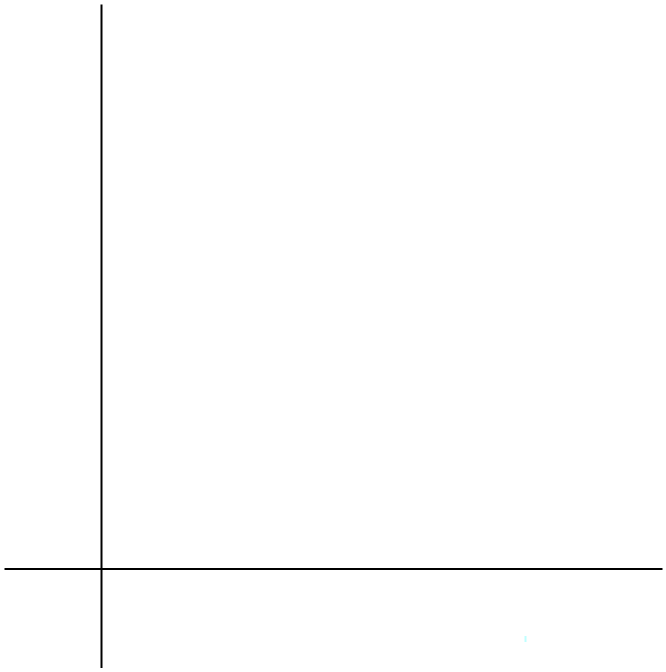
- The student will select the appropriate distributions to use in each case.
- The student will conduct hypothesis tests and interpret the results.

Television Survey

In a recent survey, it was stated that Americans watch television on average four hours per day. Assume that $\sigma = 2$. Using your class as the sample, conduct a hypothesis test to determine if the average for students at your school is lower.

1. H_o:
2. H_a:
3. In words, define the random variable. ___________ =
4. The distribution to use for the test is:
5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately.Shade the actual level of significance.

    - **a**Graph:



- **b**Determine the p-value:

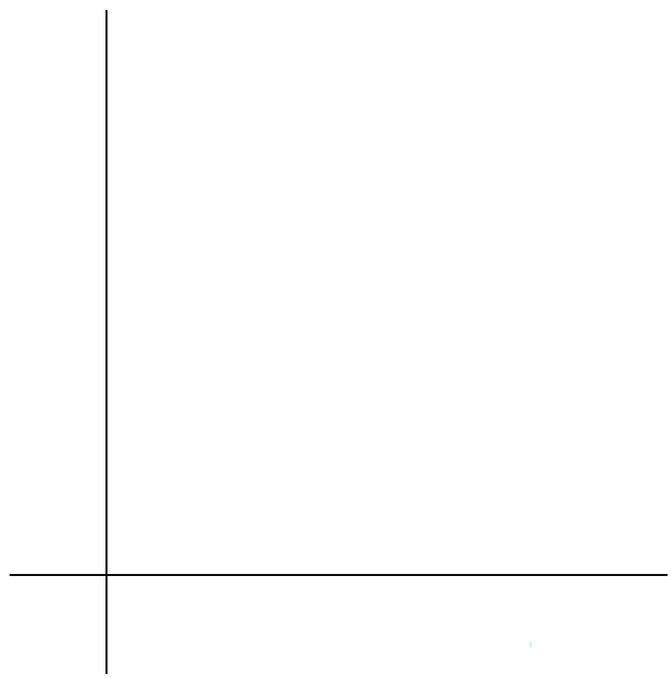
7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

Language Survey

About 42.3% of Californians and 19.6% of all Americans over age 5 speak a language other than English at home. Using your class as the sample, conduct a hypothesis test to determine if the percent of the students at your school that speak a language other than English at home is different from 42.3%. *(Source: http://www.census.gov/hhes/socdemo/language/)*

1. H_o:
2. H_a:
3. In words, define the random variable. __________ =
4. The distribution to use for the test is:
5. Determine the test statistic using your data.

6. Draw a graph and label it appropriately. Shade the actual level of significance.
    - **a** Graph:



    - **b**Determine the p-value:
7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

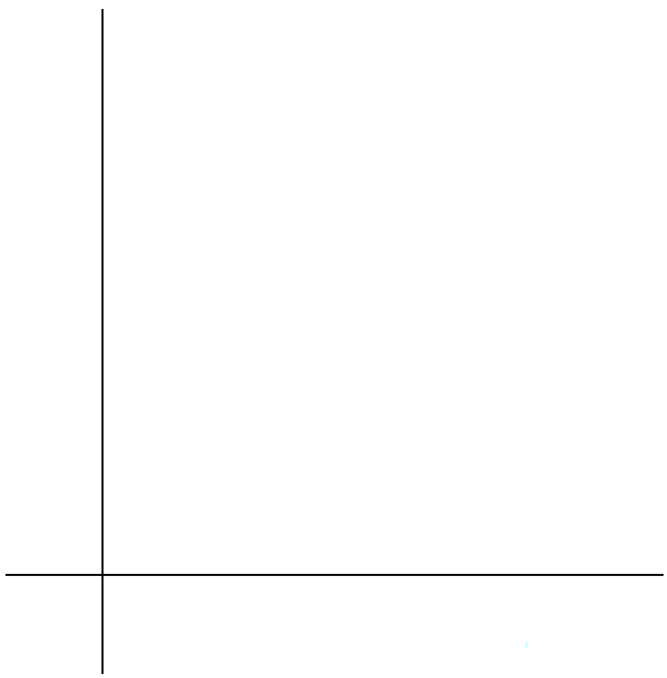
Jeans Survey

Suppose that young adults own an average of 3 pairs of jeans. Survey 8 people from your class to determine if the average is higher than 3.

1. H_o:
2. H_a:
3. In words, define the random variable. ___________ =
4. The distribution to use for the test is:

5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.

    - **a** Graph:



    - **b**Determine the p-value:

7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

Hypothesis Testing: Two Population Means and Two Population Proportions

Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Classify hypothesis tests by type.
- Conduct and interpret hypothesis tests for two population means, population standard deviations known.
- Conduct and interpret hypothesis tests for two population means, population standard deviations unknown.
- Conduct and interpret hypothesis tests for two population proportions.
- Conduct and interpret hypothesis tests for matched or paired samples.

Introduction

Studies often compare two groups. For example, researchers are interested in the effect aspirin has in preventing heart attacks. Over the last few years, newspapers and magazines have reported about various aspirin studies involving two groups. Typically, one group is given aspirin and the other group is given a placebo. Then, the heart attack rate is studied over several years.

There are other situations that deal with the comparison of two groups. For example, studies compare various diet and exercise programs. Politicians compare the proportion of individuals from different income brackets who might vote for them. Students are interested in whether SAT or GRE preparatory courses really help raise their scores.

In the previous chapter, you learned to conduct hypothesis tests on single means and single proportions. You will expand upon that in this chapter. You will compare two means or two proportions to each other. The general procedure is still the same, just expanded.

To compare two means or two proportions, you work with two groups. The groups are classified either as **independent** or **matched pairs**.

Independent groups mean that the two samples taken are independent, that is, sample values selected from one population are not related in any way to sample values selected from the other population. **Matched pairs** consist of two samples that are dependent. The parameter tested using matched pairs is the population mean. The parameters tested using independent groups are either population means or population proportions.

Note: This chapter relies on either a calculator or a computer to calculate the degrees of freedom, the test statistics, and p-values. TI-83+ and TI-84 instructions are included as well as the test statistic formulas. When using the TI-83+/TI-84 calculators, we do not need to separate two population means, independent groups, population variances unknown into large and small sample sizes. However, most statistical computer software has the ability to differentiate these tests.

This chapter deals with the following hypothesis tests:

Independent groups (samples are independent)

- Test of two population means.
- Test of two population proportions.

Matched or paired samples (samples are dependent)

- Becomes a test of one population mean.

Comparing Two Independent Population Means with Unknown Population Standard Deviations
This module provides an overview of Comparing Two Independent Population Means with Unknown Population Standard Deviations as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

1. The two independent samples are simple random samples from two distinct populations.
2. Both populations are normally distributed with the population means and standard deviations unknown unless the sample sizes are greater than 30. In that case, the populations need not be normally distributed.

Note: The test comparing two independent population means with unknown and possibly unequal population standard deviations is called the Aspin-Welch t-test. The degrees of freedom formula was developed by Aspin-Welch.

The comparison of two population means is very common. A difference between the two samples depends on both the means and the standard deviations. Very different means can occur by chance if there is great variation among the individual samples. In order to account for the variation, we take the difference of the sample means, $\overline{X_1}$ - $\overline{X_2}$, and divide by the standard error (shown below) in order to standardize the difference. The result is a t-score test statistic (shown below).

Because we do not know the population standard deviations, we estimate them using the two sample standard deviations from our independent samples. For the hypothesis test, we calculate the estimated standard deviation, or **standard error**, of **the difference in sample means**, $\overline{X_1}$ - $\overline{X_2}$
.

Equation:

The standard error is:

$$\sqrt{\frac{(S_1)^2}{n_1}+\frac{(S_2)^2}{n_2}}$$

The test statistic (t-score) is calculated as follows:

Equation:

t-score

$$\frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{\frac{(S_1)^2}{n_1}+\frac{(S_2)^2}{n_2}}}$$

where:

- s_1 and s_2, the sample standard deviations, are estimates of σ_1 and σ_2, respectively.
- σ_1 and σ_2 are the unknown population standard deviations.
- $\overline{x_1}$ and $\overline{x_2}$ are the sample means. μ_1 and μ_2 are the population means.

The **degrees of freedom (df)** is a somewhat complicated calculation. However, a computer or calculator calculates it easily. The dfs are not always a whole number. The test statistic calculated above is approximated by the student's-t distribution with dfs as follows:

Equation:

Degrees of freedom

$$\text{df}=\frac{\left[\frac{(s_1)^2}{n_1}+\frac{(s_2)^2}{n_2}\right]^2}{\frac{1}{n_1-1}\cdot\left[\frac{(s_1)^2}{n_1}\right]^2+\frac{1}{n_2-1}\cdot\left[\frac{(s_2)^2}{n_2}\right]^2}$$

When both sample sizes n_1 and n_2 are five or larger, the student's-t approximation is very good. Notice that the sample variances ${s_1}^2$ and ${s_2}^2$ are not pooled. (If the question comes up, do not pool the variances.)

Note: It is not necessary to compute this by hand. A calculator or computer easily computes it.

Example:
Independent groups
The average amount of time boys and girls ages 7 through 11 spend playing sports each day is believed to be the same. An experiment is done, data is collected, resulting in the table below. Both populations have a normal distribution.

	Sample Size	Average Number of Hours Playing Sports Per Day	Sample Standard Deviation
Girls	9	2 hours	$\sqrt{0.75}$
Boys	16	3.2 hours	1.00

Exercise:

Problem:

Is there a difference in the mean amount of time boys and girls ages 7 through 11 play sports each day? Test at the 5% level of significance.

Solution:

The population standard deviations are not known. Let g be the subscript for girls and b be the subscript for boys. Then, μ_g is the

population mean for girls and μ_b is the population mean for boys. This is a test of two **independent groups**, two population **means**.

Random variable: $\overline{X_g} - \overline{X_b}$ = difference in the sample mean amount of time girls and boys play sports each day.

H_o: $\mu_g = \mu_b$ $\qquad\qquad$ $\mu_g - \mu_b = 0$

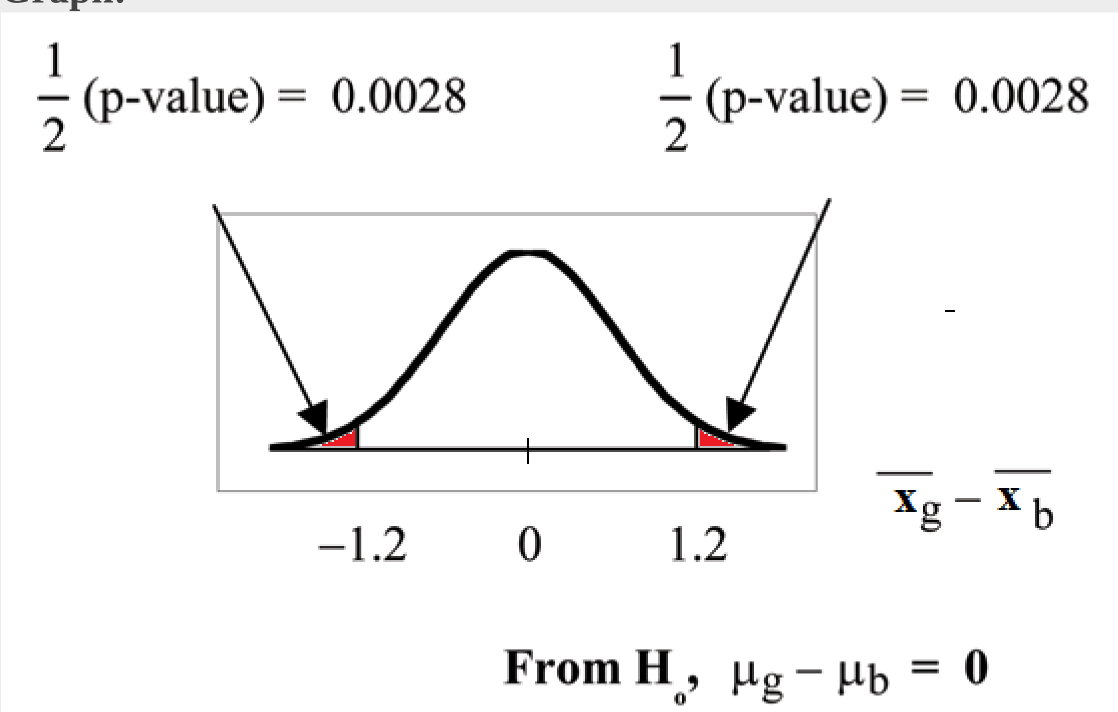
H_a: $\mu_g \neq \mu_b$ $\qquad\qquad$ $\mu_g - \mu_b \neq 0$

The words **"the same"** tell you H_o has an "=". Since there are no other words to indicate H_a, then assume **"is different."** This is a two-tailed test.

Distribution for the test: Use t_{df} where df is calculated using the df formula for independent groups, two population means. Using a calculator, df is approximately 18.8462. **Do not pool the variances.**

Calculate the p-value using a student's-t distribution: p-value = 0.0054

Graph:



$s_g = \sqrt{0.75}$

$s_b = 1$

So, $\overline{x_g} - \overline{x_b} = 2 - 3.2 = -1.2$

Half the p-value is below -1.2 and half is above 1.2.

Make a decision: Since $\alpha >$ p-value, reject H_o.

This means you reject $\mu_g = \mu_b$. The means are different.

Conclusion: At the 5% level of significance, the sample data show there is sufficient evidence to conclude that the mean number of hours that girls and boys aged 7 through 11 play sports per day is different (mean number of hours boys aged 7 through 11 play sports per day is greater than the mean number of hours played by girls OR the mean number of hours girls aged 7 through 11 play sports per day is greater than the mean number of hours played by boys).

Note:TI-83+ and TI-84: Press

`STAT`

. Arrow over to

`TESTS`

and press

`4:2-SampTTest`

. Arrow over to Stats and press

`ENTER`

. Arrow down and enter

`2`

for the first sample mean,

$\sqrt{0.75}$

for Sx1,

`9`

for n1,

`3.2`

for the second sample mean,

`1`

for Sx2, and

`16`

for n2. Arrow down to μ1: and arrow to

`does not equal`

μ2. Press

`ENTER`

. Arrow down to Pooled: and

`No`

. Press

`ENTER`

. Arrow down to

`Calculate`

and press

`ENTER`

. The p-value is p = 0.0054, the dfs are approximately 18.8462, and the test statistic is -3.14. Do the procedure again but instead of Calculate do Draw.

Example:
A study is done by a community group in two neighboring colleges to determine which one graduates students with more math classes. College A samples 11 graduates. Their average is 4 math classes with a standard deviation of 1.5 math classes. College B samples 9 graduates. Their average is 3.5 math classes with a standard deviation of 1 math class. The community group believes that a student who graduates from college A **has taken more math classes,** on the average. Both populations have a normal distribution. Test at a 1% significance level. Answer the following questions.

Exercise:

Problem: Is this a test of two means or two proportions?

Solution:

two means

Exercise:

Problem:

Are the populations standard deviations known or unknown?

Solution:

unknown

Exercise:

Problem: Which distribution do you use to perform the test?

Solution:

student's-t

Exercise:

Problem: What is the random variable?

Solution:

$\overline{X_A} - \overline{X_B}$

Exercise:

Problem: What are the null and alternate hypothesis?

Solution:

- $H_o : \mu_A \leq \mu_B$
- $H_a : \mu_A > \mu_B$

Exercise:

Problem: Is this test right, left, or two tailed?

Solution:

right

Exercise:

Problem: What is the p-value?

Solution:

0.1928

Exercise:

Problem: Do you reject or not reject the null hypothesis?

Solution:

Do not reject.

Conclusion:
At the 1% level of significance, from the sample data, there is not sufficient evidence to conclude that a student who graduates from college A has taken more math classes, on the average, than a student who graduates from college B.

Glossary

Degrees of Freedom (df)
: The number of objects in a sample that are free to vary.

Standard Deviation
: A number that is equal to the square root of the variance and measures how far data values are from their mean. Notation: s for sample standard deviation and σ for population standard deviation.

Variable (Random Variable)

A characteristic of interest in a population being studied. Common notation for variables are upper case Latin letters X, Y, Z,...; common notation for a specific value from the domain (set of all possible values of a variable) are lower case Latin letters x, y, z,.... For example, if X is the number of children in a family, then x represents a specific integer 0, 1, 2, 3, ... Variables in statistics differ from variables in intermediate algebra in two following ways.

- The domain of the random variable (RV) is not necessarily a numerical set; the domain may be expressed in words; for example, if X = hair color then the domain is {black, blond, gray, green, orange}.
- We can tell what specific value x of the Random Variable X takes only after performing the experiment.

Comparing Two Independent Population Means with Known Population Standard Deviations

This module provides an overview of hypothesis testing in situations where there are both two independent population means and known population standard deviations in statistics.

Even though this situation is not likely (knowing the population standard deviations is not likely), the following example illustrates hypothesis testing for independent means, known population standard deviations. The sampling distribution for the difference between the means is normal and both populations must be normal. The random variable is $\overline{X_1} - \overline{X_2}$. The normal distribution has the following format:

Equation:

Normal distribution

$$\overline{X_1} - \overline{X_2} \sim N\left[u_1 - u_2, \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}\right]$$

Equation:

The standard deviation is:

$$\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}$$

Equation:

The test statistic (z-score) is:

$$z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}}$$

Example:

independent groups, population standard deviations known: The mean lasting time of 2 competing floor waxes is to be compared. **Twenty floors** are randomly assigned **to test each wax**. Both populations have a normal distribution. The following table is the result.

Wax	Sample Mean Number of Months Floor Wax Last	Population Standard Deviation
1	3	0.33
2	2.9	0.36

Exercise:

Problem:

Does the data indicate that **wax 1 is more effective than wax 2**? Test at a 5% level of significance.

Solution:

This is a test of two independent groups, two population means, population standard deviations known.

Random Variable: $\overline{X_1} - \overline{X_2} =$ difference in the mean number of months the competing floor waxes last.

$H_o: \mu_1 \leq \mu_2$

$H_a: \mu_1 > \mu_2$

The words **"is more effective"** says that **wax 1 lasts longer than wax 2**, on the average. "Longer" is a $">"$ symbol and goes into H_a. Therefore, this is a right-tailed test.

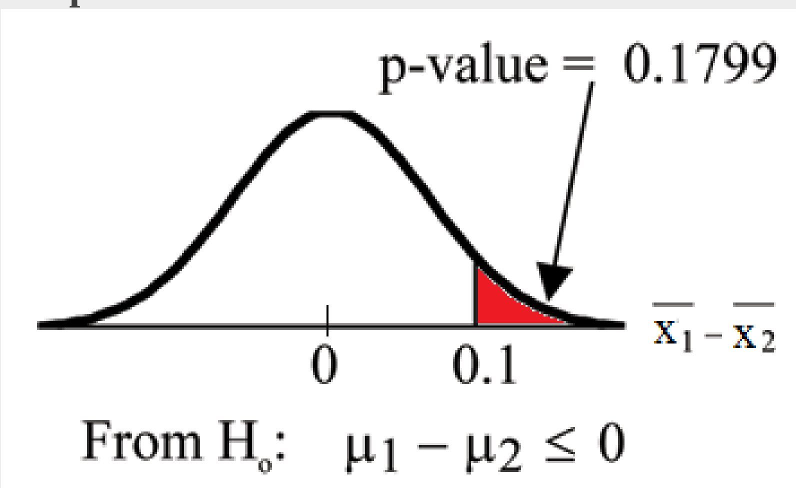
Distribution for the test: The population standard deviations are known so the distribution is normal. Using the formula above, the distribution is:

$$\overline{X_1} - \overline{X_2} \sim N\left(0, \sqrt{\frac{0.33^2}{20} + \frac{0.36^2}{20}}\right)$$

Since $\mu_1 \leq \mu_2$ then $\mu_1 - \mu_2 \leq 0$ and the mean for the normal distribution is 0.

Calculate the p-value using the normal distribution: p-value = 0.1799

Graph:



$\overline{x_1} - \overline{x_2} = 3 - 2.9 = 0.1$

Compare α and the p-value: $\alpha = 0.05$ and p-value = 0.1799. Therefore, $\alpha <$ p-value.

Make a decision: Since $\alpha <$ p-value, do not reject H_o.

Conclusion: At the 5% level of significance, from the sample data, there is not sufficient evidence to conclude that the mean time wax 1 lasts is longer (wax 1 is more effective) than the mean time wax 2 lasts.

Note:TI-83+ and TI-84: Press

`STAT`

. Arrow over to

`TESTS`

and press

`3:2-SampZTest`

. Arrow over to

`Stats`

and press

`ENTER`

. Arrow down and enter

`.33`

for sigma1,

`.36`

for sigma2,

`3`

for the first sample mean,

20

for n1,

2.9

for the second sample mean, and

20

for n2. Arrow down to μ1: and arrow to > μ2. Press

ENTER

. Arrow down to

Calculate

and press

ENTER

. The p-value is p = 0.1799 and the test statistic is 0.9157. Do the procedure again but instead of

Calculate

do

Draw

.

Comparing Two Independent Population Proportions

1. The two independent samples are simple random samples that are independent.
2. The number of successes is at least five and the number of failures is at least five for each of the samples.

Comparing two proportions, like comparing two means, is common. If two estimated proportions are different, it may be due to a difference in the populations or it may be due to chance. A hypothesis test can help determine if a difference in the estimated proportions $(P'_A - P'_B)$ reflects a difference in the population proportions.

The difference of two proportions follows an approximate normal distribution. Generally, the null hypothesis states that the two proportions are the same. That is, $H_o : p_A = p_B$. To conduct the test, we use a pooled proportion, p_c.

Equation:

The pooled proportion is calculated as follows:

$$p_c = \frac{x_A + x_B}{n_A + n_B}$$

Equation:

The distribution for the differences is:

$$P\prime_A - P\prime_B \sim N\left[0, \sqrt{p_c \cdot (1 - p_c) \cdot \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}\right]$$

Equation:

The test statistic (z-score) is:

$$z = \frac{(p\prime_A - p\prime_B) - (p_A - p_B)}{\sqrt{p_c \cdot (1 - p_c) \cdot \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

Example:
Two population proportions
Two types of medication for hives are being tested to determine if there is a **difference in the proportions of adult patient reactions. Twenty** out of a random **sample of 200** adults given medication A still had hives 30 minutes after taking the medication. **Twelve** out of another **random sample of 200 adults** given medication B still had hives 30 minutes after taking the medication. Test at a 1% level of significance.

Determining the solution

This is a test of 2 population proportions.
Exercise:

Problem: How do you know?

Solution:

The problem asks for a difference in proportions.

Let A and B be the subscripts for medication A and medication B. Then p_A and p_B are the desired population proportions.

Random Variable:
$\text{P'}_A - \text{P'}_B =$ difference in the proportions of adult patients who did not react after 30 minutes to medication A and medication B.

$H_o : p_A = p_B \qquad p_A - p_B = 0$

$H_a : p_A \neq p_B \qquad p_A - p_B \neq 0$

The words **"is a difference"** tell you the test is two-tailed.

Distribution for the test: Since this is a test of two binomial population proportions, the distribution is normal:

$p_c = \frac{x_A+x_B}{n_A+n_B} = \frac{20+12}{200+200} = 0.08 \quad 1 - p_c = 0.92$

Therefore,

$$\text{P'}_A - \text{P'}_B \sim N\left[0, \sqrt{(0.08)\cdot(0.92)\cdot\left(\frac{1}{200}+\frac{1}{200}\right)}\right]$$

$\text{P'}_A - \text{P'}_B$ follows an approximate normal distribution.

Calculate the p-value using the normal distribution: p-value = 0.1404.

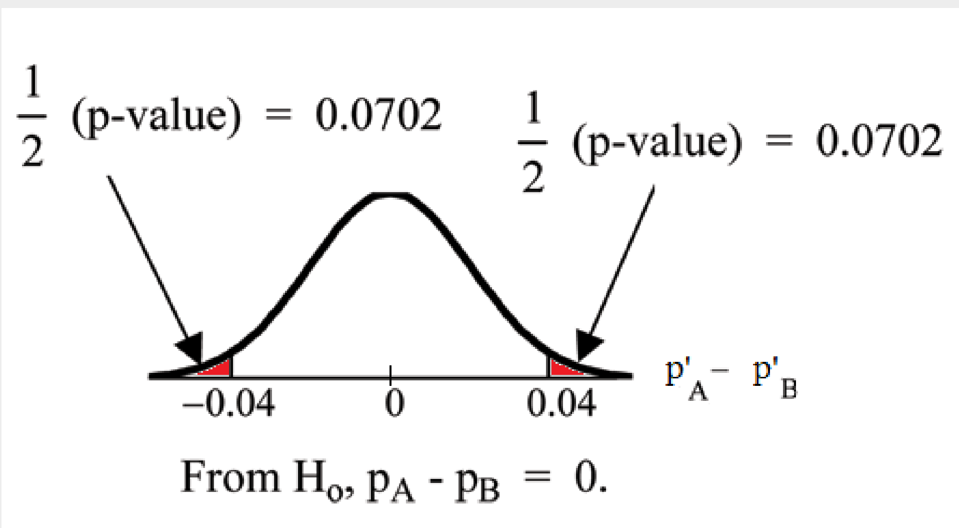
Estimated proportion for group A:

$$\text{p'}_A = \frac{x_A}{n_A} = \frac{20}{200} = 0.1$$

Estimated proportion for group B:

$$\text{p'}_B = \frac{x_B}{n_B} = \frac{12}{200} = 0.06$$

Graph:



$\text{P'}_A - \text{P'}_B = 0.1 - 0.06 = 0.04.$

Half the p-value is below -0.04 and half is above 0.04.

Compare α and the p-value: $\alpha = 0.01$ and the p-value $= 0.1404$. $\alpha <$ p-value.

Make a decision: Since $\alpha <$ p-value, do not reject H_o.

Conclusion: At a 1% level of significance, from the sample data, there is not sufficient evidence to conclude that there is a difference in the proportions of adult patients who did not react after 30 minutes to medication A and medication B.

Note:TI-83+ and TI-84: Press

`STAT`

. Arrow over to

`TESTS`

and press

`6:2-PropZTest`

. Arrow down and enter

`20`

for x1,

`200`

for n1,

`12`

for x2, and

`200`

for n2. Arrow down to

`p1`

: and arrow to

`not equal p2`

. Press

`ENTER`

. Arrow down to

`Calculate`

and press

`ENTER`

. The p-value is $p = 0.1404$ and the test statistic is 1.47. Do the procedure again but instead of

`Calculate`

do

`Draw`

.

Matched or Paired Samples
This module provides an overview of Hypothesis Testing: Matched or Paired Samples as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

1. Simple random sampling is used.
2. Sample sizes are often small.
3. Two measurements (samples) are drawn from the same pair of individuals or objects.
4. Differences are calculated from the matched or paired samples.
5. The differences form the sample that is used for the hypothesis test.
6. The matched pairs have differences that either come from a population that is normal or the number of differences is sufficiently large so the distribution of the sample mean of differences is approximately normal.

In a hypothesis test for matched or paired samples, subjects are matched in pairs and differences are calculated. The differences are the data. The population mean for the differences, μ_d, is then tested using a Student-t test for a single population mean with $n-1$ degrees of freedom where n is the number of differences.
Equation:

The test statistic (t-score) is:

$$t = \frac{x_d - \mu_d}{\left(\frac{s_d}{\sqrt{n}}\right)}$$

Example:
Matched or paired samples
A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Results for randomly selected subjects are shown in the table. The "before" value is matched to an "after" value and the differences are calculated. The differences have a normal distribution.

Subject:	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Before	6.6	6.5	9.0	10.3	11.3	8.1	6.3	11.6
After	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2.0

Exercise:

Problem:

Are the sensory measurements, on average, lower after hypnotism? Test at a 5% significance level.

Solution:

Corresponding "before" and "after" values form matched pairs. (Calculate "sfter" - "before").

After Data	Before Data	Difference
6.8	6.6	0.2
2.4	6.5	-4.1
7.4	9	-1.6
8.5	10.3	-1.8
8.1	11.3	-3.2
6.1	8.1	-2
3.4	6.3	-2.9
2	11.6	-9.6

The data **for the test** are the differences: {0.2, -4.1, -1.6, -1.8, -3.2, -2, -2.9, -9.6}

The sample mean and sample standard deviation of the differences are: $x_d = -3.13$ and $s_d = 2.91$ Verify these values.

Let μ_d be the population mean for the differences. We use the subscript d to denote "differences."

Random Variable: X_d = the mean difference of the sensory measurements
Equation:

$$H_o : \mu_d \geq 0$$

There is no improvement. (μ_d is the population mean of the differences.)
Equation:

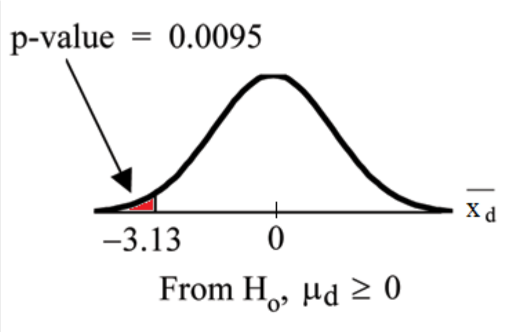
$$H_a : \mu_d < 0$$

There is improvement. The score should be lower after hypnotism so the difference ought to be negative to indicate improvement.

Distribution for the test: The distribution is a student-t with $\text{df} = n - 1 = 8 - 1 = 7$. Use t_7. **(Notice that the test is for a single population mean.)**

Calculate the p-value using the Student-t distribution: $\text{p-value} = 0.0095$

Graph:



X_d is the random variable for the differences.

The sample mean and sample standard deviation of the differences are:

$x_d = -3.13$

$s_d = 2.91$

Compare α and the p-value: $\alpha = 0.05$ and $\text{p-value} = 0.0095$. $\alpha > \text{p-value}$.

Make a decision: Since $\alpha > \text{p-value}$, reject H_o.

This means that $\mu_d < 0$ and there is improvement.

Conclusion: At a 5% level of significance, from the sample data, there is sufficient evidence to conclude that the sensory measurements, on average, are lower after hypnotism. Hypnotism appears to be effective in reducing pain.

Note: For the TI-83+ and TI-84 calculators, you can either calculate the differences ahead of time (**after - before**) and put the differences into a list or you can put the **after** data into a first list and the **before** data into a second list. Then go to a third list and arrow up to the name. Enter 1st list name - 2nd list name. The calculator will do the subtraction and you will have the differences in the third list.

Note:TI-83+ and TI-84: Use your list of differences as the data. Press

`STAT`

and arrow over to

`TESTS`

. Press

`2:T-Test`

. Arrow over to

`Data`

and press

ENTER

. Arrow down and enter

0

for μ_0, the name of the list where you put the data, and

1

for Freq:. Arrow down to

μ

: and arrow over to

<

μ_0. Press

ENTER

. Arrow down to

Calculate

and press

ENTER

. The p-value is 0.0094 and the test statistic is -3.04. Do these instructions again except arrow to

Draw

(instead of

Calculate

). Press

ENTER

.

Example:
A college football coach was interested in whether the college's strength development class increased his players' maximum lift (in pounds) on the bench press exercise. He asked 4 of his players to participate in a study. The amount of weight they could each lift was recorded before they took the strength development class. After completing the class, the amount of weight they could each lift was again measured. The data are as follows:

Weight (in pounds)	Player 1	Player 2	Player 3	Player 4
Amount of weighted lifted prior to the class	205	241	338	368
Amount of weight lifted after the class	295	252	330	360

The coach wants to know if the strength development class makes his players stronger, on average.
Exercise:

Problem:

Record the **differences** data. Calculate the differences by subtracting the amount of weight lifted prior to the class from the weight lifted after completing the class. The data for the differences are: {90, 11, -8, -8}. The differences have a normal distribution.

Using the differences data, calculate the sample mean and the sample standard deviation.

$x_d = 21.3$ $s_d = 46.7$

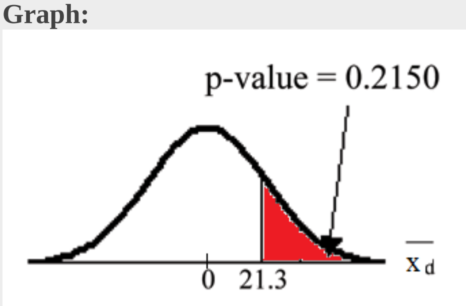
Using the difference data, this becomes a test of a single __________ (fill in the blank).

Define the random variable: $X_d =$ mean difference in the maximum lift per player.

The distribution for the hypothesis test is t_3.

$H_o : \mu_d \leq 0$ $H_a : \mu_d > 0$

Graph:



Calculate the p-value: The p-value is 0.2150

Decision: If the level of significance is 5%, the decision is to not reject the null hypothesis because $\alpha <$ p-value.

What is the conclusion?

Solution:

means; At a 5% level of significance, from the sample data, there is not sufficient evidence to conclude that the strength development class helped to make the players stronger, on average.

Example:

Seven eighth graders at Kennedy Middle School measured how far they could push the shot-put with their dominant (writing) hand and their weaker (non-writing) hand. They thought that they could push equal distances with either hand. The following data was collected.

Distance (in feet) using	**Student 1**	**Student 2**	**Student 3**	**Student 4**	**Student 5**	**Student 6**	**Student 7**
Dominant Hand	30	26	34	17	19	26	20
Weaker Hand	28	14	27	18	17	26	16

Exercise:

Problem:

Conduct a hypothesis test to determine whether the mean difference in distances between the children's dominant versus weaker hands is significant.

Note: use a t-test on the difference data. Assume the differences have a normal distribution. The random variable is the mean difference.

Note: The test statistic is 2.18 and the p-value is 0.0716.

What is your conclusion?

Solution:

H_0: μ_d equals 0; H_a: μ_d does not equal 0; Do not reject the null; At a 5% significance level, from the sample data, there is not sufficient evidence to conclude that the mean difference in distances between the children's dominant versus weaker hands is significant (there is not sufficient evidence to show that the children could push the shot-put further with their dominant hand). Alpha and the p-value are close so the test is not strong.

Summary of Types of Hypothesis Tests

Two Population Means

- Populations are independent and population standard deviations are unknown.
- Populations are independent and population standard deviations are known (not likely).

Matched or Paired Samples

- Two samples are drawn from the same set of objects.
- Samples are dependent.

Two Population Proportions

- Populations are independent.

Practice 1: Hypothesis Testing for Two Proportions
This module provides a practice of Two Population Means and Two Population Proportions as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcomes

- The student will conduct a hypothesis test of two proportions.

Given

In the recent Census, 3 percent of the U.S. population reported being two or more races. However, the percent varies tremendously from state to state. (*Source: http://www.census.gov/prod/cen2010/briefs/c2010br-02.pdf*) Suppose that two random surveys are conducted. In the first random survey, out of 1000 North Dakotans, only 9 people reported being of two or more races. In the second random survey, out of 500 Nevadans, 17 people reported being of two or more races. Conduct a hypothesis test to determine if the population percents are the same for the two states or if the percent for Nevada is statistically higher than for North Dakota.

Hypothesis Testing: Two Proportions

Exercise:

Problem: Is this a test of means or proportions?

Solution:

Proportions

Exercise:

Problem: State the null and alternative hypotheses.

- **a** H_0:

- **b** H_a:

Solution:

- **a** H_0: $p_N = p_{ND}$
- **a** H_a: $p_N > p_{ND}$

Exercise:

Problem:

Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Solution:

right-tailed

Exercise:

Problem: What is the Random Variable of interest for this test?

Exercise:

Problem: In words, define the Random Variable for this test.

Exercise:

Problem:

Which distribution (Normal or student's-t) would you use for this hypothesis test?

Solution:

Normal

Exercise:

Problem:

Explain why you chose the distribution you did for the above question.

Exercise:

Problem: Calculate the test statistic.

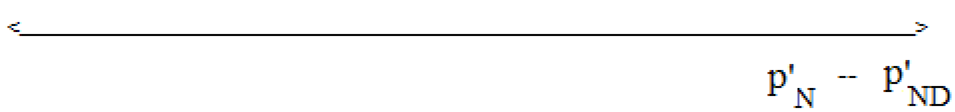
Solution:

3.50

Exercise:

Problem:

Sketch a graph of the situation. Mark the hypothesized difference and the sample difference. Shade the area corresponding to the p −value.



Exercise:

Problem: Find the p −value:

Solution:

0.0002

Exercise:

Problem: At a pre-conceived $\alpha = 0.05$, what is your:

- **a** Decision:
- **b** Reason for the decision:
- **c** Conclusion (write out in a complete sentence):

Solution:

- **a**Reject the null hypothesis

Discussion Question

Exercise:

Problem:

Does it appear that the proportion of Nevadans who are two or more races is higher than the proportion of North Dakotans? Why or why not?

Practice 2: Hypothesis Testing for Two Averages
This module provides a practice of Hypothesis Testing: Two Population Means and Two Population Proportions: as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Student Learning Outcome

- The student will conduct a hypothesis test of two means.

Given

The U.S. Center for Disease Control reports that the mean life expectancy for whites born in 1900 was 47.6 years and for nonwhites it was 33.0 years. (http://www.cdc.gov/nchs/data/dvs/nvsr53_06t12.pdf) Suppose that you randomly survey death records for people born in 1900 in a certain county. Of the 124 whites, the mean life span was 45.3 years with a standard deviation of 12.7 years. Of the 82 nonwhites, the mean life span was 34.1 years with a standard deviation of 15.6 years. Conduct a hypothesis test to see if the mean life spans in the county were the same for whites and nonwhites.

Hypothesis Testing: Two Means

Exercise:

Problem: Is this a test of means or proportions?

Solution:

Means

Exercise:

Problem: State the null and alternative hypotheses.

- **a** H_0:

- **b** H_a:

Solution:

- **a** H_0:$\mu_W = \mu_{\text{NW}}$
- **b** H_a:$\mu_W \neq \mu_{\text{NW}}$

Exercise:

Problem:

Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Solution:

two-tailed

Exercise:

Problem: What is the Random Variable of interest for this test?

Solution:

$X_W - X_{\text{NW}}$

Exercise:

Problem:

In words, define the Random Variable of interest for this test.

Solution:

The difference between the mean life spans of whites and nonwhites.

Exercise:

Problem:

Which distribution (Normal or student's-t) would you use for this hypothesis test?

Exercise:

Problem:

Explain why you chose the distribution you did for the above question.

Exercise:

Problem: Calculate the test statistic.

Solution:

5.42

Exercise:

Problem:

Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized difference and the sample difference. Shade the area corresponding to the p −value.

Exercise:

Problem: Find the p −value:

Solution:

0.0000

Exercise:

Problem: At a pre-conceived $\alpha = 0.05$, what is your:

- **a** Decision:
- **b** Reason for the decision:
- **c** Conclusion (write out in a complete sentence):

Solution:

- **a**Reject the null hypothesis

Discussion Question

Exercise:

Problem:

Does it appear that the means are the same? Why or why not?

Exercises

For questions [link] - [link], indicate which of the following choices best identifies the hypothesis test.

- **A**Independent group means, population standard deviations and/or variances known
- **B**Independent group means, population standard deviations and/or variances unknown
- **C**Matched or paired samples
- **D**Single mean
- **E**2 proportions
- **F**Single proportion

Exercise:

Problem:

A powder diet is tested on 49 people and a liquid diet is tested on 36 different people. The population standard deviations are 2 pounds and 3 pounds, respectively. Of interest is whether the liquid diet yields a higher mean weight loss than the powder diet.

Solution:

A

Exercise:

Problem:

A new chocolate bar is taste-tested on consumers. Of interest is whether the proportion of children that like the new chocolate bar is greater than the proportion of adults that like it.

Exercise:

Problem:

The mean number of English courses taken in a two–year time period by male and female college students is believed to be about the same. An experiment is conducted and data are collected from 9 males and 16 females.

Solution:

B

Exercise:

Problem:

A football league reported that the mean number of touchdowns per game was 5. A study is done to determine if the mean number of touchdowns has decreased.

Exercise:

Problem:

A study is done to determine if students in the California state university system take longer to graduate than students enrolled in private universities. 100 students from both the California state university system and private universities are surveyed. From years of research, it is known that the population standard deviations are 1.5811 years and 1 year, respectively.

Solution:

A

Exercise:

Problem:

According to a YWCA Rape Crisis Center newsletter, 75% of rape victims know their attackers. A study is done to verify this.

Exercise:

Problem:

According to a recent study, U.S. companies have an mean maternity-leave of six weeks.

Solution:

D

Exercise:

Problem:

A recent drug survey showed an increase in use of drugs and alcohol among local high school students as compared to the national percent. Suppose that a survey of 100 local youths and 100 national youths is conducted to see if the proportion of drug and alcohol use is higher locally than nationally.

Exercise:

Problem:

A new SAT study course is tested on 12 individuals. Pre-course and post-course scores are recorded. Of interest is the mean increase in SAT scores.

Solution:

C

Exercise:

Problem:

University of Michigan researchers reported in the *Journal of the National Cancer Institute* that quitting smoking is especially beneficial for those under age 49. In this American Cancer Society study, the risk (probability) of dying of lung cancer was about the same as for those who had never smoked.

Note: For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in 14. Appendix (online book version: the link is "Solution Sheets"; PDF book version: look under 14.5 Solution Sheets). Please feel free to make copies of the solution sheets. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.

Note: If you are using a student's-t distribution for a homework problem below, including for paired data, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, though.)

Exercise:

Problem:

A powder diet is tested on 49 people and a liquid diet is tested on 36 different people. Of interest is whether the liquid diet yields a higher mean weight loss than the powder diet. The powder diet group had an mean weight loss of 42 pounds with a standard deviation of 12 pounds. The liquid diet group had an mean weight loss of 45 pounds with a standard deviation of 14 pounds.

Solution:

- **d** $t_{68.44}$
- **e**-1.04
- **f**0.1519
- **h**Decision: Do not reject null

Exercise:

Problem:

The mean number of English courses taken in a two–year time period by male and female college students is believed to be about the same. An experiment is conducted and data are collected from 29 males and 16 females. The males took an average of 3 English courses with a standard deviation of 0.8. The females took an average of 4 English courses with a standard deviation of 1.0. Are the means statistically the same?

Exercise:

Problem:

A study is done to determine if students in the California state university system take longer to graduate, on average, than students enrolled in private universities. 100 students from both the California state university system and private universities are surveyed. Suppose that from years of research, it is known that the population standard deviations are 1.5811 years and 1 year, respectively. The following data are collected. The California state university system students took on average 4.5 years with a standard deviation of 0.8. The private university students took on average 4.1 years with a standard deviation of 0.3.

Solution:

Standard Normal

- **e** $z = 2.14$
- **f**0.0163
- **h**Decision: Reject null when $\alpha = 0.05$; Do not reject null when $\alpha = 0.01$

Exercise:

Problem:

A new SAT study course is tested on 12 individuals. Pre-course and post-course scores are recorded. Of interest is the mean increase in SAT scores. The following data are collected:

Pre-course score	Post-course score
1200	1300
960	920
1010	1100
840	880
1100	1070
1250	1320
860	860
1330	1370
790	770
990	1040
1110	1200

740	850

Exercise:

Problem:

A recent drug survey showed an increase in use of drugs and alcohol among local high school seniors as compared to the national percent. Suppose that a survey of 100 local seniors and 100 national seniors is conducted to see if the proportion of drug and alcohol use is higher locally than nationally. Locally, 65 seniors reported using drugs or alcohol within the past month, while 60 national seniors reported using them.

Solution:

- **e**0.73
- **f**0.2326
- **h**Decision: Do not reject null

Exercise:

Problem:

A student at a four-year college claims that mean enrollment at four–year colleges is higher than at two–year colleges in the United States. Two surveys are conducted. Of the 35 two–year colleges surveyed, the mean enrollment was 5068 with a standard deviation of 4777. Of the 35 four-year colleges surveyed, the mean enrollment was 5466 with a standard deviation of 8191. (Source: *Microsoft Bookshelf*)

Exercise:

Problem:

A study was conducted by the U.S. Army to see if applying antiperspirant to soldiers' feet for a few days before a major hike would help cut down on the number of blisters soldiers had on their feet. In the experiment, for three nights before they went on a 13-mile hike, a group of 328 West Point cadets put an alcohol-based antiperspirant on their feet. A "control group" of 339 soldiers put on a similar, but inactive, preparation on their feet. On the day of the hike, the temperature reached 83° F. At the end of the hike, 21% of the soldiers who had used the antiperspirant and 48% of the control group had developed foot blisters. Conduct a hypothesis test to see if the proportion of soldiers using the antiperspirant was significantly lower than the control group. (Source: U.S. Army study reported in *Journal of the American Academy of Dermatologists*)

Solution:

- **e**-7.33
- **f**0
- **h**Decision: Reject null

Exercise:

Problem:

We are interested in whether the proportions of female suicide victims for ages 15 to 24 are the same for the white and the black races in the United States. We randomly pick one year, 1992, to compare the races. The number of suicides estimated in the United States in 1992 for white females is 4930. 580 were aged 15 to 24. The estimate for black females is 330. 40 were aged 15 to 24. We will let female suicide victims be our population. (Source*: the National Center for Health Statistics, U.S. Dept. of Health and Human Services*)

Exercise:

Problem:

At Rachel's 11th birthday party, 8 girls were timed to see how long (in seconds) they could hold their breath in a relaxed position. After a two-minute rest, they timed themselves while jumping. The girls thought that the mean difference between their jumping and relaxed times would be 0. Test their hypothesis.

Relaxed time (seconds)	Jumping time (seconds)
26	21
47	40
30	28
22	21
23	25
45	43
37	35
29	32

Solution:

- **d** t_7
- **e**-1.51
- **f**0.1755
- **h**Decision: Do not reject null

Exercise:

Problem:

Elizabeth Mjelde, an art history professor, was interested in whether the value from the Golden Ratio formula, $\left(\frac{larger+\text{smaller dimension}}{l\text{arger dimension}}\right)$was the same in the Whitney Exhibit for works from 1900 – 1919 as for works from 1920 – 1942. 37 early works were sampled. They averaged 1.74 with a standard deviation of 0.11. 65 of the later works were sampled. They averaged 1.746 with a standard deviation of 0.1064. Do you think that there is a significant difference in the Golden Ratio calculation? (Source: *data from Whitney Exhibit on loan to San Jose Museum of Art*)

Exercise:

Problem:

One of the questions in a study of marital satisfaction of dual–career couples was to rate the statement, “I’m pleased with the way we divide the responsibilities for childcare.” The ratings went from 1 (strongly agree) to 5 (strongly disagree). Below are ten of the paired responses for husbands and wives. Conduct a hypothesis test to see if the mean difference in the husband’s versus the wife’s satisfaction level is negative (meaning that, within the partnership, the husband is happier than the wife).

Wife’s score	2	2	3	3	4	2	1	1	2	4
Husband’s score	2	2	1	3	2	1	1	1	2	4

Solution:

- **d** t_9
- **e** $t = -1.86$
- **f**0.0479
- **h**Decision: Reject null, but run another test

Exercise:

Problem:

Ten individuals went on a low–fat diet for 12 weeks to lower their cholesterol. Evaluate the data below. Do you think that their cholesterol levels were significantly lowered?

Starting cholesterol level	Ending cholesterol level
140	140
220	230
110	120
240	220
200	190
180	150
190	200
360	300
280	300
260	240

Exercise:

Problem:

Mean entry level salaries for college graduates with mechanical engineering degrees and electrical engineering degrees are believed to be approximately the same. (Source: *http:// www.graduatingengineer.com*). A recruiting office thinks that the mean mechanical engineering salary is actually lower than the mean electrical engineering salary. The recruiting office randomly surveys 50 entry level mechanical engineers and 60 entry level electrical engineers. Their mean salaries were $46,100 and $46,700, respectively. Their standard deviations were $3450 and $4210, respectively. Conduct a hypothesis test to determine if you agree that the mean entry level mechanical engineering salary is lower than the mean entry level electrical engineering salary.

Solution:

- **d** t_{108}

- **e** $t = -0.82$
- **f**0.2066
- **h**Decision: Do not reject null

Exercise:

Problem:

A recent year was randomly picked from 1985 to the present. In that year, there were 2051 Hispanic students at Cabrillo College out of a total of 12,328 students. At Lake Tahoe College, there were 321 Hispanic students out of a total of 2441 students. In general, do you think that the percent of Hispanic students at the two colleges is basically the same or different? (Source: *Chancellor's Office, California Community Colleges, November 1994*)

Exercise:

Problem:

Eight runners were convinced that the mean difference in their individual times for running one mile versus race walking one mile was at most 2 minutes. Below are their times. Do you agree that the mean difference is at most 2 minutes?

Running time (minutes)	Race walking time (minutes)
5.1	7.3
5.6	9.2
6.2	10.4
4.8	6.9
7.1	8.9
4.2	9.5

6.1	9.4
4.4	7.9

Solution:

- **d** t_7
- **e** $t = 2.9850$
- **f**0.0102
- **h**Decision: Reject null; There is sufficient evidence to conclude that the mean difference is more than 2 minutes.

Exercise:

Problem:

Marketing companies have collected data implying that teenage girls use more ring tones on their cellular phones than teenage boys do. In one particular study of 40 randomly chosen teenage girls and boys (20 of each) with cellular phones, the mean number of ring tones for the girls was 3.2 with a standard deviation of 1.5. The mean for the boys was 1.7 with a standard deviation of 0.8. Conduct a hypothesis test to determine if the means are approximately the same or if the girls' mean is higher than the boys' mean.

Exercise:

Problem:

While her husband spent 2½ hours picking out new speakers, a statistician decided to determine whether the percent of men who enjoy shopping for electronic equipment is higher than the percent of women who enjoy shopping for electronic equipment. The population was Saturday afternoon shoppers. Out of 67 men, 24 said they enjoyed the activity. 8 of the 24 women surveyed claimed to enjoy the activity. Interpret the results of the survey.

Solution:

- **e**0.22
- **f**0.4133
- **h**Decision: Do not reject null

Exercise:

Problem:

We are interested in whether children's educational computer software costs less, on average, than children's entertainment software. 36 educational software titles were randomly picked from a catalog. The mean cost was $31.14 with a standard deviation of $4.69. 35 entertainment software titles were randomly picked from the same catalog. The mean cost was $33.86 with a standard deviation of $10.87. Decide whether children's educational software costs less, on average, than children's entertainment software. (Source: *Educational Resources*, December catalog)

Exercise:

Problem:

Parents of teenage boys often complain that auto insurance costs more, on average, for teenage boys than for teenage girls. A group of concerned parents examines a random sample of insurance bills. The mean annual cost for 36 teenage boys was $679. For 23 teenage girls, it was $559. From past years, it is known that the population standard deviation for each group is $180. Determine whether or not you believe that the mean cost for auto insurance for teenage boys is greater than that for teenage girls.

Solution:

- **e** $z = 2.50$
- **f**0.0063
- **h**Decision: Reject null

Exercise:

Problem:

A group of transfer bound students wondered if they will spend the same mean amount on texts and supplies each year at their four-year university as they have at their community college. They conducted a random survey of 54 students at their community college and 66 students at their local four-year university. The sample means were $947 and $1011, respectively. The population standard deviations are known to be $254 and $87, respectively. Conduct a hypothesis test to determine if the means are statistically the same.

Exercise:

Problem:

Joan Nguyen recently claimed that the proportion of college–age males with at least one pierced ear is as high as the proportion of college–age females. She conducted a survey in her classes. Out of 107 males, 20 had at least one pierced ear. Out of 92 females, 47 had at least one pierced ear. Do you believe that the proportion of males has reached the proportion of females?

Solution:

- **e**-4.82
- **f**0
- **h**Decision: Reject null

Exercise:

Problem:

Some manufacturers claim that non-hybrid sedan cars have a lower mean miles per gallon (mpg) than hybrid ones. Suppose that consumers test 21 hybrid sedans and get a mean of 31 mpg with a standard deviation of 7 mpg. Thirty-one non-hybrid sedans get a mean of 22 mpg with a standard deviation of 4 mpg. Suppose that the population standard deviations are known to be 6 and 3, respectively. Conduct a hypothesis test to the manufacturers claim.

Questions [link] – [link] refer to the Terri Vogel's data set (see Table of Contents).

Exercise:

Problem:

Using the data from Lap 1 only, conduct a hypothesis test to determine if the mean time for completing a lap in races is the same as it is in practices.

Solution:

- **d** $t_{20.32}$
- **e**-4.70
- **f**0.0001
- **h**Decision: Reject null

Exercise:

Problem: Repeat the test in [link], but use Lap 5 data this time.

Exercise:

Problem:

Repeat the test in [link], but this time combine the data from Laps 1 and 5.

Solution:

- **d** $t_{40.94}$
- **e**-5.08
- **f**0
- **h**Decision: Reject null

Exercise:

Problem:

In 2 – 3 complete sentences, explain in detail how you might use Terri Vogel's data to answer the following question. "Does Terri Vogel drive faster in races than she does in practices?"

Exercise:

Problem:

Is the proportion of race laps Terri completes slower than 130 seconds less than the proportion of practice laps she completes slower than 135 seconds?

Solution:

- **e**-0.9223
- **f**0.1782
- **h**Decision: Do not reject null

Exercise:

Problem:"To Breakfast or Not to Breakfast?" by Richard Ayore

In the American society, birthdays are one of those days that everyone looks forward to. People of different ages and peer groups gather to mark the 18th,

20th, … birthdays. During this time, one looks back to see what he or she had achieved for the past year, and also focuses ahead for more to come.

If, by any chance, I am invited to one of these parties, my experience is always different. Instead of dancing around with my friends while the music is booming, I get carried away by memories of my family back home in Kenya. I remember the good times I had with my brothers and sister while we did our daily routine.

Every morning, I remember we went to the shamba (garden) to weed our crops. I remember one day arguing with my brother as to why he always remained behind just to join us an hour later. In his defense, he said that he preferred waiting for breakfast before he came to weed. He said, “This is why I always work more hours than you guys!”

And so, to prove his wrong or right, we decided to give it a try. One day we went to work as usual without breakfast, and recorded the time we could work before getting tired and stopping. On the next day, we all ate breakfast before going to work. We recorded how long we worked again before getting tired and stopping. Of interest was our mean increase in work time. Though not sure, my brother insisted that it is more than two hours. Using the data below, solve our problem.

Work hours with breakfast	Work hours without breakfast
8	6
7	5
9	5
5	4
9	7
8	7

10	7
7	5
6	6
9	5

Try these multiple choice questions.

For questions [link] – [link], use the following information.

A new AIDS prevention drugs was tried on a group of 224 HIV positive patients. Forty-five (45) patients developed AIDS after four years. In a control group of 224 HIV positive patients, 68 developed AIDS after four years. We want to test whether the method of treatment reduces the proportion of patients that develop AIDS after four years or if the proportions of the treated group and the untreated group stay the same.

Let the subscript t= treated patient and ut= untreated patient.

Exercise:

Problem: The appropriate hypotheses are:

- **A** H_o:$p_t < p_{\text{ut}}$ and H_a:$p_t \geq p_{\text{ut}}$
- **B**H_o:$p_t \leq p_{\text{ut}}$ and H_a:$p_t > p_{\text{ut}}$
- **C** H_o:$p_t = p_{\text{ut}}$ and H_a:$p_t \neq p_{\text{ut}}$
- **D** H_o:$p_t = p_{\text{ut}}$ and H_a:$p_t < p_{\text{ut}}$

Solution:

D

Exercise:

Problem: If the p -value is 0.0062 what is the conclusion (use $\alpha = 0.05$)?

- **A**The method has no effect.

- **B**There is sufficient evidence to conclude that the method reduces the proportion of HIV positive patients that develop AIDS after four years.
- **C**There is sufficient evidence to conclude that the method increases the proportion of HIV positive patients that develop AIDS after four years.
- **D**There is insufficient evidence to conclude that the method reduces the proportion of HIV positive patients that develop AIDS after four years.

Solution:

B

Exercise:

Problem:

Lesley E. Tan investigated the relationship between left-handedness and right-handedness and motor competence in preschool children. Random samples of 41 left-handers and 41 right-handers were given several tests of motor skills to determine if there is evidence of a difference between the children based on this experiment. The experiment produced the means and standard deviations shown below. Determine the appropriate test and best distribution to use for that test.

	Left-handed	Right-handed
Sample size	41	41
Sample mean	97.5	98.1
Sample standard deviation	17.5	19.2

- **A**Two independent means, normal distribution
- **B**Two independent means, student's-t distribution
- **C**Matched or paired samples, student's-t distribution
- **D**Two population proportions, normal distribution

Solution:

B

For questions [link] – [link], use the following information.

An experiment is conducted to show that blood pressure can be consciously reduced in people trained in a "biofeedback exercise program." Six (6) subjects were randomly selected and the blood pressure measurements were recorded before and after the training. The difference between blood pressures was calculated $(\text{after} - \text{before})$ producing the following results: $x_d = -10.2$ $s_d = 8.4$. Using the data, test the hypothesis that the blood pressure has decreased after the training,

Exercise:

Problem: The distribution for the test is

- **A** t_5
- **B** t_6
- **C** $N(-10.2{,}8.4)$
- **D** $N(-10.2{,}\frac{8.4}{\sqrt{6}})$

Solution:

A

Exercise:

Problem: If $\alpha = 0.05$, the p-value and the conclusion are

- **A**0.0014; There is sufficient evidence to conclude that the blood pressure decreased after the training
- **B**0.0014; There is sufficient evidence to conclude that the blood pressure increased after the training
- **C**0.0155; There is sufficient evidence to conclude that the blood pressure decreased after the training
- **D**0.0155; There is sufficient evidence to conclude that the blood pressure increased after the training

---

Solution:

C

For questions [link]– [link], use the following information.

The Eastern and Western Major League Soccer conferences have a new Reserve Division that allows new players to develop their skills. Data for a randomly picked date showed the following annual goals.

Western	Eastern
Los Angeles 9	D.C. United 9
FC Dallas 3	Chicago 8
Chivas USA 4	Columbus 7
Real Salt Lake 3	New England 6
Colorado 4	MetroStars 5
San Jose 4	Kansas City 3

Conduct a hypothesis test to determine if the Western Reserve Division teams score, on average, fewer goals than the Eastern Reserve Division teams. Subscripts: **1** Western Reserve Division (**W**); **2** Eastern Reserve Division (**E**)

Exercise:

Problem: The **exact** distribution for the hypothesis test is:

- **A**The normal distribution.
- **B**The student's-t distribution.
- **C**The uniform distribution.
- **D**The exponential distribution.

Solution:

B

Exercise:

Problem: If the level of significance is 0.05, the conclusion is:

- **A**There is sufficient evidence to conclude that the **W** Division teams score, on average, fewer goals than the **E** teams.
- **B**There is insufficient evidence to conclude that the **W** Division teams score, on average, more goals than the **E** teams.
- **C**There is insufficient evidence to conclude that the **W** teams score, on average, fewer goals than the **E** teams score.
- **D**Unable to determine.

Solution:

C

Questions [link] – [link] refer to the following.

Neuroinvasive West Nile virus refers to a severe disease that affects a person's nervous system . It is spread by the Culex species of mosquito. In the United States in 2010 there were 629 reported cases of neuroinvasive West Nile virus out of a total of 1021 reported cases and there were 486 neuroinvasive reported cases out of a total of 712 cases reported in 2011. Is the 2011 proportion of neuroinvasive West Nile virus cases more than the 2010 proportion of neuroinvasive West Nile virus cases? Using a 1% level of significance, conduct an appropriate hypothesis test. (Source: *http:// http://www.cdc.gov/ncidod/dvbid/westnile/index.htm*)

- "2011" subscript: 2011 group.
- "2010" subscript: 2010 group

Exercise:

Problem: This is:

- **A**a test of two proportions
- **B**a test of two independent means

- **C**a test of a single mean
- **D**a test of matched pairs.

Solution:

A

Exercise:

Problem: An appropriate null hypothesis is:

- **A** $p_{2011} \leq p_{2010}$
- **B** $p_{2011} \geq p_{2010}$
- **C** $\mu_{2011} \leq \mu_{2010}$
- **D** $p_{2011} > p_{2010}$

Solution:

A

Exercise:

Problem:

The p-value is 0.0022. At a 1% level of significance, the appropriate conclusion is

- **A**There is sufficient evidence to conclude that the proportion of people in the United States in 2011 that got neuroinvasive West Nile disease is less than the proportion of people in the United States in 2010 that got neuroinvasive West Nile disease.
- **B**There is insufficient evidence to conclude that the proportion of people in the United States in 2011 that got neuroinvasive West Nile disease is more than the proportion of people in the United States in 2010 that got neuroinvasive West Nile disease.
- **C**There is insufficient evidence to conclude that the proportion of people in the United States in 2011 that got neuroinvasive West Nile disease is less than the proportion of people in the United States in 2010 that got neuroinvasive West Nile disease.
- **D**There is sufficient evidence to conclude that the proportion of people in the United States in 2011 that got neuroinvasive West Nile disease is

more than the proportion of people in the United States in 2010 that got neuroinvasive West Nile disease.

Solution:

D

Questions [link] and [link] refer to the following:

A golf instructor is interested in determining if her new technique for improving players' golf scores is effective. She takes four (4) new students. She records their 18-holes scores before learning the technique and then after having taken her class. She conducts a hypothesis test. The data are as follows.

	Player 1	Player 2	Player 3	Player 4
Mean score before class	83	78	93	87
Mean score after class	80	80	86	86

Exercise:

Problem: This is:

- **A**a test of two independent means
- **B**a test of two proportions
- **C**a test of a single proportion
- **D**a test of matched pairs.

Solution:

D

Exercise:

Problem: The correct decision is:

- **A**Reject H_o
- **B**Do not reject H_o

Solution:

B

Questions [link] and [link] refer to the following:

Suppose a statistics instructor believes that there is no significant difference between the mean class scores of statistics day students on Exam 2 and statistics night students on Exam 2. She takes random samples from each of the populations. The mean and standard deviation for 35 statistics day students were 75.86 and 16.91. The mean and standard deviation for 37 statistics night students were 75.41 and 19.73. The "day" subscript refers to the statistics day students. The "night" subscript refers to the statistics night students.

Exercise:

Problem: An appropriate alternate hypothesis for the hypothesis test is:

- **A** $\mu_{\text{day}} > \mu_{\text{night}}$
- **B** $\mu_{\text{day}} < \mu_{\text{night}}$
- **C** $\mu_{\text{day}} = \mu_{\text{night}}$
- **D** $\mu_{\text{day}} \neq \mu_{\text{night}}$

Solution:

D

Exercise:

Problem: A concluding statement is:

- **A**There is sufficient evidence to conclude that statistics night students mean on Exam 2 is better than the statistics day students mean on Exam

2.

- **B**There is insufficient evidence to conclude that the statistics day students mean on Exam 2 is better than the statistics night students mean on Exam 2.
- **C**There is insufficient evidence to conclude that there is a significant difference between the means of the statistics day students and night students on Exam 2.
- **D**There is sufficient evidence to conclude that there is a significant difference between the means of the statistics day students and night students on Exam 2.

Solution:

C

Review

The next three questions refer to the following information:

In a survey at Kirkwood Ski Resort the following information was recorded:

	0 – 10	11 - 20	21 - 40	40+
Ski	10	12	30	8
Snowboard	6	17	12	5

Sport Participation by Age

Suppose that one person from of the above was randomly selected.

Exercise:

Problem:

Find the probability that the person was a skier or was age 11 – 20.

Solution:

$\frac{77}{100}$

Exercise:

Problem:

Find the probability that the person was a snowboarder given he/she was age 21 – 40.

Solution:

$\frac{12}{42}$

Exercise:

Problem: Explain which of the following are true and which are false.

- **a**Sport and Age are independent events.
- **b**Ski and age 11 – 20 are mutually exclusive events.
- **c** $P(\text{Ski and age } 21-40) < P(\text{Ski} \mid \text{age } 21-40)$
- **d** $P(\text{Snowboard or age } 0-10) < P(\text{Snowboard} \mid \text{age } 0-10)$

Solution:

- **a**False
- **b**False
- **c**True
- **d**False

Exercise:

Problem:

The average length of time a person with a broken leg wears a cast is approximately 6 weeks. The standard deviation is about 3 weeks. Thirty people who had recently healed from broken legs were interviewed. State the distribution that most accurately reflects total time to heal for the thirty people.

Solution:

$N(180{,}16.43)$

Exercise:

Problem:

The distribution for X is Uniform. What can we say for certain about the distribution for X when $n = 1$?

- **A**The distribution for X is still Uniform with the same mean and standard dev. as the distribution for X.
- **B**The distribution for $X$is Normal with the different mean and a different standard deviation as the distribution for X.
- **C**The distribution for X is Normal with the same mean but a larger standard deviation than the distribution for X.
- **D**The distribution for X is Normal with the same mean but a smaller standard deviation than the distribution for X.

Solution:

A

Exercise:

Problem:

The distribution for X is uniform. What can we say for certain about the distribution for $\sum X$ when $n = 50$?

- **A**The distribution for $\sum X$is still uniform with the same mean and standard deviation as the distribution for X.
- **B**The distribution for $\sum X$ is Normal with the same mean but a larger standard deviation as the distribution for X.
- **C**The distribution for $\sum X$ is Normal with a larger mean and a larger standard deviation than the distribution for X.
- **D**The distribution for $\sum X$ is Normal with the same mean but a smaller standard deviation than the distribution for X.

Solution:

C

The next three questions refer to the following information:

A group of students measured the lengths of all the carrots in a five-pound bag of baby carrots. They calculated the average length of baby carrots to be 2.0 inches with a standard deviation of 0.25 inches. Suppose we randomly survey 16 five-pound bags of baby carrots.

Exercise:

Problem:

State the approximate distribution for X, the distribution for the average lengths of baby carrots in 16 five-pound bags. X ~

Solution:

$N(2, \frac{.25}{\sqrt{16}})$

Exercise:

Problem:

Explain why we cannot find the probability that one individual randomly chosen carrot is greater than 2.25 inches.

Exercise:

Problem: Find the probability that x is between 2 and 2.25 inches.

Solution:

0.5000

The next three questions refer to the following information:

At the beginning of the term, the amount of time a student waits in line at the campus store is normally distributed with a mean of 5 minutes and a standard deviation of 2 minutes.

Exercise:

Problem: Find the 90th percentile of waiting time in minutes.

Solution:

7.6

Exercise:

Problem: Find the median waiting time for one student.

Solution:

5

Exercise:

Problem:

Find the probability that the average waiting time for 40 students is at least 4.5 minutes.

Solution:

0.9431

Symbols and their Meanings

This module defines symbols used throughout the Collaborative Statistics textbook.

Chapter (1st used)	Symbol	Spoken	Meaning
Sampling and Data	$\surd$	The square root of	same
Sampling and Data	π	Pi	3.14159… (a specific number)
Descriptive Statistics	Q1	Quartile one	the first quartile
Descriptive Statistics	Q2	Quartile two	the second quartile
Descriptive Statistics	Q3	Quartile three	the third quartile
Descriptive Statistics	IQR	inter-quartile range	Q3-Q1=IQR
Descriptive Statistics	x	x-bar	sample mean
Descriptive Statistics	μ	mu	population mean

Chapter (1st used)	**Symbol**	**Spoken**	**Meaning**
Descriptive Statistics	s s_x sx	s	sample standard deviation
Descriptive Statistics	s^2 s_x^2	s-squared	sample variance
Descriptive Statistics	σ σ_x σx	sigma	population standard deviation
Descriptive Statistics	σ^2 σ_x^2	sigma-squared	population variance
Descriptive Statistics	Σ	capital sigma	sum
Probability Topics	$\{\}$	brackets	set notation
Probability Topics	S	S	sample space
Probability Topics	A	Event A	event A
Probability Topics	$P(A)$	probability of A	probability of A occurring

Chapter (1st used)	**Symbol**	**Spoken**	**Meaning**
Probability Topics	$P(A \mid B)$	probability of A given B	prob. of A occurring given B has occurred
Probability Topics	$P(A \text{ or } B)$	prob. of A or B	prob. of A or B or both occurring
Probability Topics	$P(A \text{ and } B)$	prob. of A and B	prob. of both A and B occurring (same time)
Probability Topics	A'	A-prime, complement of A	complement of A, not A
Probability Topics	$P(\text{A'})$	prob. of complement of A	same
Probability Topics	G_1	green on first pick	same
Probability Topics	$P(G_1)$	prob. of green on first pick	same
Discrete Random Variables	PDF	prob. distribution function	same

Chapter (1st used)	**Symbol**	**Spoken**	**Meaning**
Discrete Random Variables	X	X	the random variable X
Discrete Random Variables	X~	the distribution of X	same
Discrete Random Variables	B	binomial distribution	same
Discrete Random Variables	G	geometric distribution	same
Discrete Random Variables	H	hypergeometric dist.	same
Discrete Random Variables	P	Poisson dist.	same
Discrete Random Variables	λ	Lambda	average of Poisson distribution
Discrete Random Variables	$\geq$	greater than or equal to	same

Chapter (1st used)	Symbol	Spoken	Meaning
Discrete Random Variables	$\leq$	less than or equal to	same
Discrete Random Variables	$=$	equal to	same
Discrete Random Variables	$\neq$	not equal to	same
Continuous Random Variables	$f\left(x\right)$	f of x	function of x
Continuous Random Variables	pdf	prob. density function	same
Continuous Random Variables	U	uniform distribution	same
Continuous Random Variables	Exp	exponential distribution	same
Continuous Random Variables	k	k	critical value

Chapter (1st used)	**Symbol**	**Spoken**	**Meaning**
Continuous Random Variables	$f(x) =$	f of x equals	same
Continuous Random Variables	m	m	decay rate (for exp. dist.)
The Normal Distribution	N	normal distribution	same
The Normal Distribution	z	z-score	same
The Normal Distribution	Z	standard normal dist.	same
The Central Limit Theorem	CLT	Central Limit Theorem	same
The Central Limit Theorem	X	X-bar	the random variable X-bar
The Central Limit Theorem	μ_x	mean of X	the average of X
The Central Limit Theorem	μ_x	mean of X-bar	the average of X-bar

Chapter (1st used)	**Symbol**	**Spoken**	**Meaning**
The Central Limit Theorem	σ_x	standard deviation of X	same
The Central Limit Theorem	σ_x	standard deviation of X-bar	same
The Central Limit Theorem	ΣX	sum of X	same
The Central Limit Theorem	Σx	sum of x	same
Confidence Intervals	CL	confidence level	same
Confidence Intervals	CI	confidence interval	same
Confidence Intervals	EBM	error bound for a mean	same
Confidence Intervals	EBP	error bound for a proportion	same
Confidence Intervals	t	student-t distribution	same
Confidence Intervals	df	degrees of freedom	same

Chapter (1st used)	Symbol	Spoken	Meaning
Confidence Intervals	$t_{\frac{\alpha}{2}}$	student-t with a/2 area in right tail	same
Confidence Intervals	p’ $\widehat{p}$	p-prime; p-hat	sample proportion of success
Confidence Intervals	q’ $\hat{q}$	q-prime; q-hat	sample proportion of failure
Hypothesis Testing	H_0	H-naught, H-sub 0	null hypothesis
Hypothesis Testing	H_a	H-a, H-sub a	alternate hypothesis
Hypothesis Testing	H_1	H-1, H-sub 1	alternate hypothesis
Hypothesis Testing	α	alpha	probability of Type I error
Hypothesis Testing	β	beta	probability of Type II error
Hypothesis Testing	$X1 - X2$	X1-bar minus X2-bar	difference in sample means

Chapter (1st used)	**Symbol**	**Spoken**	**Meaning**
	$\mu_1 - \mu_2$	mu-1 minus mu-2	difference in population means
	$P'_1 - P'_2$	P1-prime minus P2-prime	difference in sample proportions
	$p_1 - p_2$	p1 minus p2	difference in population proportions
Chi-Square Distribution	X^2	Ky-square	Chi-square
	O	Observed	Observed frequency
	E	Expected	Expected frequency
Linear Regression and Correlation	$y = a + \text{bx}$	y equals a plus b-x	equation of a line
	$\hat{y}$	y-hat	estimated value of y
	r	correlation coefficient	same

Chapter (1st used)	**Symbol**	**Spoken**	**Meaning**
	ε	error	same
	SSE	Sum of Squared Errors	same
	$1.9s$	1.9 times s	cut-off value for outliers
F-Distribution and ANOVA	F	F-ratio	F ratio

Symbols and their Meanings

Formulas

This module provides an overview of Statistics Formulas used as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

Formula

Factorial

$$n! = n(n-1)(n-2)\ldots(1)$$

$$0! = 1$$

Formula

Combinations

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Formula

Binomial Distribution

$$X\text{~}B(n,p)$$

$P\left(X = x\right) = \binom{n}{x}p^{x}q^{n-x}$, for $x = 0,1,2,\ldots,n$

Formula

Geometric Distribution

$$X\text{~}G(p)$$

$P\left(X = x\right) = q^{x-1}p$, for $x = 1,2,3,\ldots$

Formula

Hypergeometric Distribution

X~H (r,b,n)

$$P\left(X = x\right) = \left(\frac{\binom{r}{x}\binom{b}{n-x}}{\binom{r+b}{n}}\right)$$

Formula

Poisson Distribution

X~P (μ)

$P\left(X = x\right) = \frac{\mu^{x}e^{-\mu}}{x!}$

Formula

Uniform Distribution

X~$U(a,b)$

$f\left(X\right) = \frac{1}{b - a}, a < x < b$

Formula

Exponential Distribution

X~ $\text{Exp}(m)$

$f(x) = me^{-\text{mx}}, m > 0, x \geq 0$

Formula

Normal Distribution

X~$N\left(\mu,\sigma^{2}\right)$

$f\left(x\right) = \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x - \mu)^{2}}{2\sigma^{2}}}, \quad -\infty < x < \infty$

Formula

Gamma Function

$\Gamma\left(z\right) = \int_{0}^{\infty}x^{z - 1}e^{-x}\,\text{dx}\ z > 0$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$\Gamma(m + 1) = m!$ for m, a nonnegative integer

otherwise: $\Gamma(a + 1) = \text{a}\Gamma(a)$

Formula

Student-t Distribution

X~t_{df}

$$f\left(x\right) = \frac{\left(1+\frac{x^2}{n}\right)^{\frac{-(n+1)}{2}}\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)}$$

$$X = \frac{Z}{\sqrt{\frac{Y}{n}}}$$

Z~$N(0,1)$, Y~X_{df}^2 ,n = degrees of freedom

Formula

Chi-Square Distribution

X~X_{df}^2

$f\left(x\right) = \frac{x^{\frac{n-2}{2}}e^{\frac{-x}{2}}}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)}$, $x > 0$, n = positive integer and degrees of freedom

Formula

F Distribution

X~$F_{\mathrm{df}(n),\mathrm{df}(d)}$

$\mathrm{df}(n) =$degrees of freedom for the numerator

$\mathrm{df}(d) =$degrees of freedom for the denominator

$$f\left(x\right) = \frac{\Gamma\left(\frac{u+v}{2}\right)}{\Gamma\left(\frac{u}{2}\right)\Gamma\left(\frac{v}{2}\right)}\left(\frac{u}{v}\right)^{\frac{u}{2}}x^{\left(\frac{u}{2}-1\right)}\left[1 + \left(\frac{u}{v}\right)x^{-.5(u+v)}\right]$$

$X = \frac{Y_u}{W_v}$, Y, W are chi-square

Notes for the TI-83, 83+, 84 Calculator
Notes and tips for using TI-83, TI-83+, and TI-84 calculators for statistics applications.

Quick Tips

Legend

- 

  represents a button press
- [ ] represents yellow command or green letter behind a key
- < > represents items on the screen

To adjust the contrast
Press

, then hold

to increase the contrast or

to decrease the contrast.

To capitalize letters and words
Press

to get one capital letter, or press

, then

ALPHA

to set all button presses to capital letters. You can return to the top-level button values by pressing

ALPHA

again.

To correct a mistake
If you hit a wrong button, just hit

CLEAR

and start again.

To write in scientific notation
Numbers in scientific notation are expressed on the TI-83, 83+, and 84 using E notation, such that...

- 4.321 E 4 = 4.321×10^4
- 4.321 E -4 = 4.321×10^{-4}

To transfer programs or equations from one calculator to another:
Both calculators: Insert your respective end of the link cable cable and press

2nd

, then `[LINK]`.
Calculator receiving information:

Use the arrows to navigate to and select `<RECEIVE>`
Press
ENTER

Calculator sending information:

Press appropriate number or letter.

Use up and down arrows to access the appropriate item.
Press ENTER to select item to transfer.

Press right arrow to navigate to and select <TRANSMIT>.
Press ENTER

> **Note:** ERROR 35 LINK generally means that the cables have not been inserted far enough.

Both calculators: Insert your respective end of the link cable cable Both calculators: press

2nd

, then [QUIT] To exit when done.

Manipulating One-Variable Statistics

> **Note:** These directions are for entering data with the built-in statistical program.

Data	Frequency
-2	10

Data	Frequency
-1	3
0	4
1	5
3	8

Sample DataWe are manipulating 1-variable statistics.

To begin:

Turn on the calculator.

ON

Access statistics mode.

STAT

Select `<4:ClrList>` to clear data from lists, if desired.

4

,

ENTER

Enter list `[L1]` to be cleared.

2nd

, `[L1]` ,

ENTER

Display last instruction.

2nd

, [ENTRY]

Continue clearing remaining lists in the same fashion, if desired.

◀

,

2nd

, [L2] ,

ENTER

Access statistics mode.

STAT

Select <1:Edit . . .>

ENTER

Enter data. Data values go into [L1]. (You may need to arrow over to [L1])

- Type in a data value and enter it. (For negative numbers, use the negate (-) key at the bottom of the keypad)

(−)

,

9

,

ENTER

- Continue in the same manner until all data values are entered.

In [L2], enter the frequencies for each data value in [L1].

- Type in a frequency and enter it. (If a data value appears only once, the frequency is "1")

4

,

ENTER

- Continue in the same manner until all data values are entered.

Access statistics mode.

STAT

Navigate to <CALC>
Access <1:1-var Stats>

ENTER

Indicate that the data is in [L1]...

2nd

, [L1] ,

,

...and indicate that the frequencies are in [L2].

2nd

, [L2] ,

ENTER

The statistics should be displayed. You may arrow down to get remaining statistics. Repeat as necessary.

Drawing Histograms

Note: We will assume that the data is already entered

We will construct 2 histograms with the built-in STATPLOT application. The first way will use the default ZOOM. The second way will involve customizing a new graph.

Access graphing mode.

2nd

, [STAT PLOT]

Select <1:plot 1> To access plotting - first graph.

ENTER

Use the arrows navigate go to <ON> to turn on Plot 1.

<ON> .

,

ENTER

Use the arrows to go to the histogram picture and select the histogram. ENTER

Use the arrows to navigate to `<Xlist>`
If "L1" is not selected, select it.

2nd

, `[L1]` ,

ENTER

Use the arrows to navigate to `<Freq>`.
Assign the frequencies to `[L2]`.

2nd

, `[L2]` ,

ENTER

Go back to access other graphs.

2nd

, `[STAT PLOT]`

Use the arrows to turn off the remaining plots.
Be sure to deselect or clear all equations before graphing.

To deselect equations:

Access the list of equations.

Y=

Select each equal sign (=).

▼

▶

ENTER

Continue, until all equations are deselected.

To clear equations:

Access the list of equations.

Y=

Use the arrow keys to navigate to the right of each equal sign (=) and clear them.

▼

▶

CLEAR

Repeat until all equations are deleted.

To draw default histogram:

Access the ZOOM menu.

ZOOM

Select `<9:ZoomStat>`

9

The histogram will show with a window automatically set.

To draw custom histogram:

Access WINDOW to set the graph parameters.

- $X_{\text{min}} = -2.5$
- $X_{\text{max}} = 3.5$
- $X_{\text{scl}} = 1$ (width of bars)
- $Y_{\text{min}} = 0$
- $Y_{\text{max}} = 10$
- $Y_{\text{scl}} = 1$ (spacing of tick marks on y-axis)
- $X_{\text{res}} = 1$

Access GRAPH to see the histogram.

To draw box plots:

Access graphing mode.

2nd , [STAT PLOT]

Select <1:Plot 1> to access the first graph.

ENTER

Use the arrows to select <ON> and turn on Plot 1.

ENTER

Use the arrows to select the box plot picture and enable it.

ENTER

Use the arrows to navigate to`<Xlist>`
If "L1" is not selected, select it.

2nd

, `[L1]` ,

ENTER

Use the arrows to navigate to`<Freq>`.
Indicate that the frequencies are in`[L2]`.

2nd

, `[L2]` ,

ENTER

Go back to access other graphs.

2nd

, `[STAT PLOT]`

Be sure to deselect or clear all equations before graphing using the method mentioned above.

View the box plot.

GRAPH

, `[STAT PLOT]`

Linear Regression

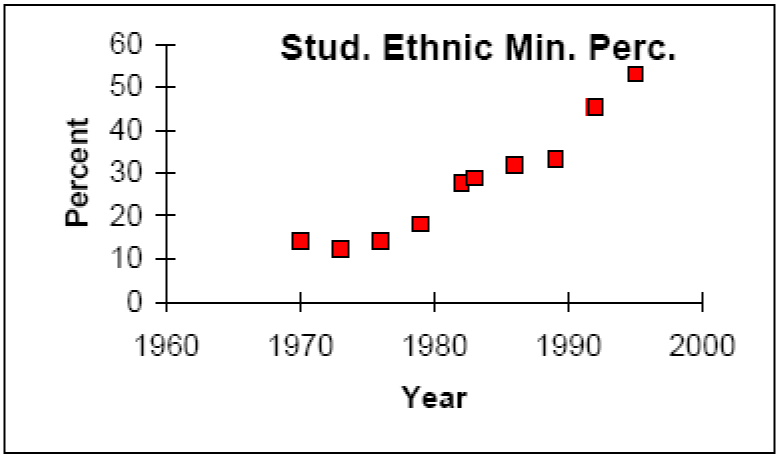
Sample Data

The following data is real. The percent of declared ethnic minority students at De Anza College for selected years from 1970 - 1995 was:

Year	**Student Ethnic Minority Percentage**
1970	14.13
1973	12.27
1976	14.08
1979	18.16
1982	27.64
1983	28.72
1986	31.86
1989	33.14
1992	45.37
1995	53.1

The independent variable is "Year," while the independent variable is "Student Ethnic Minority Percent."

Student Ethnic Minority Percentage



By hand, verify the scatterplot above.

Note: The TI-83 has a built-in linear regression feature, which allows the data to be edited.The x-values will be in

`[L1]`

; the y-values in

`[L2]`

.

To enter data and do linear regression:

ON Turns calculator on

Before accessing this program, be sure to turn off all plots.

- Access graphing mode

mode.

2nd

, `[STAT PLOT]`

- Turn off all plots.

4

,

ENTER

Round to 3 decimal places. To do so:

- Access the mode menu.

MODE

, `[STAT PLOT]`

- Navigate to `<Float>` and then to the right to `<3>`.

▼

▶

- All numbers will be rounded to 3 decimal places until changed.

ENTER

Enter statistics mode and clear lists [L1] and [L2], as describe above.

STAT , 4

Enter editing mode to insert values for x and y.

STAT , ENTER

Enter each value. Press ENTER to continue.

To display the correlation coefficient:

Access the catalog.

2nd , [CATALOG]

Arrow down and select <DiagnosticOn>

▼ … , ENTER , ENTER

r and r^2 will be displayed during regression calculations.
Access linear regression.

STAT

▶

Select the form of $y = a + bx$

8

,

ENTER

The display will show:
LinReg

- $y = a + bx$
- $a = -3176.909$
- $b = 1.617$
- $r^2 = 0.924$
- $r = 0.961$

This means the Line of Best Fit (Least Squares Line) is:

- $y = -3176.909 + 1.617x$
- $\text{Percent} = -3176.909 + 1.617(\text{year } \#)$

The correlation coefficient $r = 0.961$
To see the scatter plot:

Access graphing mode.

2nd

, `[STAT PLOT]`

Select `<1:plot 1>` To access plotting - first graph.

ENTER

Navigate and select `<ON>` to turn on Plot 1.

`<ON>`

ENTER

Navigate to the first picture.
Select the scatter plot.

ENTER

Navigate to `<Xlist>`
If `[L1]` is not selected, press 2nd , `[L1]` to select it.

Confirm that the data values are in `[L1]`.

`<ON>`

ENTER

Navigate to `<Ylist>`
Select that the frequencies are in `[L2]`.

2nd

, `[L2]` ,

ENTER

Go back to access other graphs.

2nd

`[STAT PLOT]`

, [STAT PLOT]

Use the arrows to turn off the remaining plots.

Access WINDOW to set the graph parameters.

- $X_{\text{min}} = 1970$
- $X_{\text{max}} = 2000$
- $X_{\text{scl}} = 10$ (spacing of tick marks on x-axis)
- $Y_{\text{min}} = -0.05$
- $Y_{\text{max}} = 60$
- $Y_{\text{scl}} = 10$ (spacing of tick marks on y-axis)
- $X_{\text{res}} = 1$

Be sure to deselect or clear all equations before graphing, using the instructions above.

Press GRAPH to see the scatter plot.

To see the regression graph:

Access the equation menu. The regression equation will be put into Y1.

Y=

Access the vars menu and navigate to `<5: Statistics>`

VARS

,

5

Navigate to `<EQ>`.

`<1: RegEQ>` contains the regression equation which will be entered in Y1.

ENTER

Press GRAPH. The regression line will be superimposed over scatter plot.

To see the residuals and use them to calculate the critical point for an outlier:

Access the list. RESID will be an item on the menu.
Navigate to it.

2nd, [LIST], <RESID>

Confirm twice to view the list of residuals. Use the arrows to select them.

ENTER, ENTER

The critical point for an outlier is: $1.9V\frac{\text{SSE}}{n-2}$ where:

- n = number of pairs of data
- SSE = sum of the squared errors
- $\sum \text{residual}^2$

Store the residuals in [L3].

STO▶, 2nd, [L3], ENTER

Calculate the $\frac{(\text{residual})^2}{n-2}$. Note that $n-2=8$

2nd

, [L3] ,

x²

,

÷

,

8

Store this value in [L4].

STO▶

,

2nd

, [L4] ,

ENTER

Calculate the critical value using the equation above.

1

,

.

,

9

,

X

,

2nd

, [V] ,

2nd

, [LIST]

▶

,

▶

,

5

,

2nd

, [L4] ,

)

,

)

,

ENTER

Verify that the calculator displays: 7.642669563. This is the critical value.

Compare the absolute value of each residual value in `[L3]` to 7.64 . If the absolute value is greater than 7.64, then the (x, y) corresponding point is an outlier. In this case, none of the points is an outlier.

To obtain estimates of y for various x-values:
There are various ways to determine estimates for "y". One way is to substitute values for "x" in the equation. Another way is to use the

TRACE

on the graph of the regression line.

TI-83, 83+, 84 instructions for distributions and tests

Distributions

Access `DISTR` (for "Distributions").

For technical assistance, visit the Texas Instruments website at http://www.ti.com and enter your calculator model into the "search" box.
Binomial Distribution

- `binompdf(n,p,x)` corresponds to P(X = x)
- `binomcdf(n,p,x)` corresponds to P(X ≤ x)
- To see a list of all probabilities for x: 0, 1, . . . , n, leave off the "`x`" parameter.

Poisson Distribution

- `poissonpdf(λ,x)` corresponds to P(X = x)
- `poissoncdf(λ,x)` corresponds to P(X ≤ x)

Continuous Distributions (general)

- $-\infty$ uses the value -1EE99 for left bound
- ∞ uses the value 1EE99 for right bound

Normal Distribution

- `normalpdf(x,μ,σ)` yields a probability density function value (only useful to plot the normal curve, in which case "`x`" is the variable)
- `normalcdf(left bound, right bound, μ,σ)` corresponds to P(left bound < X < right bound)
- `normalcdf(left bound, right bound)` corresponds to P(left bound < Z < right bound) - standard normal
- `invNorm(p,μ,σ)` yields the critical value, k: P(X < k) = p
- `invNorm(p)` yields the critical value, k: P(Z < k) = p for the standard normal

Student-t Distribution

- `tpdf(x,df)` yields the probability density function value (only useful to plot the student-t curve, in which case "`x`" is the variable)
- `tcdf(left bound, right bound, df)` corresponds to P(left bound < t < right bound)

Chi-square Distribution

- `X²pdf(x,df)` yields the probability density function value (only useful to plot the chi^2 curve, in which case "`x`" is the variable)
- `X²cdf(left bound, right bound, df)` corresponds to P(left bound < X^2 < right bound)

F Distribution

- `Fpdf(x,dfnum,dfdenom)` yields the probability density function value (only useful to plot the F curve, in which case "`x`" is the variable)
- `Fcdf(left bound,right bound,dfnum,dfdenom)` corresponds to P(left bound < F < right bound)

Tests and Confidence Intervals

Access `STAT` and `TESTS`.

For the Confidence Intervals and Hypothesis Tests, you may enter the data into the appropriate lists and press `DATA` to have the calculator find the sample means and standard deviations. Or, you may enter the sample means and sample standard deviations directly by pressing `STAT` once in the appropriate tests.

Confidence Intervals

- `ZInterval` is the confidence interval for mean when σ is known
- `TInterval` is the confidence interval for mean when σ is unknown; s estimates σ.
- `1-PropZInt` is the confidence interval for proportion

> **Note:** The confidence levels should be given as percents (ex. enter "`95`" or "`.95`" for a 95% confidence level).

Hypothesis Tests

- `Z-Test` is the hypothesis test for single mean when σ is known
- `T-Test` is the hypothesis test for single mean when σ is unknown; s estimates σ.
- `2-SampZTest` is the hypothesis test for 2 independent means when both σ's are known
- `2-SampTTest` is the hypothesis test for 2 independent means when both σ's are unknown
- `1-PropZTest` is the hypothesis test for single proportion.
- `2-PropZTest` is the hypothesis test for 2 proportions.
- X^2`-Test` is the hypothesis test for independence.
- X^2`GOF-Test` is the hypothesis test for goodness-of-fit (TI-84+ only).
- `LinRegTTEST` is the hypothesis test for Linear Regression (TI-84+ only).

Note: Input the null hypothesis value in the row below "`Inpt`." For a test of a single mean, "`μ∅`" represents the null hypothesis. For a test of a single proportion, "`p∅`" represents the null hypothesis. Enter the alternate hypothesis on the bottom row.

Tables

Note: When you are finished with the table link, use the back button on your browser to return here.

Tables (NIST/SEMATECH e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook/, January 3, 2009)

- Student-t table
- Normal table
- Chi-Square table
- F-table
- All four tables can be accessed by going to http://www.itl.nist.gov/div898/handbook/eda/section3/eda367.htm

95% Critical Values of the Sample Correlation Coefficient Table

- 95% Critical Values of the Sample Correlation Coefficient

Note: The url for this table is http://cnx.org/content/m17098/latest/